The Prudent Principal

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Abstract

This paper re-examines managerial incentive compensation, using a principal-agent model in which the principal is downside risk averse, or prudent (as a number of empirical facts and scholarly works suggest), instead of risk neutral (as it has been commonly assumed so far in the literature). We find that, when the principal is 'prudent enough' in relation to the agent, optimal incentive pay should be concave in performance; in this case, the agent should face high-powered incentives while in the bad states but be given weaker incentives when things are going well. This runs counter to current evidence that most incentive compensation packages are actually convex in performance. We show that this disparity can be attributed to certain limited liability and taxation regimes. Implications for public policy and financial regulation are briefly discussed.

Keywords: Executive compensation, downside risk aversion, prudence, limited liability, individual and corporate taxation

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1 Introduction

The main normative framework to deal with managerial/executive incentive compensation is the principal-agent model. In this context, the 'principal,' who stands for shareholders or the corporate board, is commonly viewed as being risk-neutral. This assumption, however, can be challenged on at least two grounds. First, since Roy (1952) and Markowitz (1959), a number of economists have contended that investors react asymmetrically to gains and losses. Corroborating this, Harvey and Siddique (2000), Ang et al. (2006), and others report that stock returns do reflect a premium for bearing downside risk. Second, corporate law and jurisprudence endow corporate board members with a fiduciary duty towards their corporation (Clark 1985; Adams et al 2010; Lan and Herakleous 2010). This notably confers board directors and officers a key role in preventing and managing crisis situations (Mace 1971; Williamson 2007; Adams et al 2010). Directors and officers should accordingly "(...) exercise that degree of care, skill, and diligence which an ordinary, prudent man would exercise in the management of his own affairs." (Clark 1985, p. 73). This requirement should again drive corporate boards (which are chiefly responsible in setting top executives' compensation) to weigh differently the risks correlated with downside losses versus those linked to upside gains. Such behavior is of course inconsistent with risk neutrality, so one might reasonably question some of the prescriptions from current and past principal-agent analyses of incentive pay.

In this paper, we re-examine managerial compensation using a principal-agent model in which the principal is 'prudent,' in the sense introduced in economics by Kimball (1990). This indeed portrays the principal as a downside risk averse entity.[1] A prudent decision maker dislikes mean and variance-preserving transformations that skew the distribution of outcomes to the left (Menezes et al 1980; Crainich and Eeckhoudt 2008). Equivalently, she prefers additional volatility to be associated with good rather than bad outcomes (Eeckhoudt and Schlesinger 2006; Denuit et al 2010). Formally, someone is prudent when her marginal utility function is strictly convex (it is of course constant in the risk neutral case). As a characteristic of the agent's (not the principal's) preferences - the agent standing here for an executive or a top manager, prudence has already been dealt with and found relevant in the literature, especially in contingent monitoring (Fagart and Sinclair-Desgagné 2007) and background risk (Ligon and Thistle 2008) situations.[2] To our knowledge, this is the first time prudence is taken to also be an attribute of the principal.

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[1] An alternative way to capture loss aversion would be to assume that utility declines sharply (albeit at a decreasing rate) below some reference point, as in prospect theory (Kahneman and Tversky 1979). Dittman et al (2010), for instance, use this representation of an agent's preferences (while assuming a risk neutral principal) to analyze executive compensation.

[2] Empirical evidence that executives are prudent can be found in McAnally et al (2011), Garvey and Milbourn (2006), and the references therein. A revealing indirect indication, moreover, is Garvey and Milbourn (2006, p. 198)'s observation that "(...) the average executive loses 25-45% less pay from bad luck than is gained from good luck."

In a benchmark model, we will now show that incentive compensation should be *concave* in performance when the principal is prudent enough (in a sense to be made precise) compared with the agent. This extends Hemmer et al (2000)'s proposition that remuneration will be convex in outcome if the agent is prudent and the principal is risk neutral. The principle underneath these statements seems straightforward: whoever is relatively more prudent should bear less downside risk. A convex incentive scheme, being very sensitive to performance in upbeat situations and rather flat in the range where results are mediocre, shelters a more prudent agent against downside volatility which must then be born by the principal. A concave scheme, by contrast, rewards performance improvements much more strongly under adverse circumstances and makes the agent bear significant downside risk; a more prudent principal thereby decreases her own exposure to downside risk by firmly pushing her agent to get away from dangerous territory.

Whether the principal is more or less prudent relative to the agent seems therefore to be an important practical matter in setting optimal compensation contracts. Considering in particular the just-mentioned fiduciary duties of corporate boards, we submit that there is no reason to believe it is the agent/executive who would always be the more prudent player. Yet, empirical studies confirm that managerial incentive contracts are generally convex in performance, notably through the inclusion of stock options (see, e.g., Hall and Murphy 2003). One might impute this state of affairs to managerial power (e.g., Bebchuck and Fried 2003) and other well-documented governance failures (e.g., Jensen and Murphy. 2004). In this paper, sticking to a normative outlook, we rather bring up some institutional features such as limited liability and taxation regimes.[3] In cases where the agent/manager cannot be inflicted negative revenues or the principal/corporation can be refunded when net profits are negative (which can be seen as a rough proxy for the recent TARP - Trouble Asset Relief Program - rescue of financial institutions), we find that concave contracts are no longer optimal even if the principal is very prudent. A similar conclusion holds when the principal's profits are taxed (which corresponds to the British government's 2008 proposal concerning banks' profits). When the manager's income is subject to progressive taxation (which roughly reproduces the suggestion, actively discussed in the U.S. and France, to tax traders' bonuses), the upshot is even more radical: a prudent principal should nevertheless offer convex rewards in order to properly encourage the agent to pursue the better states of nature.

The rest of the paper unfolds as follows. Section 2 presents the benchmark model - a static principal-agent model where the agent is effort and risk averse while the principal is both risk averse and prudent; we assume throughout that the first-order approach, as justified in Rogerson (1985), is valid. Our central proposition - that the optimal contract must seek a balance between the agent's and the principal's

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[3] Tax laws have been pointed our by many, including Hall and Murphy (2003), to be one explanation of the sudden wave of executive option grants which convexify incentive compensation schemes. In a more recent study, however, Kadan and Swinkels (2008) find mitigated evidence of this. Concerning liability regimes, Dittman and Maug (2007)'s theoretical and empirical work suggests that bankruptcy risks tend to reduce the convexity of incentive schemes. These findings are further discussed in Sections 4 and 5.

respective prudence - is established in Section 3. Sections 4 and 5 next examine whether limited liability and taxation can respectively support the empirical predominance of convex incentive schemes. Section 6 contains some concluding remarks and policy recommendations. All proofs are in the Appendix.

2   The benchmark model

Consider an agent - standing for a CEO or a top executive/manager - whose preferences can be represented by a Von Neumann-Morgenstern utility function $u(\cdot)$ defined over monetary payments. We assume this function is increasing and strictly concave, formally $u'(\cdot) > 0$ and $u''(\cdot) < 0$, so the agent is risk averse.

This agent can work for a principal - that is, a company's investors, shareholders or corporate board - whose preferences are represented by the Von Neumann-Morgenstern utility function $v(\cdot)$ defined over net final wealth. We suppose this function is increasing and strictly concave, i.e. $v'(\cdot) > 0$ and $v''(\cdot) < 0$, so the principal is risk averse. Moreover, let the marginal utility $v'(\cdot)$ be convex, i.e. $v'''(\cdot) > 0$, which means that the principal is downside risk averse or (equivalently) prudent. The principal will thus make some profits which depend stochastically on the agent's effort level a. The latter cannot be observed, however, and the agent incurs a cost of effort $c(a)$ that is increasing and convex ($c'(a) > 0$ and $c''(a) \geq 0$). The principal only gets a signal s, drawn from a compact subset S of $\mathbb{R}$, which is positively correlated with the agent's effort a through the conditional probability distribution $F(s; a)$ with density $f(s; a)$. Based on s, she can infer a realized profit $\pi(s)$, which we suppose increasing and concave in s ($\pi'(s) > 0$ and $\pi''(s) \leq 0$), and pays the agent a compensation $w(s)$.

The principal's problem is then to find a reward schedule or incentive scheme $w(s)$ that maximizes net profit, under the constraints that the agent maximizes his own expected utility (the incentive compatibility constraint) and must receive an expected payoff that is not inferior to some external one U_0 (the participation constraint). This can be written formally as follows:

$$\max_{w(s), a^*} \int_{s \in S} v(\pi(s) - w(s)) dF(s; a^*) \tag{1}$$

subject to

$$a^* \in \arg\max_a \int_{s \in S} u(w(s)) dF(s; a) - c(a)$$

$$\int_{s \in S} u(w(s)) dF(s; a^*) - c(a^*) \geq U_0$$

As it is commonly done in the literature, for tractability reasons, we replace the incentive compatibility constraint by the first-order necessary condition on the payoff-maximizing effort a^*. This transforms the

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principal's initial problem into the following one:

$$\max_{w(s),a} \int_{s \in S} v(\pi(s) - w(s))dF(s;a) \tag{2}$$

subject to

$$\int_{s \in S} u(w(s))dF_a(s;a) - c'(a) \geq 0, \qquad (\delta)$$

$$\int_{s \in S} u(w(s))dF(s;a) - c(a) \geq U_0, \qquad (\mu)$$

where δ and μ are the constraints' respective Lagrange multipliers. This so-called 'first-order approach' will deliver a valid solution under the following sufficient conditions (see Rogerson 1985).[4]

Assumption 1 [Concave Monotone Likelihood Ratio Property]: The ratio $\frac{f_a(s;a^*)}{f(s;a^*)}$ is non decreasing and concave in s for each value of a.

Assumption 2 [Convexity of the Distribution Function Condition]: At every a and s, we have that $F_{aa}(s,a) \geq 0$.

Before ending this section, let us write $R_u = -\frac{u''}{u'}$ and $R_v = -\frac{v''}{v'}$ the Arrow-Pratt measures of risk aversion corresponding to the agent's and the principal's utility functions u and v respectively, and $P_u = -\frac{u'''}{u''}$, $P_v = -\frac{v'''}{v''}$ the analogous measures of prudence proposed by Kimball (1990).

This completes the description of our benchmark model. We shall now proceed to characterize the optimal incentive scheme in this context.

3   Optimal concave incentive schemes

This section will now establish that a principal who is sufficiently downside risk averse (in a sense to be made precise very soon) should set an incentive compensation package that is concave in outcome. The implications of such a contract are discussed below. To first derive this contract, note that the Kuhn-Tucker necessary and sufficient conditions require that a solution to program (2) meet the well-known equation

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu + \delta \frac{f_a(s;a^*)}{f(s;a^*)}, \qquad \forall s \tag{3}$$

The multipliers μ and δ being non-negative by construction, the right-hand-side of (3) is increasing and concave in the signal s, by Assumption 1. This allows to state the following.

Lemma 1 *The optimal reward schedule $w^*(s)$ is increasing in the performance signal s.*

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[4]Assumption 1 is actually due to Jewitt (1988); Rogerson (1985)'s article does not suppose that the likelihood ratio is concave. For economic interpretations and examples of distributions that satisfy the above two assumptions, see Li Calzi and Spaeter (2003).

The proof can be found in the Appendix. It consists in taking the first derivative of the left-hand-side of expression (3), knowing it must be positive. Similarly taking the second derivative, which must be negative, yields another key preliminary result.

Lemma 2 *The optimal reward schedule $w^*(s)$ is concave in the signal s when the following condition holds*

$$\frac{P_u R_u}{P_v R_v} < \left(\frac{\pi' - w'}{w'}\right)^2 \quad .$$

Proof. See the Appendix. ∎

The latter expression possesses a nice economic interpretation. First, observe that the product $P_v R_v = \frac{v'''}{v'} = d_v$, a coefficient introduced by Modica and Scarsini (2005) to measure someone's degree of local downside risk aversion (or local prudence). A higher coefficient d_v means the principal would be ready to pay more to insure against a risk with greater negative skewness. As shown by Crainich and Eeckhoudt (2008), furthermore, d_v increases if the utility function v becomes more concave while the marginal utility v' becomes more convex.[5] If the ratio of the net profit gradient over the wage gradient is also bounded away from 0, i.e. $\inf_s \frac{\pi' - w'}{w'} \geq \frac{1}{k} > 0$ for some integer k - a rather reasonable supposition, which amounts to assuming that the wage gradient is smooth and the principal's net income is increasing in s,[6] then the lemma's proviso is fulfilled when

$$k^2 \cdot d_u < d_v \ .$$

Our main result is now at hand.

Theorem 1 *Suppose the ratio $\frac{\pi' - w'}{w'} \geq \frac{1}{k} > 0$ for some integer k. The optimal wage schedule $w^*(s)$ is concave in s when the principal is always more prudent than the agent by a factor larger than k^2.*

The theorem's conclusion holds vacuously - hence the optimal incentive scheme is concave - when $u''' < 0$ so the agent is not prudent. If the agent is prudent (i.e. $u''' > 0$), the theorem says that he may still have to bear significant downside risk when the principal exhibits enough local prudence. In this case, incentive compensation will be concave, so more responsive to performance under unfavorable than under positive circumstances. By offering such a contract, the prudent principal motivates the agent to keep away from, not only the bad, but indeed the very bad outcomes.

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[5] A somewhat different measure is the 'index of downside risk aversion' $S_v = d_v - \frac{3}{2} R_v^2$ due to Keenan and Snow (2005). This index does not have the properties d_v has, but it recalls the Arrow-Pratt measure of risk aversion in the sense that its value increases under monotonic downside risk averse transformations of the utility function v.

[6] A proof that the principal's net income is nondecreasing in s, i.e. $\pi'(s) - w'(s) \geq 0$ for all s, can be found in Rogerson (1985).

Note incidentally that a concave contract may not render the agent less eager to take risks. As Ross (2004) pointed out, the overall effect of an incentive scheme $w(s)$ compared to an alternative $z(s)$ on the agent's behavior towards risk depends on whether the utility function $u(w(s))$ displays more or less risk aversion than the utility function $u(z(s))$. Suppose, for instance, that the latter scheme takes the form of a call option (a convex contract) $z(s) = \max\{s - r, 0\}$ with r the exercise price, while the former is the put option (a concave contract) $w(s) = \min\{b - r + s, b\}$ with b a fixed fee and r the exercise price.[7] An agent whose risk aversion decreases with wealth (prudence is a necessary condition for this) will then be *less* locally risk averse at the exercise price r under contract $w(\cdot)$ than under contract $z(\cdot)$.

While Theorem 1 recommends to set concave contracts under certain conditions, non-concave or even convex compensation modes seem rather prevalent in practice. Hall and Murphy (2003, p. 49), for instance, report that: "In 1992, firms in the Standard & Poor's 500 granted their employees options worth a total of \$11 billion at the time of grant; by 2000, option grants in S&P 500 firms increased to \$119 billion." This phenomenon per se does not invalidate our result, since we adopt here a normative standpoint. The current model may simply not capture key elements of the corporate landscape that would make non-concave incentive schemes optimal. In the following sections, we successively examine two sets of reasons which, when added to the benchmark model, might indeed justify why concave incentive schemes should have become the exception rather than the rule.

4 Limited liability and non-concavity

As a first departure from our benchmark model, let's allow either the agent or the principal to bear limited losses. In the first subsection, the agent will always earn nonnegative revenue. In the second subsection, the principal will be rescued whenever net profits are falling below zero.

4.1 The judgment-proof agent

Suppose the agent's revenue is bounded from below, so he cannot bear very high penalties when performance is bad. Management remuneration is frequently subject to this type of constraint.[8] An agent with limited wealth, for instance, can file for bankruptcy if he cannot afford paying some penalty. Golden parachutes and other devices (like retirement benefits) have also been introduced to compensate top managers in case employment is terminated. Executives who own large amounts of their company's stock can now often hedge their holdings to contain losses if the value of the stock plunges (Gao 2010).

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[7] This example is taken from Ross (2004, p. 209-211).

[8] Hence, since Holmstrom (1979) and especially Sappington (1983)'s seminal works, analyzing the impact of the agent's limited liability remains a rather well-covered topic in the principal-agent literature. For a recent account of this literature, see Poblete and Spulber (2011). In most articles, however, both the principal and the agent are assumed to be risk neutral.

Recently, CEOs have even been offered insurance to offset the costs of investigations for certain criminal mischiefs such as foreign corruption and bribery.[9] And in certain contexts, institutions that prevent an agent from breaching his contract under bad circumstances might simply not exist.

Without loss of generality, let us then normalize the agent's minimum revenue to zero. The principal's optimization problem now becomes:

$$\max_{w(s),a} \int_{s \in S} v(\pi(s) - w(s)) dF(s; a) \tag{4}$$

subject to

$$\int_{s \in S} u(w(s)) dF_a(s; a) - c'(a) \geq 0, \qquad (\delta)$$

$$\int_{s \in S} u(w(s)) dF(s; a) - c(a) \geq U_0, \qquad (\mu)$$

$$w(s) \geq 0, \forall s \qquad (\lambda(s))$$

where $\lambda(s)$ is the Lagrange multiplier associated with the nonnegative wage constraint in state s.

Let a^{**} denote the agent's new choice of effort (to be soon compared with a^*). The Kuhn-Tucker conditions for a solution to this problem (4) lead this time to the equation

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu + \delta \frac{f_a(s; a^{**})}{f(s; a^{**})} + \frac{\lambda(s)}{f(s; a^{**}) u'(w(s))}, \qquad \forall s \tag{5}$$

with $\lambda(s)w(s) = 0$ at all s. This, and some extra computation, entail the following conclusions.

Proposition 1 *Suppose that the agent is protected by limited liability, and that the principal is 'more prudent' than the agent in the sense made precise in Theorem 1. Then:*

i) The optimal wage schedule is such that

$$w(s) \;\; = \;\; 0 \;\; \text{for any signal } s \text{ lower than some threshold } s_0$$

$$w'(s) \;\; > \;\; 0 \;\; \text{and } w''(s) < 0 \;\; \text{for } s > s_0 \;.$$

ii) For any wage schedule, $a^{**} < a^*$ so the agent's effort is lower.

Proof. See the Appendix. ∎

Hence, allowing the agent to have limited liability should not change the increasing and concave pay-performance relationship established in Theorem 1, but only in the range where performance signals induce strictly positive rewards. Some convexity is introduced through the floor payment $w(s) = 0$ when $s \leq s_0$. The upshot is a decrease in the agent's effort relative to the benchmark situation without limited liability.

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[9] Marsh & McLennan created such a policy, in order to allow people and businesses to cover the cost of investigations under the U.S. Foreign Corrupt Practices Act and the U.K.'s Bribery Act.

4.2 The sheltered principal

Consider now a situation where it is the principal's losses which are limited. Many countries actually possess, implicitly or explicitly, rescue programs aimed at supporting their so-called 'strategic' or 'too big to fail' enterprises when they are on the verge of collapse. In 2008, for example, at the heart of the financial crisis, the United States government - under the Troubled Assets Relief Program (TARP) - purchased hundreds of billions of dollars in assets and equity from distressed financial institutions. In 2004, the engineering and manufacturing company Alstom, which had experienced a string of business disasters, received 2.5 billion euros in rescue money from the French government (as part of a plan previously approved by the European Commission). Such state interventions usually raise concerns that they will fuel moral hazard from the sheltered firms. As we shall see, they might at first have an effect on the incentive contracts of executives and CEOs.

Suppose there is a state of nature s_1 in which $\pi(s_1) = 0$; profits being increasing in s by assumption, we have that $\pi(s) < 0$ when $s < s_1$ and $\pi(s) \geq 0$ for $s \geq s_1$. We postulate that the principal will be rescued after she has compensated the agent.[10] Let's assume (rather safely) that $|w(s)| < |\pi(s)|$ for any $s \in S$, so the principal never pays the agent an amount larger than her gains or lower than her losses; net profits $\pi(s) - w(s)$ remain therefore positive for any $s > s_1$, equal to zero at $s = s_1$ and negative for any $s < s_1$.

Consider now the two subsets:

$$\overline{S_1} = \{s \in S; \pi(s) - w(s) \geq 0\}$$
$$\underline{S_1} = \{s \in S; \pi(s) - w(s) < 0\},$$

A sheltered principal must then solve the following problem:

$$\max_{w(s)} \int_{s \in \overline{S_1}} v(\pi(s) - w(s)) dF(s; a^*) + v(0).F(s_1; a^*) \tag{6}$$

subject to

$$\int_{s \in S} u(w(s)) dF_a(s; a^*) - c'(a^*) \geq 0, \qquad (\delta)$$
$$\int_{s \in S} u(w(s)) dF(s; a^*) - c(a^*) \geq U_0, \qquad (\mu)$$

The Kuhn-Tucker conditions applied to this problem give rise to two distinct expressions. That is:

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)}, \quad \forall s \in \overline{S_1} \tag{7}$$

and

$$u'(w(s)) \cdot f(s; a^*) \left(\mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)} \right) \geq 0 \text{ for } s_0 < s \leq s_1 \text{ and } \leq 0 \text{ for } s \leq s_0 \tag{8}$$

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[10] As an illustration, recall that, during the 2008 financial crisis, some banks disclosed their financial losses after having set aside provisions to pay their traders.

Condition (7) is the same as in the benchmark case while condition (8) entails a constant lower wage. The optimal incentive scheme is thus similar to the one described in Proposition 1.

Proposition 2 *Suppose the principal's net profits can never be negative, and that the principal is 'more prudent' than the agent in the sense of Theorem 1. The optimal incentive schemes will then be such that*

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\begin{cases}
(i) & w'(s) > 0, w''(s) < 0 \quad \forall s \in \overline{S_1} \\
(ii) & w(s) = w(s_1) = 0 \quad \forall s \in \underline{S_1}
\end{cases}
\tag{9}
$$

Proof. See Appendix. ■

Limiting the principal's losses induces therefore some convexity in the pay-performance relationship of the agent's wage schedule, as the agent will partly benefit (when profits $\pi(s)$ would be negative) from the principal's protection.

Let us now turn to examining the effect of taxation.

5 Taxation and convexity

The second departure from our benchmark model is to introduce personal and corporate taxes. Tax laws are often pointed out as a key rationale for the popularity of stock options and other components of executive compensation (see, e.g., Murphy 1999, p. 20; Hall and Murphy 2003; Dittman and Maug 2007). Taxation - progressive taxation notably - is also periodically mentioned as a means to moderate what many people regard as excessive rewards to some corporate members (see Rose and Wolfram 2002 for an empirical study): in 2009, for example, U.S. President Barack Obama and U.K. Prime Minister Gordon Brown respectively gazed at taxing traders or banks to precisely meet this concern. In what follows, we investigate the ramifications such proposals could have for the agent's incentive scheme.

5.1 Income taxation

Assume that the agent has to pay a tax $\tau(s) \cdot w(s)$ when her income $w(s)$ is positive. Let the tax rate be positive and nondecreasing in the state of nature, i.e. $\tau(s) > 0$ and $\tau'(s) \geq 0$ at all s. Such progressive taxation actually exists in several countries. In the Netherlands, for example, the first 200,000 euros of taxable income are subject to a tax rate of 20%, while the rate on further income is 25,5%. To keep matters simple, we suppose that $\tau''(s) = 0$.

Using the notation $S^+ = \{s \in S; w(s) > 0\}$ and $S^- = \{s \in S; w(s) \leq 0\}$, the optimal incentive scheme

must now solve the following problem

$$\max_{w(s),a} \int_{s\in S} v(\pi(s) - w(s))dF(s;a) \tag{10}$$

subject to

$$\int_{s\in S^+} u((1-\tau(s))w(s))dF_a(s;a) + \int_{s\in S^-} u(w(s))dF_a(s;a^*) - c'(a) \geq 0, \qquad (\delta)$$

$$\int_{s\in S^+} u((1-\tau(s))w(s))dF(s;a) + \int_{s\in S^-} u(w(s))dF(s;a) - c(a) \geq U_0, \qquad (\mu)$$

The first-order conditions are then given by

$$\frac{v'(\pi(s) - w(s))}{(1-\tau(s))u'((1-\tau(s))w(s))} = \mu + \delta\frac{f_a(s;a^*)}{f(s;a^*)}, \quad \forall s \in S^+ \tag{11}$$

and

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu + \delta\frac{f_a(s;a^*)}{f(s;a^*)}, \quad \forall s \in S^- \tag{12}$$

Taking the first and second derivatives of the left-hand side of the latter expressions leads to a perhaps surprising (albeit intuitive) conclusion.

Proposition 3 *Assume that a nondecreasing linear tax rate $\tau(s)$ applies to the agent's positive revenues, and that the agent's net income $(1 - \tau(s))w(s)$ is nondecreasing in s. Then:*

(i) The optimal wage schedule $w(s)$ remains concave for any state s in S if the tax rate is constant and the agent is risk neutral.

(ii) The pay-performance relationship can be convex on S^+ (while it remains concave on S^-), whether the agent is prudent or not.

Proof. See the Appendix. ∎

This proposition compares a situation with (i) a constant tax rate to one (ii) where it is progressive. Each fiscal policy has of course a specific impact on the agent's behavior and affects therefore the optimal compensation scheme set by the principal. In the former case ($\tau'(s) = 0$), the principal's prudence prevails and the pay-performance relationship should remain concave provided the agent is not too prudent. For any remuneration package, however, an increasing tax function ($\tau'(s) > 0$) weakens more and more the agent's incentives as his efforts yield better results. This might induce even a prudent principal to find concave incentive pay inappropriate and offer instead a reward function that becomes steeper as s goes up. One possible design is illustrated in Figure 1.

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Insert Figure 1 about here

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Progressive taxation might thus bring about convex reward schemes, despite the fact the principal is prudent, and despite the often-explicit intent of such fiscal policy to curb executive revenues.

5.2 Profit taxation

Suppose now that the principal's positive net profit is subject to a tax rate $\gamma(s)$ which is non decreasing and linear in s, i.e.$\gamma(s) > 0$, $\gamma'(s) \geq 0$ and $\gamma''(s) = 0$. An optimal incentive scheme must then solve

$$\max_{w(s)} \int_{s \in \overline{S_1}} v((1 - \gamma(s))(\pi(s) - w(s)))dF(s; a^*) + \int_{s \in \underline{S_1}} v(\pi(s) - w(s))dF(s; a^*) \qquad (13)$$

subject to

$$\int_{s \in S} u(w(s))dF_a(s; a^*) - c'(a^*) \geq 0, \qquad (\delta)$$

$$\int_{s \in S} u(w(s))dF(s; a^*) - c(a^*) \geq U_0, \qquad (\mu)$$

where $\overline{S_1} = \{s \in S; \pi(s) - w(s) \geq 0\}$ and $\underline{S_1} = \{s \in S; \pi(s) - w(s) < 0\}$.

The first-order conditions in this case are given by

$$\frac{(1 - \gamma(s))v'[(1 - \gamma(s))(\pi(s) - w(s))]}{u'(w(s))} = \mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)}, \forall s \in \overline{S_1} \qquad (14)$$

and

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)}, \forall s \in \underline{S_1} \qquad (15)$$

Proceeding as before yields our last result.

Proposition 4 *Assume that a nondecreasing linear tax rate $\gamma(s)$ applies to the principal's positive net profit, and that the principal's after-tax net profit $(1 - \gamma(s))(\pi(s) - w(s))$ is nondecreasing in s.*

(i) If the agent is risk-neutral, the optimal reward schedule remains concave whatever the taxation policy.

(ii) For a risk-averse agent, the optimal incentive scheme can become convex over the range of performance signals where the principal's net profit is positive. However, there exists a positive threshold $\widehat{\gamma}$ such that remuneration is concave for any tax rate higher than $\widehat{\gamma}$, and this conclusion does not depend on the principal's prudence.

Proof. See the Appendix. ■

Statement (i) is analogous to part (i) of Proposition 3. Part (ii) raises again the possibility of obtaining convex incentive schemes. Figure 2 illustrates such a case.

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Insert Figure 2 about here

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Intuitively, taxing positive net profit might increase the principal's willingness to give out more and more revenue as the firm's performance gets better and better, especially if this motivates the agent further

without changing the expected tax bill by much. This rationale (meant here to be normative) can be supported empirically. Jensen and Murphy (2004, p. 30), for example, report that:

> In 1994, the Clinton tax act (the Omnibus Budget Reconciliation Act of 1993) defined non-performance related compensation in excess of \$1 million as "unreasonable" and therefore not deductible as an ordinary business expense for corporate income tax purposes.

> Ironically, although the populist objective was to reduce "excessive" CEO pay levels, the ultimate outcome of the controversy (similar to what happened in response to the Golden Parachute restrictions) was a significant increase in executive compensation, driven by *an escalation in option grants* that satisfied the new IRS regulations and allowed pay significantly in excess of \$1 million to be tax deductible to the corporation. (emphasis added)

Under highly progressive tax rates, however, the proposition says it is possible to induce concave incentive schemes *whatever* the principal's degree of prudence. This reveals that the progressive taxation of net profit can, under some conditions, have the same effect as having a prudent principal! The difference, of course, is that taxation policy can be manipulated by political lobbies, while a principal's risk preferences (in principle) cannot.

6   Concluding remarks

The 2008 financial crisis has put again the spotlight on executive pay. One highlighted feature is the increasing convexification over the past decades - through more and more widespread use of call options, notably - of the pay-performance relationship in incentive packages: in other words, managerial rewards have generally become very responsive to upside gains but relatively immune to poor results. This asymmetry is now being criticized by several scholars (see, e.g., Jensen and Murphy 2004; Boyer 2011). In its January 2011 report, the National Commission in charge of investigating the causes of the financial and economic downturn maintains that:

> Compensation systems - designed in an environment of cheap money, intense competition, and light regulation - too often rewarded the quick deal, the short-term gain - without considerations of long-term consequences. Often those systems encouraged the big bet - where *the payoff on the upside could be huge and the downside limited.* This was the case up and down the line - from the corporate boardroom to the mortgage broker on the street. (emphasis added)

This paper showed indeed that the optimal incentive scheme to be set by a sufficiently prudent 'principal' - this word standing here for shareholders or the corporate board - should rather be *concave* with

respect to firm performance: in this case, remuneration would be much more sensitive to improved performance at the lower levels than across the highpoints.

The term prudence is now understood and formally defined in economics as aversion to downside risk (Menezes et al. 1980; Eeckhoudt and Schlesinger 2006). Assuming a prudent principal surely departs from the traditional principal-agent literature, in which the principal is typically taken as being risk-neutral. Yet, this attribute corresponds to well-documented behavioral characteristics of investors (Harvey and Siddique 2000; Ang et al. 2006). It also seems to render rather well the fact that corporate board members possess a fiduciary duty to protect their corporation's interest (Lan and Herakleous 2010). Intuitively, a prudent principal will then set a concave contract in order to have the agent (who stands for a manager or executive) bear more downside risk and thereby motivate him to keep the firm away from the worst.

In the utility sector, where corporate boards' fiduciary duties are usually upheld quite strongly (hence where a 'principal' is likely to be quite prudent), the pay-performance relation is actually concave (Murphy 1999). But it remains convex in most other industries. Sections 4 and 5 above argued that limited liability (of the agent or the firm) or progressive taxation (of income or profit) might induce even a prudent principal to avoid concave incentive schemes. This might call for a re-evaluation of these policies. Progressive taxation of a manager's income, in particular, which common wisdom often presents as a means to curb excessive managerial rewards, can in fact entail *convex* schemes, as it might overly downgrade the incentives a concave scheme provides on the upside.

Reviewing liability regimes and tax laws might of course not be enough to allow the prudence of principals to prevail, and to restore (when needed) concavity in pay-performance relationships. The process of setting incentive contracts involves negotiations and third-party inputs (from consultants, other employees, etc.) which are exposed to manipulations, power struggles and conflicts of interest. Agency problems also exist between boards and shareholders, so it might not be clear whose risk preferences are being taken into account after all. The choice of an incentive scheme is furthermore subject to other criteria, such as attracting and retaining talented people. Finally, as pointed out by Jensen and Murphy (2004), concave schemes do have drawbacks as well: managers subject to concave bonuses, for instance, are encouraged to smooth performance across periods, so they might hide superior results at one time in order to use them later when facing harsher circumstances (hence Jensen and Murphy recommend to use linear schemes, but they do not support this using a formal principal-agent model). This paper thus represents only a small step towards building a complete, integrated and operational normative framework for the analysis of executive compensation.

Proof of Lemma 1.

Risk aversion of at least one player is sufficient to obtain that $w^{*\prime}(s) \geq 0$. Indeed we have, with $v(\pi(s) - w(s))$ denoted as $v(\cdot)$ and $u(w(s))$ denoted as $u(\cdot)$:

$$\frac{\partial}{\partial s}\left(\frac{v'(\pi(s) - w(s))}{u'(w(s))}\right)$$

$$= \frac{(\pi'(s) - w'(s)).v''(\cdot)u'(\cdot) - v'(\cdot).u''(\cdot).w'(s)}{(u'(\cdot))^2}$$

$$= \frac{-w'(s).(v''(\cdot).u'(\cdot) + v'(\cdot).u''(\cdot)) + \pi'(s).v''(\cdot).u'(\cdot)}{(u'(\cdot))^2}$$

A necessary condition for this last equality to be positive is $w'(s) \geq 0$. ♦

Proof of Lemma 2.

Let us now compute the second derivative:

$$\frac{\partial^2}{\partial s^2}\left(\frac{v'(\pi(s) - w(s))}{u'(w(s))}\right)$$

$$= \frac{1}{(u'(.))^4} \cdot \left(\{-w''.(v''u' + v'u'') - w'. [(\pi' - w')v'''u' + w'v''u'' + (\pi' - w')v''u'' + w'v'u''']\right.$$

$$\left. + \pi''v''u' + \pi'[(\pi' - w')v'''u' + v''w'u'']\}.u'^2 + 2u''u'.w'. [w'(v''u' + v'u'') - \pi'v''u']\right)$$

$$= \frac{1}{u'^3} \cdot \left(\{-w''.(v''u' + v'u'') + (\pi' - w')^2 v'''u' + (\pi' - w')w'v''u'' - w'^2 v'u''' + \pi''v''u'\} .u'\right.$$

$$\left. -(\pi' - w')v''u''w'u'(u' + 2) + 2u''^2 w'^2 v'\right)$$

$$= \frac{1}{u'^3} \cdot \left(\{-w''.(v''u' + v'u'') + (\pi' - w')^2 v'''u' - w'^2 v'u''' + \pi''v''u'\} .u'\right.$$

$$\left. -2w'u''. ((\pi' - w')v''u' - v'u''w')\right)$$

$$= \frac{1}{u'^2} \cdot \left[-w''.(v''u' + v'u'') + (\pi' - w')^2 v'''u' - w'^2 v'u''' + \pi''v''u'\right]$$

$$+ 2w'R_u. \frac{((\pi' - w')v''u' - v'u''w')}{u'^2}$$

The last term is in fact $\frac{\partial}{\partial s}\left(\frac{v'(\pi(s)-w(s))}{u'(w(s))}\right)$, which must be positive by Assumption 1. Then:

$$
\begin{aligned}
&\frac{\partial^2}{\partial s^2}\left(\frac{v'(\pi(s)-w(s))}{u'(w(s))}\right) \\
=\; &2w' R_u.\frac{\partial}{\partial s}\left(\frac{v'(\pi(s)-w(s))}{u'(w(s))}\right) \\
&+\frac{1}{u'^2}.\left[-w''.(v''u'+v'u'')+(\pi'-w')^2 v'''u'-w'^2 v'u'''+\pi''v''u'\right] \\
=\; &2w' R_u.\frac{\partial}{\partial s}\left(\frac{v'(\pi(s)-w(s))}{u'(w(s))}\right) \\
&+\frac{v'}{u'}.\left[w''.(R_v+R_u)+(\pi'-w')^2 P_v R_v - w'^2 P_u R_u - \pi'' R_v\right] \\
=\; &2w' R_u.\frac{\partial}{\partial s}\left(\frac{v'(\pi(s)-w(s))}{u'(w(s))}\right) \\
&+\frac{v'}{u'}.\left[w''.(R_v+R_u)-\pi'' R_v +(\pi'-w')^2 P_v R_v - w'^2 P_u R_u\right]
\end{aligned}
$$

The sign of this last expression depends on the sign of $(\pi'-w')^2 P_v R_v - w'^2 P_u R_u$. If it is negative, which is the case when

$$
\frac{P_u R_u}{P_v R_v}<\frac{(\pi'-w')^2}{w'^2},
$$

then $w''(s)$ must be negative in order to make $\frac{\partial^2}{\partial s^2}\left(\frac{v'(\pi(s)-w(s))}{u'(w(s))}\right)<0$. ♦

Proof of Proposition 1.

First consider the states in which the optimal wages are strictly positive. In all these states, we have $\lambda(s)=0$ and the optimal reward function is identical to the one obtained in the previous section. It is then increasing and concave in s when the principal is more prudent than the agent by a factor larger than k^2 (see Theorem 1).

Let s_0 be the signal value for which a zero wage is optimal. Since the reward function is increasing on the positive subspace, this signal is unique. Moreover, the limited liability constraint is binding for all signals lower than s_0. This proves (i).

To show (ii), consider the following two subsets

$$
\begin{aligned}
\overline{S_0} &= \{s \in S; s > s_0\} \\
\underline{S_0} &= \{s \in S; s \le s_0\},
\end{aligned}
$$

and let us compare a^* and a^{**}. The agent subject to limited liability computes the following program for any reward schedule $w(s)$:

$$
\max_a U = \int_{s\in\overline{S_0}} u(w(s))dF(s;a) + u(0).F(s_0;a) - c(a)
$$

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The first-order condition is given by

$$a^{**} \quad / \quad \int\limits_{s \in \overline{S_0}} u(w(s))dF_a(s; a^{**}) + u(0).F_a(s_0; a^{**}) - c'(a^{**}) = 0 \tag{16}$$

Recall now the first-order condition under unlimited liability:

$$a^* \quad / \quad \int\limits_{s \in S} u(w(s))dF(s; a^*) - c'(a^*) = 0 \tag{17}$$

From Assumption 1, we have that $F_a(s_0; a) < 0$. The utility function $u(\cdot)$ being increasing, it is also true that $u(0) \geq u(w(s))$ for any negative $w(s)$, with at least one strict inequality. Hence, comparing (17) and (16), we can conclude that $a^* > a^{**}$ for any given reward function $w(s)$. ♦

Proof of Proposition 2.

The Lagrangian function for this problem is given by

$$\mathcal{L} \quad = \quad \int\limits_{s \in \overline{S_1}} v(\pi(s) - w(s))dF(s; a^*) + v(0).F(s_1; a^*)$$

$$+\delta \left(\int\limits_{s \in S} u(w(s))dF_a(s; a^*) - c'(a^*) \right) + \mu \left(\int\limits_{s \in S} u(w(s))dF(s; a^*) - c(a^*) - U_0 \right)$$

Two expressions must be considered when verifying the first-order conditions for maximizing $\mathcal{L}$. If $s \in \overline{S_1}$, we have

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)}, \quad \forall s \in \overline{S_1} \tag{18}$$

and, in particular,

$$\lim_{s \downarrow s_1} \frac{v'(\pi(s) - w(s))}{u'(w(s))} = \lim_{s \downarrow s_1} \left(\mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)} \right)$$

$$\frac{v'(\pi(s_1) - w(s_1))}{u'(w(s_1))} = \mu + \delta \frac{f_a(s_1; a^*)}{f(s_1; a^*)} \tag{19}$$

For $s \in \underline{S_1}$, we have

$$\frac{\partial \mathcal{L}}{\partial w(s)} = u'(w(s)).f(s; a^*) \left(\mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)} \right), \quad \forall s \in \underline{S_1} \tag{20}$$

Expression (18) is identical to (3) in the unlimited case. Thus, the reward schedule corresponds to the one described in (i) if the principal is 'more prudent' than the agent.

>From (19) we have that $\mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)} > 0$. This expression being continuous and non decreasing in s (Assumption 1), there exists a state $s_0 < s_1$ such that $\mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)} > 0 \ \forall s \in \,]s_0, s_1[$ and $\mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)} \leq 0$ $\forall s \leq s_0$. With $\frac{\partial^2 \mathcal{L}}{\partial w(s)^2} = u''(w(s)).f(s; a^*) \left(\mu + \delta \frac{f_a(s; a^*)}{f(s; a^*)} \right)$, this implies that

$$\frac{\partial \mathcal{L}}{\partial w(s)} \quad > \quad 0 \ \text{ and } \ \frac{\partial^2 \mathcal{L}}{\partial w(s)^2} < 0, \quad \forall s \in \,]s_0, s_1[\tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial w(s)} \quad \leq \quad 0 \ \text{ and } \ \frac{\partial^2 \mathcal{L}}{\partial w(s)^2} > 0, \quad \forall s \leq s_0 \tag{22}$$

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The Lagrangian function $\mathcal{L}$ being concave in $w(s)$ on $]s_0, s_1[$ and convex otherwise, the optimal revenue is the maximum possible one for any $s \leq s_1$. More precisely, $w(s)$ being continuous and increasing on $\overline{S_1}$, the reward schedule satisfies $w^*(s) = w(s_1) = \pi(s_1) = 0$ for any $s \leq s_1$. $\blacklozenge$

Proof of Proposition 3.

Consider condition (9). Let $A = \frac{\partial}{\partial s}((1 - \tau(s))w(s))$, which is assumed to be positive. We have that

$$\frac{\partial}{\partial s}\left(\frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'((1 - \tau(s))w(s))}\right)$$

$$= \frac{(\pi' - w')v''(1 - \tau)u' - v'(u''A(1 - \tau) - \tau'u')}{(1 - \tau)^2 u'^2}$$

$$= \frac{(\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau))}{(1 - \tau)u'}$$

Now, write $B = \frac{\partial}{\partial s}(\frac{\tau'(s)}{(1 - \tau(s))}) = \frac{\tau''(1 - \tau) + 2\tau'}{(1 - \tau)^2}$. With $\tau'' = 0$ by assumption, the latter simplifies to $B = \frac{2\tau'}{(1 - \tau)^2}$. Then:

$$\frac{\partial^2}{\partial s^2}\left(\frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'((1 - \tau(s))w(s))}\right)$$

$$= \frac{1}{(1 - \tau)u'} \cdot \left[(\pi'' - w'').v'' + (\pi' - w')^2 v'''\right.$$

$$+v''(\pi' - w')(R_u A + \tau'/(1 - \tau)) + v'(R'_u A + R_u A' + B)]$$

$$-\frac{1}{(1 - \tau)^2 u'^2} \cdot [((1 - \tau)Au'' - \tau'u')((\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau)))]$$

$$= \frac{1}{(1 - \tau)u'} \cdot \left[(\pi'' - w'').v'' + (\pi' - w')^2 v'''\right.$$

$$+v''(\pi' - w')(R_u A + \tau'/(1 - \tau)) + v'(R'_u A + R_u A' + B)$$

$$+ (AR_u + \tau'/(1 - \tau))((\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau)))]$$

Note that $A' = (1 - \tau)w'' - 2\tau'w'$. Using the fact that $\tau''(s) = 0$, the latter expression becomes

$$\frac{\partial^2}{\partial s^2}\left(\frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'((1 - \tau(s))w(s))}\right)$$

$$= \frac{1}{(1 - \tau)u'} \cdot \left[(\pi''v'' + w''(v'R_u(1 - \tau) - v'') + (\pi' - w')^2 v'''\right. \tag{23}$$

$$+v''(\pi' - w')(R_u A + \tau'/(1 - \tau)) + v'(R'_u A - R_u 2\tau'w' + B)$$

$$+ (AR_u + \tau'/(1 - \tau))((\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau)))]$$

$$= \frac{1}{(1 - \tau)u'} \cdot \left[(\pi''v'' + w''v'(R_u(1 - \tau) + R_v) + (\pi' - w')^2 v'''\right. \tag{24}$$

$$+v''(\pi' - w')(R_u A + \tau'/(1 - \tau)) + v'(R'_u A - R_u 2\tau'w' + B) \tag{25}$$

$$+ (AR_u + \tau'/(1 - \tau))((\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau)))] \tag{26}$$

The first term in line (24), namely $\pi''v''$, is positive by assumption. The third term, $(\pi' - w')^2 v'''$, is positive for a prudent principal. Lines (25) and (26) now remain to be signed. The sum of these lines can

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be rewritten:

$$. \quad v''(\pi' - w')(R_u A + \tau'/(1-\tau)) + v'(R'_u A - R_u 2\tau' w' + B)$$

$$+ (AR_u + \tau'/(1-\tau))((\pi' - w')v'' + v'(R_u A + \tau'/(1-\tau)))$$

$$= v'(AR_u + \tau'/(1-\tau))^2 + 2v''(AR_u + \tau'/(1-\tau))(\pi' - w') + v'(R'_u A - R_u 2\tau' w' + B)$$

$$= v'. \left[(AR_u + \tau'/(1-\tau))^2 - 2R_v(AR_u + \tau'/(1-\tau))(\pi' - w') + (R'_u A - R_u 2\tau' w' + B) \right]$$

$$= v'. \left[((AR_u + \tau'/(1-\tau)) - R_v(\pi' - w'))^2 - R_v^2(\pi' - w')^2 + (R'_u A - R_u 2\tau' w' + B) \right]$$

With $R'_u = \frac{\partial R_u}{\partial s} = AR_u(R_u - P_u)$ it finally becomes

$$v'. \left[((AR_u + \tau'/(1-\tau)) - R_v(\pi' - w'))^2 - R_v^2(\pi' - w')^2 + A^2 R_u(R_u - P_u) - R_u 2\tau' w' + B \right] \quad (27)$$

If the Agent is risk neutral, given that $B = \frac{2\tau'}{(1-\tau)^2}$, the latter reduces to

$$v'. \left[(\tau'/(1-\tau) - R_v(\pi' - w'))^2 - R_v^2(\pi' - w')^2 + B \right]$$

$$= v'. \left[\tau'^2/(1-\tau)^2 - 2R_v(\pi' - w')\tau'/(1-\tau) + \frac{2\tau'}{(1-\tau)^2} \right]$$

$$= \frac{2\tau' v'}{(1-\tau)} . \left[\frac{(\tau'+2)}{(1-\tau)} - 2R_v(\pi' - w') \right] \quad (28)$$

which is equal to zero for a constant tax rate. Therefore, since all expressions in (24), (25) and (26) are positive, except $w''v'(R_u(1-\tau) + R_v)$, it is necessary that $w''(s) < 0$ in order to have $\frac{\partial^2}{\partial s^2} \left(\frac{v'(\pi(s) - w(s))}{(1-\tau(s))u'((1-\tau(s))w(s))} \right) < 0$. This demonstrates (i).

When the Agent is risk averse and taxes increase with the signal s, expression (27) can be negative even if $P_u \leq 0$. In this case, $w''(s)$ can be either positive or negative on S^+. On S^-, condition (10) holds; one obtains therefore that $w''(s) < 0$. ♦

Proof Proposition 4.

Write $C = \frac{\partial}{\partial s}[(1 - \gamma(s))(\pi(s) - w(s))]$. We have that

$$\frac{\partial}{\partial s} \left(\frac{v'[(1-\gamma(s))(\pi(s) - w(s))]}{u'(w(s))} \right)$$

$$= \frac{Cv''u' - v'u''w'}{u'^2}$$

$$= \frac{Cv'' + R_u v'w'}{u'} = \frac{v'(R_u w' - CR_v)}{u'}$$

Net profits after taxation are nondecreasing in s by assumption, so $C \geq 0$. This entails that $w'(s) > 0$ if we want $\frac{\partial}{\partial s} \left(\frac{v'[(1-\gamma(s))(\pi(s) - w(s))]}{u'(w(s))} \right) > 0$.

Computing now the second derivative, we have

$$\frac{\partial^2}{\partial s^2}\left(\frac{v'\left[(1-\gamma(s))(\pi(s)-w(s))\right]}{u'(w(s))}\right)$$

$$=\frac{C'v''+C^2v'''+(R_u'v'+R_uCv'')\,w'+R_uv'w''}{u'}$$

$$-\frac{(Cv''+R_uv'w')\,u''w'}{u'^2}$$

$$=\frac{C'v''+C^2v'''+(R_u'v'+R_uCv'')\,w'+R_uv'w''+(Cv''+R_uv'w')\,R_uw'}{u'}$$

With $C=(1-\gamma)(\pi'-w')-\gamma'(\pi-w)>0$ and $C'=-2\gamma'(\pi'-w')+(1-\gamma)(\pi''-w'')$, knowing that $\gamma''(s)=0$ by assumption, we get

$$\frac{\partial^2}{\partial s^2}\left(\frac{v'\left[(1-\gamma(s))(\pi(s)-w(s))\right]}{u'(w(s))}\right)$$

$$=\frac{1}{u'}\cdot\left[(-2\gamma'(\pi'-w')+(1-\gamma)\pi'')v''+C^2v'''+(R_u'v'+R_uCv'')\,w'\right.$$

$$\left.+(R_uv'-(1-\gamma)v'')\,w''+(Cv''+R_uv'w')\,R_uw'\right]$$

$$=\frac{v'}{u'}\cdot\left[(R_u+(1-\gamma)R_v)\,w''-(-2\gamma'(\pi'-w')+(1-\gamma)\pi'')R_v\right.$$

$$\left.+C^2P_vR_v+(R_u'-R_uCR_v)\,w'\right]+\frac{v'\cdot(R_uw'-CR_v)\,R_uw'}{u'}$$

The last term, $R_uw'\cdot\frac{\partial}{\partial s}\left(\frac{v'[(1-\gamma(s))(\pi(s)-w(s))]}{u'(w(s))}\right)$ is positive. The second term is also positive since $(\pi'-w')$ must be positive to ensure that $C\geq 0$ for any tax structure. We have to sign the third term. With $R_u'=w'R_u(R_u-P_u)$, this third term is

$$C^2P_vR_v+(R_u'-R_uCR_v)\,w'$$

$$=\;C^2P_vR_v+(w'R_u(R_u-P_u)-R_uCR_v)\,w'$$

$$=\;(C^2P_vR_v-w'^2P_uR_u)+R_uR_vw'(w'.\frac{R_u}{R_v}-C)\qquad(29)$$

For $R_u=0$, this expression is equal to zero. This proves (i).

If $R_u>0$, then the first term is positive if $\frac{P_uR_u}{P_vR_v}\leq\frac{C^2}{w'^2}$ or if $\frac{w'R_u}{CR_v}\leq\frac{C.P_v}{w'.P_u}$. The second term is positive if $1\leq\frac{w'R_u}{CR_v}$. Thus the whole expression is positive if $1\leq\frac{w'R_u}{CR_v}\leq\frac{C.P_v}{w'.P_u}$. Only in this case must the optimal schedule $w(s)$ be concave in s in order to obtain $\frac{\partial^2}{\partial s^2}\left(\frac{v'[(1-\gamma(s))(\pi(s)-w(s))]}{u'(w(s))}\right)<0$. On the contrary, if expression (29) is negative, which is possible even with a prudent principal and a non prudent agent, then $w(s)$ can be convex. This is point (ii).

To show (iii), notice that the more progressive the tax rate (i.e. the higher the marginal rate γ'), the lower the value of C. The smallest value of C is of course zero. When the agent is non prudent (so $P_u\leq 0$), we have

$$\lim_{C\downarrow 0}(C^2P_vR_v-w'^2P_uR_u)+R_uR_vw'(w'.\frac{R_u}{R_v}-C)$$

$$=\;-w'^2P_uR_u+R_u^2w'^2>0$$

Because all the functions involved in (29) are continuous, there exists a strictly positive value $\widehat{\gamma} = \gamma'(s)$ (recall that $\gamma''(s) = 0$ by assumption) such that expression (29) is strictly positive for any marginal rate higher than $\widehat{\gamma}$ and that ensures the positivity of C. In this specific situation, $w''(s) < 0$ is again a necessary condition for $\frac{\partial^2}{\partial s^2} \left(\frac{v'[(1-\gamma(s))(\pi(s)-w(s))]}{u'(w(s))} \right) < 0$. This holds whatever the principal's degree of prudence P_v.

♦

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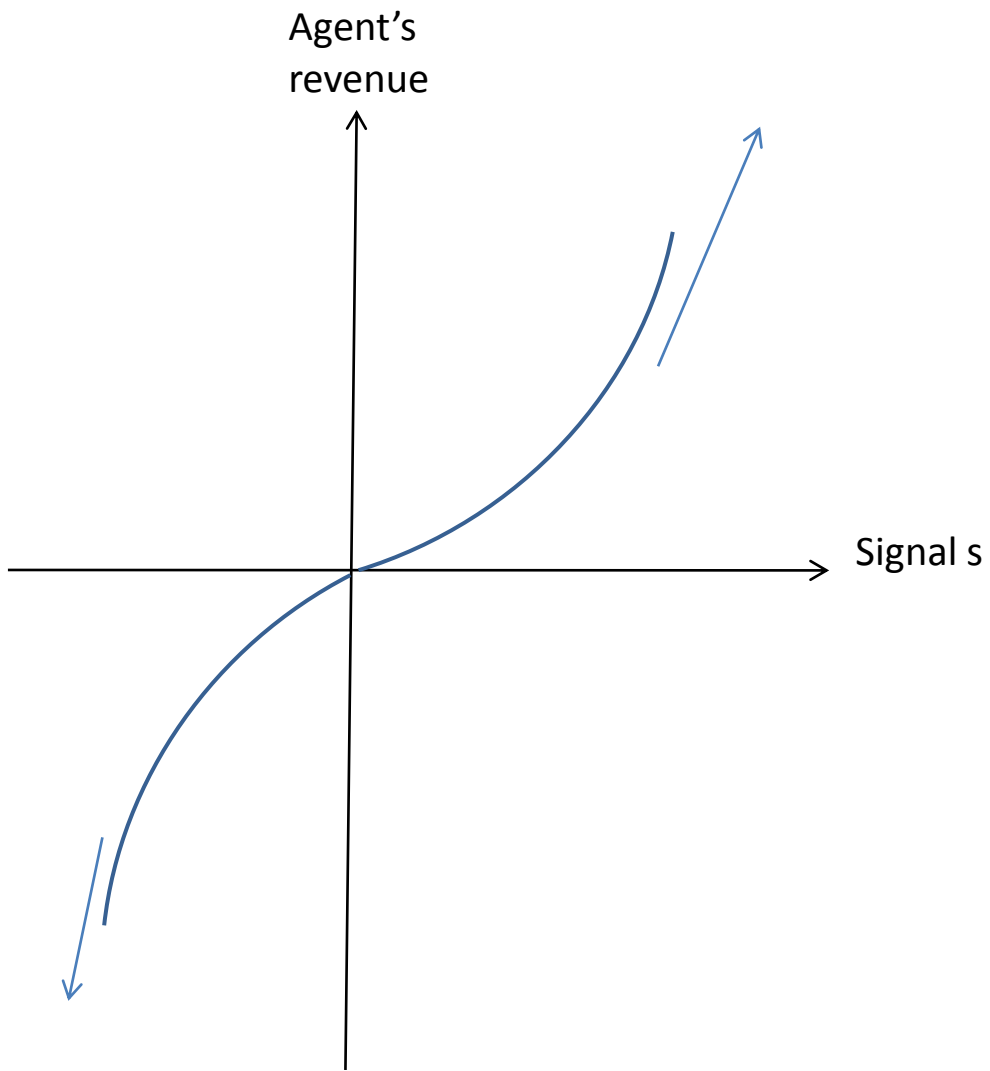
Figure 1.
Optimal revenue function when an increasing tax rate is applied to the Agent's revenue
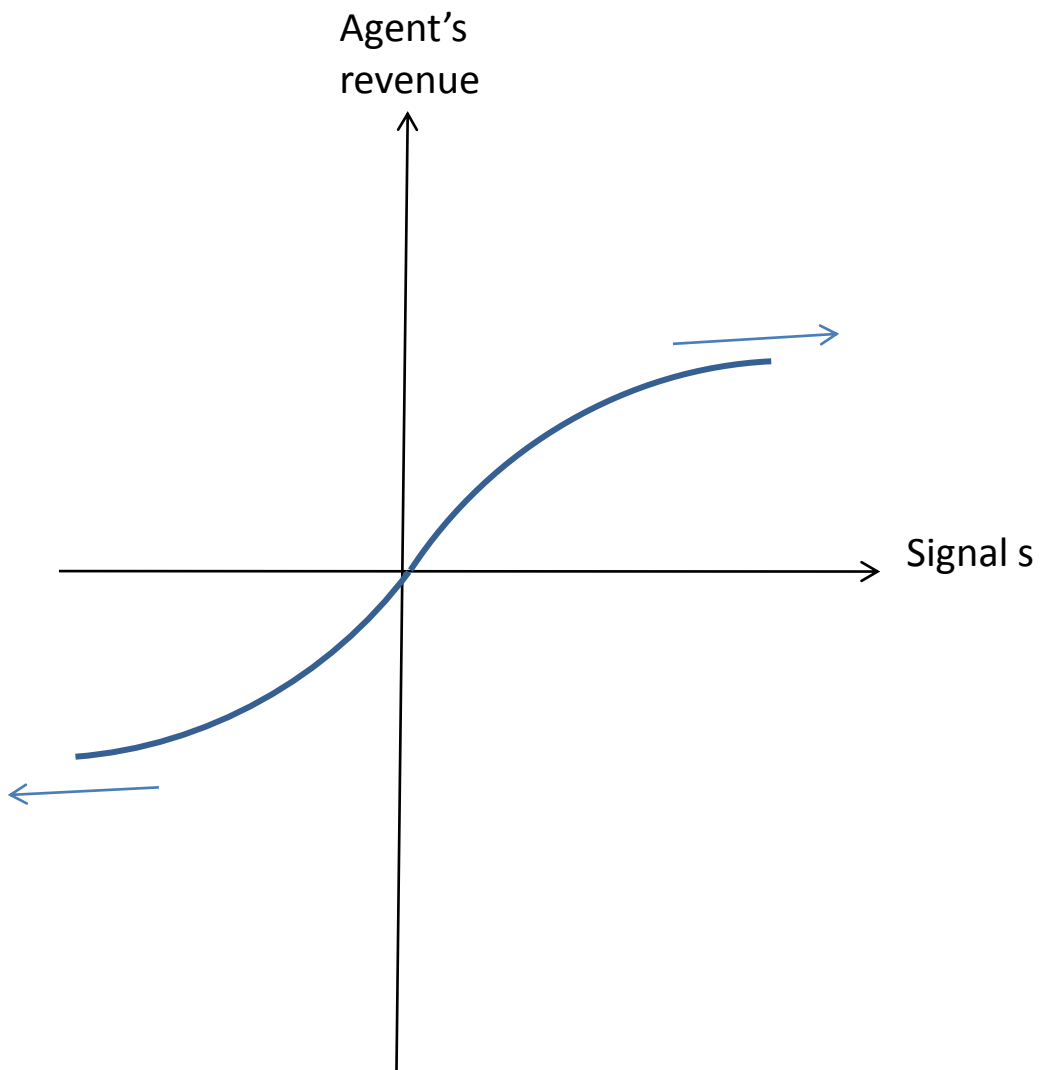
Figure 2.
Example of optimal revenue function when an increasing tax rate is applied to the Principal's revenue