

Unconventional Fiscal Policy at the Zero Bound*

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Abstract

When the zero lower bound on nominal interest rates binds, monetary policy cannot provide appropriate stimulus. We show that in the standard New Keynesian model, tax policy can deliver such stimulus at no cost and in a time-consistent manner. There is no need to use inefficient policies such as wasteful public spending or future commitments to low interest rates.

Key words: Zero Bound; Fiscal policy; Monetary policy; Sticky prices.

JEL classification: E31; E40; E52; E58; E62; E63.

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1 Introduction

Arbitrage between money and bonds restricts nominal interest rates from becoming negative. One can imagine circumstances in which, in the event of a potential recession, it is desirable for the Central Bank to lower the policy rate. If the interest rate is very close to zero to begin with, the constraint may be binding. This "zero bound" constraint gives rise to a macroeconomic situation known as a liquidity trap. It constitutes a difficult challenge for stabilization policy.

There is a well developed Keynesian view on the subject. In a liquidity trap, monetary policy is ineffective—increasing the money supply is like “pushing on a string”. The standard Keynesian prescription is therefore use fiscal policy in order to stimulate the economy, in the form of tax cuts or government spending increases. This recommendation has been very influential in shaping policy. Yet its validity can be questioned and refined. Indeed, the absence of microfoundations, lack of dynamics, and neglect of expectations formation, of the basic IS-LM model creates difficulties for normative analysis (how to think about welfare?), as well as for positive analysis (how to think about the adjustment of prices, about the effects of different taxes vs. government expenditures, or about the effects of future policy commitments?).

Recently, a literature has emerged that revisits the Keynesian analysis in the context of explicitly micro-founded, dynamic, rational expectations New Keynesian models that do not suffer from these shortcomings. There is now a New Keynesian view of liquidity traps. Krugman (1998), and Eggertsson and Woodford (2003) have characterized optimal monetary policy at the zero bound. Their work emphasizes the role of policy commitments. They show that it is optimal to commit to keeping the interest rate at zero for longer than under the no-commitment solution. This increases output and inflation both in the present and in the future—optimally trading off the mitigation of a recession in the present and the creation of a boom in the future. This literature has also emphasized the beneficial effects of fiscal policy. For example, Christiano, Eichenbaum, Rebelo (2011), Eggertsson (2009), and Woodford (2010) have shown that government spending multipliers can be very large at the zero bound, and that increasing government spending can be welfare improving.

In this paper we study the liquidity trap in the context of a standard New Keynesian model. We characterize jointly optimal monetary and fiscal policy. Our main result is to demonstrate how distortionary taxes can be used to replicate the effects of negative nominal interest rates and completely circumvent the zero bound problem. We label this scheme *unconventional fiscal policy*. It involves engineering over time an increasing path for consumption taxes, a decreasing path for labor taxes, coupled with a temporary investment tax credit or a temporary cut in capital income taxes.

Whether or not the first-best allocation can be implemented depends on the set of available instruments, and in particular on the existence of lump sum taxes. In the simple New Keynesian model of Eggertsson (2009), where lump sum taxes are allowed, we show that the first-best allocation can be implemented at the zero bound with unconventional fiscal policy (Section 2). The first best can also be attained in a model with capital (Section 3), and also in a model with sticky prices and sticky wages (Section 4). In more general

set ups, full efficiency cannot be attained. That is the case in a Ramsey model in which lump sum taxes cannot be raised. However, as it turns out, the zero bound is not a constraint in that second best problem: Unconventional fiscal policy can perfectly replicate the effects of negative nominal interest rates at the second best (Section 5). This point is more general: The full set of implementable allocations is unrestricted by the zero bound constraint. And that is the case, whether the first best can be achieved or not. We show this in a model where productivity shocks are firm specific and the initial distribution of prices is nondegenerate, so that the relative price distortions created by sticky prices cannot be removed (Section 6). We also quantify the tax policy that would be necessary to avoid a major recession at the zero bound (Section 7).

The intuition why tax policy can neutralize the effects of the zero bound constraint is simple. Suppose real rates ought to be negative. Since the nominal interest rate cannot be negative, the only way to achieve negative real interest rates is to generate inflation. But producer price inflation is costly. Indeed, in the New Keynesian, sticky price, literature, price setting decisions are staggered. Producer price inflation then necessarily leads to dispersion in relative prices—a real economic distortion. Is it possible to achieve negative real interest rates without incurring this economic cost?

It turns out that the prices that matter for intertemporal decisions are consumer prices, which are gross of consumption taxes. The idea is to induce inflation in consumer prices, while keeping producer price inflation at zero. The result is negative real interest rates, and the distortions associated with producer price inflation are altogether avoided. This can be achieved by simultaneously adjusting consumption and labor taxes. Imagine first that producer price inflation is zero. Then an increasing path of consumption taxes over time generates inflation in consumer prices. The problem is that this change in consumption taxes introduces undesirable variations in the marginal cost of firms over time—creating incentives for producers to change their prices. This effect must therefore be counteracted with a decreasing path for labor taxes. Overall, this policy achieves a negative interest rate in the consumer price numeraire.

In a model with capital, this policy must be supplemented with a temporary capital subsidy—in the form of a temporary investment tax credit or a temporary cut in capital income taxes. This is because an increasing path of consumption taxes acts as a tax on capital. This tax on capital is undesirable and must be counteracted with a corresponding subsidy in order to adequately channel savings to investment.

In our basic setup with lump sum taxes, the optimal policy that we characterize implements the first-best allocation. It is therefore time-consistent: if a future planner were given an opportunity to revise this policy in the future, it would choose not to do so. This should be contrasted with the policy recommendations involving future commitments to low interest rates in Krugman (1998) and Eggertsson and Woodford (2003). It is also the case that under more reasonable assumptions preventing lump sum taxation, optimal policy is in general time inconsistent. However, there is a sense in which the time-inconsistency that arises is unrelated to the zero bound, since indeed the zero bound does not restrict the second best problem.

Literature review. There is a vast literature on the zero bound in New Keynesian models. There is work on public spending multipliers, showing that these can be very large at the zero bound (see Christiano et al. (2011), Eggertsson (2009), Woodford (2010), Mertens and Ravn (2010)¹). Eggertsson (2009) also considers different alternative taxes and assesses which one is the most desirable to deal with the zero bound. The zero bound is also a key component in the numerical work presented in the evaluation of the American Recovery and Reinvestment Plan by Romer and Bernstein (2009). It is also a main concern in Blanchard, Dell’Ariccia and Mauro (2010) who argue for a better integration between monetary and fiscal policy.

There is also earlier work on the implications of the zero bound for monetary and fiscal policy, motivated by the prolonged recession in Japan where overnight rates have been close to zero for the last fifteen years, as well as by the low targets for the Fed funds rate in the US in 2003 and 2004.² Krugman (1998), Eggertsson and Woodford (2003 and 2004a) show that there may be downturns that could, and should, be avoided if it was not for the zero bound. They also show how monetary policy can be adjusted so that the costs of those downturns may be reduced. In particular they propose policies that keep the interest rate for a longer period at zero in order to generate inflation. Eggertsson and Woodford (2004b) consider both monetary and fiscal policy in a Ramsey taxation model with consumption taxes only.³ All this work is done in the context of standard sticky price models, where the zero bound on interest rates can be a serious challenge to policy. That is indeed the general conclusion, justifying the use of inefficient policies, such as wasteful government spending, leading to undesirable inflation. With a different, more general focus, Correia, Nicolini, and Teles (2008) show that fiscal policy can be used to neutralize the effects of price stickiness. They consider an optimal Ramsey taxation model without capital and with a monetary distortion, similar to the one in Lucas and Stokey (1983) and Chari, Christiano and Kehoe (1991), but with sticky prices. They show that under sticky prices it is possible to implement the same allocations as under flexible prices, and that it is optimal to do so. Since the zero bound is the optimal policy under flexible prices, it must also be the optimal one under sticky prices⁴. In this sense, the zero bound is not a constraint to policy. These results and the pressing relevance of the policy question were the motivation for this work.

There have also been recent policy proposals by Robert Hall and Susan Woodward⁵, and earlier on, by Feldstein (2003), intended at Japan, that go in the direction of the unconventional fiscal policy that we characterize.⁶ Both of them suggested lowering consumption taxes as a way to fight the crisis. Our model

¹Mertens and Ravn show that multipliers can be low if the economy is close to an alternative, liquidity trap, steady state.

²In 2003 and 2004, the Fed funds rate fell down to 1%, and remained there for more than year.

³They also consider the case of two consumption taxes, such that prices are set after one and before the other. In that case it would be possible to implement the same allocation, as if the zero bound did not bind. They find those taxes to be highly unrealistic and move on to analyze the case of a single consumption tax.

⁴In a model with cash and credit goods, additional conditions are needed for that to be the case. But, the optimality of the Friedman rule is a robust result in other monetary models with flexible prices.

⁵An article by Justin Lahart in the Wall Street Journal, January 5, 2009, "State Sales-Tax Cuts: Get Another Look", comments on the proposals of Hall and Woodward in their blog. See also the speech by Narayana Kocherlakota, President of the Minneapolis Fed, "Monetary Policy Actions and Fiscal Policy Substitutes," November 18, 2010.

⁶"The Japanese government could announce that it will raise the current 5 percent value added tax by 1 percent per quarter

formalizes these proposals and highlights the way other taxes must be jointly used.

2 A simple model

The model we analyze is a standard new-Keynesian model, similar to the one analyzed by Eggertsson and Woodford (2003) and (2004b), and Eggertsson (2009). As has become standard in the New Keynesian literature, the economy is cashless.

The uncertainty in period $t \geq 0$ is described by the random variable $s_t \in S_t$, where S_t is the set of possible events at t , and the history of its realizations up to period t is denoted by $s^t \in S^t$. For simplicity we index by t the variables that are functions of s^t .

We assume that there is a representative household with preferences described over aggregate consumption C_t and leisure L_t ,

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t, \xi_t) \quad (1)$$

with

$$C_t = \left[\int_0^1 c_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

where c_{it} is private consumption of variety $i \in [0, 1]$, and ξ_t is a preference shock.

Aggregate government consumption G_t is exogenous. It is also a Dixit-Stiglitz aggregator of public consumption of different varieties g_{it} ,

$$G_t = \left[\int_0^1 g_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (3)$$

where the elasticity of substitution between varieties is $\theta > 1$.

The production function of each good i , uses labor, n_{it} , according to

$$c_{it} + g_{it} = A_t n_{it}, \quad (4)$$

where A_t is an aggregate productivity shock. Total labor $N_t = 1 - L_t$ is

$$N_t = \int_0^1 n_{it} di. \quad (5)$$

2.1 Government

As is standard in the new-Keynesian literature, we allow for lump-sum taxes, T_t , which is a residual variable that adjusts so that the government budget constraint is satisfied. There are also taxes on consumption, τ_t^c , labor income, τ_t^n , and profits, τ_t^d .

The government minimizes the expenditure on the individual goods, given G_t . If we let

$$P_t \equiv \left[\int_0^1 p_{it}^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad (6)$$

and simultaneously reduce the income tax rates to keep revenue unchanged, continuing this for several years until the VAT reaches 20 percent." Feldstein (2003).

where p_{it} is the price of variety i , then, the minimization of expenditure on the individual goods, implies

$$\frac{g_{it}}{G_t} = \left(\frac{p_{it}}{P_t} \right)^{-\theta}. \quad (7)$$

2.2 Households

The representative household also minimizes spending on aggregate C_t , by choosing the consumption of different varieties according to

$$\frac{c_{it}}{C_t} = \left(\frac{p_{it}}{P_t} \right)^{-\theta}. \quad (8)$$

The budget constraint can then be written in terms of the aggregates as

$$\begin{aligned} \frac{1}{1+i_t} \bar{B}_t + E_t Q_{t,t+1} B_{t,t+1} &= \bar{B}_{t-1} + B_{t-1,t} + (1 - \tau_t^n) W_t N_t \\ &+ (1 - \tau_t^d) \Pi_t - (1 + \tau_t^c) P_t C_t - T_t, \quad t \geq 0 \end{aligned} \quad (9)$$

together with a no-Ponzi games condition. $B_{t,t+1}$ represent the quantity of state contingent bonds that pay one unit of money at time $t+1$, in state s^{t+1} and \bar{B}_t are risk free nominal bonds paying one unit of money at $t+1$. The price of the state contingent bonds, normalized by the probability of occurrence of the state at $t+1$, is $Q_{t,t+1}$. Consequently $E_t Q_{t,t+1} = \frac{1}{1+i_t}$ is the price of the riskless bond, where $1+i_t$ is the gross nominal interest rate. The nominal wage is W_t and $\Pi_t = \int_0^1 \Pi_{it} di$ are total profits.

The first order conditions of the household problem that maximizes utility (1) subject to the budget constraint (9) with respect to the aggregates are

$$\frac{u_C(C_t, N_t, \xi_t)}{u_L(C_t, N_t, \xi_t)} = \frac{(1 + \tau_t^c) P_t}{(1 - \tau_t^n) W_t} \quad (10)$$

and

$$Q_{t,t+1} = \frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{u_C(C_t, N_t, \xi_t)} \frac{(1 + \tau_t^c) P_t}{(1 + \tau_{t+1}^c) P_{t+1}}, \quad (11)$$

$$\frac{u_C(C_t, N_t, \xi_t)}{(1 + \tau_t^c) P_t} = (1 + i_t) E_t \frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{(1 + \tau_{t+1}^c) P_{t+1}}. \quad (12)$$

2.3 Firms

Each variety is produced by a monopolist. Prices are set as in Calvo (1983). Every period, a firm is able to revise the price with probability $1 - \alpha$. The lottery that assigns rights to change prices is *i.i.d.* over time and across firms. Since there is a continuum of firms, $1 - \alpha$ is also the share of firms that are able to revise prices. Those firms choose the price p_t to maximize profits net of taxes,

$$E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} (1 - \tau_{t+j}^d) [p_t y_{t+j} - W_{t+j} n_{t+j}]$$

where $Q_{t,t+j}$ is the nominal price at t of one unit of money at a particular state in period $t+j$, output $y_{t+j} = c_{t+j} + g_{t+j}$ must satisfy the technology constraint and the demand function

$$y_{t+j} = \left(\frac{p_t}{P_{t+j}} \right)^{-\theta} Y_{t+j}.$$

obtained from (8) and (7), where $Y_{t+j} = C_{t+j} + G_{t+j}$.

The optimal price set by these firms is⁷

$$p_t = \frac{\theta}{(\theta - 1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{A_{t+j}}, \quad (13)$$

where

$$\eta_{t,j} = \frac{(\alpha\beta)^j \frac{(1-\tau_{t+j}^d)u_C(t+j)}{(1+\tau_{t+j}^c)} (P_{t+j})^{\theta-1} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{(1-\tau_{t+j}^d)u_C(t+j)}{(1+\tau_{t+j}^c)} (P_{t+j})^{\theta-1} Y_{t+j}}. \quad (14)$$

The price level in (6) can be written as

$$P_t = [(1 - \alpha)p_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \quad (15)$$

2.4 Equilibria

Using the demand functions (7) and (8), it follows that

$$C_t + G_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di \right]^{-1} A_t N_t. \quad (16)$$

An equilibrium for $\{C_t, N_t\}$, $\{p_t, P_t, W_t\}$, and $\{i_t, \tau_t^c, \tau_t^n, \tau_t^d\}$ is characterized by conditions (10), (12), the zero bound constraint $i_t \geq 0$, (13), (15), and (16) rewritten as

$$C_t + G_t = \left[\sum_{j=0}^{t+1} \varpi_j \left(\frac{p_{t-j}}{P_t} \right)^{-\theta} \right]^{-1} A_t N_t, \quad (17)$$

in which ϖ_j is the share of firms that have set prices j periods before, $\varpi_j = (\alpha)^j(1 - \alpha)$, $j = 0, 1, 2, \dots, t$, and $\varpi_{t+1} = (\alpha)^{t+1}$, which is the share of firms that have never set prices so far. We assume that they all charge an exogenous price p_{-1} .⁸

2.5 First best allocations

The first best allocation is the one that maximizes utility (1) subject to the technology constraints (2), (3), (4) and (5), above.

From (4) and (5), it follows that the marginal rate of transformation between any two varieties is equal to one. Because the marginal rate of substitution is $\left(\frac{c_{it}}{c_{jt}} \right)^{-\frac{1}{\theta}}$, it must be that a first best allocation satisfies

$$c_{it} = C_t, \text{ all } i, t.$$

⁷Notice that we are assuming that firms set prices before consumption taxes. This is an important assumption. We base this on the extensive evidence of very high pass-through of consumption taxes even in the cases in which the usual practice is to quote after tax prices, as is the case for the value added tax in Europe.

⁸We do not need to keep track of the budget constraints, since lump sum taxes adjust to satisfy the budget.

A similar argument applies to public consumption of the different varieties, so that

$$g_{it} = G_t, \text{ all } i, t.$$

The efficiency conditions for the aggregates (C_t, N_t) are fully determined by:

$$\frac{u_C(C_t, N_t, \xi_t)}{u_L(C_t, N_t, \xi_t)} = \frac{1}{A_t} \quad (18)$$

and

$$C_t + G_t = A_t N_t. \quad (19)$$

We now show that there are policies and prices that support the first best allocation, both away from and at the zero bound. We do this by showing that there are policies and prices satisfying all the equilibrium conditions, above, for the first best allocation, taking into account the zero bound constraint on the nominal interest rate.

2.6 Optimal policy

In the simple model that we consider in this section as well as in Sections 3 and 4, profit taxes are irrelevant. The first best allocation can be implemented in the exact same way irrespective of profit taxes $\{\tau_t^d\}$,⁹ which only impact the level of lump sum taxes T_t . This will not be true in the environments considered in Sections 5, 6 and 7. Until then, to fix ideas, and without any loss of generality, we can imagine that profit are fully taxed, in the sense that we consider the limiting case in which the tax converges to one.

Policy away from the zero bound. Monetary policy can implement the first best allocation with constant taxes on consumption τ^c and labor τ^n . In order to achieve production efficiency, conditions (7) and (8) imply that prices must be the same across firms $\frac{p_t}{P_t} = 1$. That can only be the case if firms start at time zero with a common price, p_{-1} ,¹⁰ as we assume, and if firms that can subsequently change prices choose that common price, so that $p_t = P_t = p_{-1}$. This means that the price level must be constant across time and states. The reason is simple. Because price setting decisions are staggered, inflation necessarily comes at the cost of dispersion in relative prices. This represents an economic distortion.

It follows that the aggregate resource constraint (17) becomes (19). Calvo's price setting condition (13) implies that

$$p_t = \eta_{t,0} \frac{\theta}{(\theta - 1)} \frac{W_t}{A_t} + (1 - \eta_{t,0}) E_t p_{t+1}.$$

So, when $p_t = p_{-1}$, it must be that

$$p_t = p_{-1} = P = \frac{\theta}{(\theta - 1)} \frac{W_t}{A_t}, \quad (20)$$

⁹The profit tax is useful to show the more general result that we develop in Section 6. We thank a referee for pointing this out to us.

¹⁰This is the standard assumption in the literature. Yun (2005) is an exception.

as under flexible prices. Thus, the nominal wage must move with productivity so as to maintain the nominal marginal cost constant.

From (12), with constant consumption taxes, we have

$$u_C(C_t, N_t, \xi_t) = (1 + i_t) E_t [\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})]$$

so the nominal interest rate i_t must equal the natural rate of interest $u_C(C_t, N_t, \xi_t) / E_t [\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})] - 1$ —the real interest rate that prevails at the first best allocation.

From (10) and (20), it must be that

$$\frac{u_C(C_t, N_t, \xi_t)}{u_L(C_t, N_t, \xi_t)} = \frac{(1 + \tau^c) \frac{\theta}{(\theta-1)}}{(1 - \tau^n) A_t}, \quad (21)$$

implying that $1 - \tau^n = (1 + \tau^c) \frac{\theta}{\theta-1}$. One possibility is to set consumption taxes to zero, $\tau^c = 0$. The resulting labor subsidy, $\tau^n = \frac{-1}{\theta-1}$, that eliminates the mark up distortion, is constant over time.

As long as the natural rate of interest is nonnegative, i.e as long as $u_C(C_t, N_t, \xi_t) \geq E_t [\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})]$, the zero bound constraint is not binding and the first best allocation is implemented with constant taxes and flexible interest rate policy. In this model, in normal times, monetary policy achieves perfect economic stabilization. We now look at the more interesting case where the natural rate of interest is negative.

Policy at the zero bound We have seen that, in order to implement the first best allocation with constant taxes, the nominal interest rate must equal the natural rate of interest, and prices must be constant. This implementation breaks down when the natural rate of interest turns negative, because of the zero lower bound. With constant taxes, this failure is unavoidable and optimal monetary policy can only achieve a second best allocation. Here we explain how flexible taxes can be used to completely circumvent the zero lower bound and implement the first best allocation.

The intertemporal condition (12) can be satisfied with zero nominal interest rates $i_t = 0$ and constant prices with the appropriate choice of consumption taxes over time:

$$\frac{u_C(C_t, N_t, \xi_t)}{(1 + \tau_t^c)} = E_t \frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{(1 + \tau_{t+1}^c)} \quad (22)$$

The intratemporal condition

$$\frac{u_C(C_t, N_t, \xi_t)}{u_L(C_t, N_t, \xi_t)} = \frac{(1 + \tau_t^c)}{(1 - \tau_t^n)} \frac{\theta}{(\theta - 1)} \frac{1}{A_t}, \quad (23)$$

obtained from (10) and (20) can then be satisfied by the choice of the labor income tax that sets $\frac{(1 + \tau_t^c)}{(1 - \tau_t^n)} \frac{\theta}{(\theta - 1)} = 1$.¹¹

Hence, as long as consumption and labor income taxes are flexible instruments, the first best can be achieved, and the zero bound is not a constraint to policy. In the rest of the paper, we will use the term *unconventional fiscal policy* to refer to the combination of taxes that circumvents the zero bound.

¹¹The taxes that implement the first best at the zero bound do not have to respond to contemporaneous information. Consumption and labor income taxes can be predetermined.

In order to build intuition for the required behavior of taxes, we now consider a special case of the model—the same considered by Eggertsson (2009) and Christiano et al. (2011) – and describe optimal tax policy following a shock that lowers the natural rate of interest to the point where the zero bound constraint would be binding.

2.7 Illustration: using fiscal policy to avoid a recession

As in Eggertsson (2009) and Christiano et al. (2011) we consider the particular preferences

$$u(C_t, N_t, \xi_t) = u(C_t, N_t) \xi_t. \quad (24)$$

In this way, the preference shock does not affect the marginal rate of substitution between consumption and leisure. It does, however, affect the marginal rate of substitution between consumption at time t and consumption at time $t + 1$. We also assume that $G_t = G$, $A_t = 1$, so that the only shock is the preference shock.

Note that in this case, the conditions (18) and (19) imply that the first best allocation is unaffected by the preference shock, and is constant.

Let us consider a particular example, a deterministic version of an example in Eggertsson (2009) and Christiano et al. (2011). In their models, it is this shock - interacting with the zero bound - that generates a potentially big recession. Assume that ξ_t evolves exogenously according to $\xi_t/\xi_{t+1} < \beta$ for $t = 0, 1, \dots, T - 1$, and $\xi_t/\xi_{t+1} = 1$ for $t \geq T$. The natural rate of interest is $\xi_t/\beta\xi_{t+1} - 1 < 0$ if $t \leq T - 1$ and $(1 - \beta)/\beta > 0$ for $t \geq T$.

We set the nominal interest rate to $i_t = 0$ for $t \leq T - 1$ and $i_t = \beta^{-1} - 1$ for $t \geq T$. We set the path of consumption taxes according to

$$\frac{1 + \tau_{t+1}^c}{1 + \tau_t^c} = \beta \frac{\xi_{t+1}}{\xi_t}, \text{ for } t = 0, 1, 2, \dots, T - 1,$$

and we set labor taxes as follows $\frac{(1 + \tau_t^c)}{(1 - \tau_t^n)} \frac{\theta}{(\theta - 1)} = 1$, for all t . For $t \geq T$, the taxes are constant.

Note that, in this deterministic case, there is one degree of freedom in the choice of tax policy: the initial level of the consumption tax, τ_0^c . Given an initial consumption tax, the equations above completely determine the paths of consumption and labor taxes. Consumption taxes increase over time for $t \leq T - 1$ and then stabilize at some level τ^c for $t \geq T$. Labor taxes follow the opposite pattern: they decrease over time for $t \leq T - 1$ and then stabilize at some level τ^n for $t \geq T$.

The key is that the prices that matter for intertemporal decisions are consumer prices, which are gross of consumption taxes. The idea is to induce inflation in consumer prices, while keeping producer price inflation at zero. The result is negative real interest rates, and the distortions associated with producer price inflation are altogether avoided.

This can be achieved by a simultaneous adjustment in consumption and labor taxes. An increasing path of consumption tax generates inflation in consumer prices. Why does the labor tax need to be adjusted? The

changes in the consumption tax introduce undesirable variations in the marginal cost of firms which increases over time in the absence of any adjustment in the labor tax. This effect must therefore be counteracted by engineering a decreasing path of labor taxes.

This policy resembles the sales tax holiday proposal by Hall and Woodward at the end of 2008 and Feldstein in 2003 addressing the Japanese stagnation in the nineties. To implement the first best, however, it is important to note that labor taxes must be adjusted in the opposite direction of consumption taxes so as not to distort the intratemporal margin.

We now make explicit some important properties of the optimal policy solution that we have characterized. We show that it has two important desirable features. First, it is time-consistent. Second, it is revenue-neutral. These properties enhance its practical applicability, and further distinguishes it from some alternative second-best policies that have been suggested in the literature.

2.8 Time-consistency

Importantly, because our policy implements the first best allocation, it is time-consistent. If a future planner were given an opportunity to revise this policy in the future, it would choose not to do so. This should be contrasted with the policy recommendations involving future commitments to low interest rates in Krugman (1998) and Eggertsson and Woodford (2003 and 2004a). These policies involve commitments to "being irresponsible" in the future by keeping the nominal interest rate below the natural rate of interest even when the latter turns back positive. When the future comes, a planner is tempted to renege on these commitments and raise interest rates as soon as the natural rate of interest turns positive.

This represents an additional advantage of unconventional fiscal policy. Not only does it deliver a better allocation (the first best one), it also has the benefit of not requiring costly commitments that might be difficult to make credible.

2.9 Revenue neutrality.

The tax policy that implements the first best allocation does not involve net taxing or subsidizing. Notice that the present value budget constraint of the household, with $\tau_t^d = 1$, can be written as

$$E_0 \sum_{t=0}^{\infty} Q_{0,t} [(1 + \tau_t^c) P_t C_t - (1 - \tau_t^n) W_t N_t] = \bar{B}_0 + B_{-1,0}.$$

Replacing prices and taxes from the household marginal conditions (10) and (11), as

$$E_0 \sum_{t=0}^{\infty} \beta^t [u_C(t) C_t - u_L(t) N_t] + E_0 \sum_{t=0}^{\infty} \beta^t u_C(t) \frac{T_t}{P_t} = \frac{\mathbb{W}_0 u_C(0)}{P_0 (1 + \tau_0^c)}. \quad (25)$$

The first best allocation is such that $C_t = A_t N_t - G_t$, and $u_L(t) = A_t u_C(t)$. This implies that¹²

$$E_0 \sum_{t=0}^{\infty} \beta^t u_C(t) \left(G_t - \frac{T_t}{P_t} \right) = -\frac{W_0 u_C(0)}{P_0 (1 + \tau_0^c)}.$$

Notice that τ_0^c is unrestricted by the implementation of the first best allocation whether at the zero bound, or away from it. It is a lump sum tax on the initial nominal wealth of households. The present value of lump sum taxes is equal to the present value of government spending plus the value of initial liabilities. It follows that the present value of the other taxes, used to implement the first best allocation, is zero. This is the case whether the allocation is implemented with interest rates away from the zero bound, or with consumption and labor income taxes. In this sense, tax policy that implements the first best allocation at the zero bound is revenue neutral.

There is another sense in which unconventional fiscal policy is revenue neutral. Away from the zero bound, if the natural rate of interest is positive, the allocation can be implemented either with time varying nominal interest rates for a constant tax policy, or with a time varying tax policy for a constant nominal interest rate. The budget constraint (25) is the same regardless of the particular implementation.

3 A model with capital

The model can easily be extended to allow for capital accumulation. However, time varying consumption taxes of the type we propose to circumvent the zero bound, will distort capital accumulation. To undo this distortion and achieve the first best, policy must include taxes that directly affect the marginal condition for capital accumulation. We consider taxes that exist in the current tax codes, such as a capital income tax and an investment tax credit. As we show below, while either one suffices for the theoretical results, they do have different quantitative implications.

We assume that total investment, I_t , is also an aggregate of the individual varieties,

$$I_t = \left[\int_0^1 i_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \quad (26)$$

Aggregate investment increases the capital stock according to

$$K_{t+1} = (1 - \delta) K_t + I_t. \quad (27)$$

Capital is accumulated by the representative household, that minimizes the expenditure on the individual investment goods by choosing

$$\frac{i_{it}}{I_t} = \left(\frac{p_{it}}{P_t} \right)^{-\theta}. \quad (28)$$

¹²Notice that the discounting of government consumption is the same as the one of private consumption in (25). This is the case at the first best.

We assume that the investment tax credit, s_t^I , applies to the gross investment made by the household, $P_t [K_{t+1} - (1 - \delta)K_t]$. In addition, the tax τ_t^k is levied on capital income, $(U_t K_t - \delta P_t K_t)$, where U_t is the rental cost of capital, with an allowance for depreciation. As before, there is a tax on pure profits¹³.

The period-by period budget constraint can, thus, be written as

$$\begin{aligned} & \frac{1}{1 + i_t} \bar{B}_{t+1} + E_t Q_{t,t+1} B_{t,t+1} + (1 - s_t^I) P_t K_{t+1} \\ \leq & \bar{B}_t + B_{t-1,t} + (1 - \delta) (1 - s_t^I) P_t K_t + \\ & (1 - \tau_t^k) U_t K_t + \tau_t^k \delta P_t K_t + (1 - \tau_t^n) W_t N_t + (1 - \tau_t^d) \Pi_t - (1 + \tau_t^c) P_t C_t - T_t. \end{aligned} \quad (29)$$

The marginal condition for capital accumulation by households is

$$P_t = E_t Q_{t,t+1} \left[\frac{(1 - s_{t+1}^I)}{(1 - s_t^I)} (1 - \delta) P_{t+1} + \frac{(1 - \tau_{t+1}^k) U_{t+1} + \tau_{t+1}^k \delta P_{t+1}}{(1 - s_t^I)} \right], t \geq 0. \quad (30)$$

A constant investment tax credit is equivalent to a constant subsidy to capital income gross of depreciation. However, a variable investment tax credit can have a larger impact on the return from capital accumulation. For instance, note that if the subsidy only lasts for one period -period t -, so that $s_{t+1}^I = 0$, the direct impact of the subsidy s_t^I also raises the value of the undepreciated capital.

The production of each good i , y_{it} , uses labor, n_{it} , and capital, k_{it} , and is given by

$$y_{it} = A_t F(k_{it}, n_{it}), \quad (31)$$

where A_t is an aggregate productivity shock and the production function is constant returns to scale. The demand for each variety is

$$\frac{y_{it}}{Y_t} = \left(\frac{p_{it}}{P_t} \right)^{-\theta}, \quad (32)$$

obtained from (7), (8), and (28), where $y_{it} = c_{it} + i_{it} + g_{it}$, and $Y_t = C_t + I_t + G_t$.

The firm choices must satisfy $\frac{k_{it}}{n_{it}} = \frac{K_t}{N_t}$ and

$$\frac{U_t}{W_t} = \frac{F_k \left(\frac{K_t}{N_t} \right)}{F_n \left(\frac{K_t}{N_t} \right)}. \quad (33)$$

The corresponding cost function is $C(y_{it}; U_t, W_t) = \frac{U_t}{F_k \left(\frac{K_t}{N_t} \right)} y_{it} = \frac{W_t}{F_n \left(\frac{K_t}{N_t} \right)} y_{it}$, so that marginal cost is a function of the aggregates only¹⁴. The optimal price set by the firms that are able to reset prices is

$$p_t = \frac{\theta}{(\theta - 1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{A_{t+j} F_n \left(\frac{K_{t+j}}{N_{t+j}} \right)}, \quad (34)$$

where $\eta_{t,j}$ are the same as in the model without capital, (14).

¹³In practice, it may be difficult to distinguish capital income from pure profits. However, a tax on profits can be replicated by a tax on total income from capital and profits, together with an investment subsidy.

¹⁴Since the production function is constant returns to scale, $F(k_{it}, n_{it}) = F_k \left(\frac{k_{it}}{n_{it}} \right) k_{it} + F_n \left(\frac{k_{it}}{n_{it}} \right) n_{it}$ and $\frac{k_{it}}{n_{it}} = \frac{K_t}{N_t}$ is the same across firms.

Market clearing for each variety implies that

$$c_{it} + g_{it} + i_{it} = A_t F(n_{it}, k_{it}), \quad (35)$$

while for capital and labor it must be that

$$K_t = \int_0^1 k_{it} di, \quad (36)$$

and (5) hold, respectively.

Using the demand functions (8), (7) and (28), as well as (27), it follows that

$$C_t + G_t + K_{t+1} - (1 - \delta) K_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di \right]^{-1} A_t F(K_t, N_t). \quad (37)$$

An equilibrium for $\{C_t, N_t, K_t\}$, $\{p_t, P_t, W_t, U_t\}$, and $\{i_t \geq 0, \tau_t^c, \tau_t^n, \tau_t^k, s_t^I\}$ is characterized by (10), (12), (15), (33), (34) and (14),

$$\frac{u_C(C_t, N_t, \xi_t)}{(1 + \tau_t^c)} = E_t \frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{(1 + \tau_{t+1}^c)} \left[\frac{(1 - s_{t+1}^I)}{(1 - s_t^I)} (1 - \delta) + \frac{(1 - \tau_{t+1}^k) \frac{U_{t+1}}{P_{t+1}} + \tau_{t+1}^k \delta}{(1 - s_t^I)} \right], \quad (38)$$

obtained from (11) and (30), together with

$$C_t + G_t + K_{t+1} - (1 - \delta) K_t = \left[\sum_{j=0}^{t+1} \varpi_j \left(\frac{p_{t-j}}{P_t} \right)^{-\theta} \right]^{-1} A_t F(K_t, N_t). \quad (39)$$

As before, we do not need to keep track of the budget constraints, since lump sum taxes adjust to satisfy the budget.

Efficient allocations. At the first best allocation, the marginal rate of technical substitution between any two varieties must be equal to one, so

$$c_{it} = C_t; g_{it} = G_t; i_{it} = I_t.$$

The efficiency conditions for the aggregates are:

$$\frac{u_C(C_t, N_t, \xi_t)}{u_L(C_t, N_t, \xi_t)} = \frac{1}{A_t F_n \left(\frac{K_t}{N_t} \right)}, \quad (40)$$

$$u_C(C_t, N_t, \xi_t) = E_t \beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1}) \left[A_{t+1} F_k \left(\frac{K_{t+1}}{N_{t+1}} \right) + 1 - \delta \right] \quad (41)$$

and

$$C_t + G_t + K_{t+1} - (1 - \delta) K_t = A_t F(K_t, N_t). \quad (42)$$

Policy away from the zero bound. We first set $\tau_t^c = 0$. As before, so as to achieve production efficiency, the price level must be constant. Since there is no price dispersion, the aggregate resource constraint (39) becomes (42). When $P_t = P$, (34) becomes

$$P = \frac{\theta}{(\theta - 1)} \frac{W_t}{A_t F_n \left(\frac{K_t}{N_t} \right)}, \quad (43)$$

so that nominal marginal cost must be constant, meaning that the nominal wage must be proportional to marginal productivity. From (10) and (43), it must be that

$$\frac{u_C(C_t, N_t, \xi_t)}{u_L(C_t, N_t, \xi_t)} = \frac{\frac{\theta}{(\theta-1)}}{(1 - \tau_t^n) A_t F_n \left(\frac{K_t}{N_t} \right)}, \quad (44)$$

implying that $\tau_t^n = \frac{-1}{\theta-1}$. The nominal interest rate must move with the real rate to satisfy

$$u_C(C_t, N_t, \xi_t) = (1 + i_t) E_t [\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})].$$

The rental cost of capital satisfies (33).

Finally, for a given s_t^I , the tax rate on capital income, τ_{t+1}^k , and the investment tax credit, s_{t+1}^I , must be chosen to satisfy the marginal condition for capital (38), repeated here,

$$\begin{aligned} & u_C(C_t, N_t, \xi_t) \\ = & E_t \beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1}) \left[\frac{(1 - s_{t+1}^I)}{(1 - s_t^I)} (1 - \delta) + \frac{(1 - \tau_{t+1}^k) \frac{\theta-1}{\theta} A_{t+1} F_k \left(\frac{K_{t+1}}{N_{t+1}} \right) + \tau_{t+1}^k \delta}{(1 - s_t^I)} \right]. \end{aligned} \quad (45)$$

If the capital income tax was not used, $\tau_t^k = 0$, a constant investment tax credit, $s^I = \frac{1}{\theta}$, would satisfy the condition at the first best.

Instead, if $s_t^I = 0$ for all t , then the capital income tax, τ_{t+1}^k , must be moving with shocks in order to implement the first best allocation. It is no longer the case that the first best can be implemented with constant taxes, away from the zero bound¹⁵. This is the case because we assume, as is standard, that firms can deduct depreciation expenses from the capital income tax/subsidy, i.e., the tax is paid on $(U_t - \delta P_t) K_t$. If, instead, the tax was paid on the gross return $U_t K_t$, the last term in the right hand side of (45), $\frac{\tau_{t+1}^k \delta}{(1 - s_t^I)}$, would not be there, and setting a constant tax, τ^k , with $(1 - \tau^k) \frac{\theta-1}{\theta} = 1$, would be consistent with the optimal allocation.

Policy at the zero bound. When the natural rate of interest is negative, the first best can be implemented with time-varying taxes, as in the simple model with labor only. The intertemporal condition for noncontingent bonds with a constant price level, (22), when $i_t = 0$, imposes the same restrictions on the path of consumption taxes. As before, the labor income tax will have to move to compensate for the movements

¹⁵Standard New Keynesian models usually have labor only and assume taxes are not flexible. If instead they considered capital, the nonflexibility of taxes would be costly.

in the consumption tax, satisfying

$$\frac{u_C(C_t, N_t, \xi_t)}{u_L(C_t, N_t, \xi_t)} = \frac{(1 + \tau_t^c) \frac{\theta}{(\theta-1)}}{(1 - \tau_t^n) A_t F_n \left(\frac{K_t}{N_t} \right)}, \quad (46)$$

which is the analog to (23), in the model with labor only. Both taxes can be predetermined.

Either the capital income tax or the investment tax credit can be used to compensate for the changes in the consumption tax. For example, if we set $\tau_t^k = 0$ for all t , the investment tax credit must move so as to satisfy the condition for capital accumulation:

$$\frac{u_C(C_t, N_t, \xi_t)}{(1 + \tau_t^c)} = E_t \left\{ \frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{(1 + \tau_{t+1}^c)} \left[\frac{(1 - s_{t+1}^I)}{(1 - s_t^I)} (1 - \delta) + \frac{1}{(1 - s_t^I)} \frac{\theta - 1}{\theta} A_{t+1} F_k \left(\frac{K_{t+1}}{N_{t+1}} \right) \right] \right\}. \quad (47)$$

Note that while efficient investment tax policy can be done, away from the zero bound, with a constant subsidy, at the zero bound the investment tax credit cannot be constant. However, it can be predetermined.

To summarize, when the zero bound is temporarily binding, we must supplement the flexibility in consumption and labor income taxes with flexible taxes on investment or on capital income. The reason is simple. When capital is introduced in the model, the increasing path of consumption taxes that is necessary to circumvent the zero bound constraint, acts as an undesirable tax on capital. Its effects on capital accumulation must therefore be counteracted with an offsetting capital income subsidy. This subsidy must remain in place as long as consumption taxes are increasing.

4 Sticky wages

We have assumed so far that prices are sticky but wages are fully flexible. But, what if the relevant nominal friction was sticky wages rather than sticky prices? Would it still be possible to do efficient policy at the zero bound? What fiscal instruments would be necessary? We answer these questions below.

An environment with sticky wages. In order to allow for sticky wages, we now consider a single household with a continuum of members indexed by $h \in [0, 1]$, each supplying a differentiated labor input n_{ht} . Preferences of the household are described by (1), where leisure is equal to total time, normalized to one, minus time spent working $L_t \equiv 1 - \int_0^1 n_{ht} dh$.

The differentiated labor varieties aggregate up to the labor input N_t , used in production, according to the Dixit-Stiglitz aggregator

$$N_t = \left[\int_0^1 n_{ht}^{\frac{\theta^w - 1}{\theta^w}} dh \right]^{\frac{\theta^w}{\theta^w - 1}}, \quad \theta^w > 1. \quad (48)$$

There is a single good produced, Y_t , that uses labor, N_t , and capital, K_t , and that can be used for private or public consumption and investment, according to

$$C_t + G_t + K_{t+1} - (1 - \delta) K_t = Y_t = A_t F(K_t, N_t). \quad (49)$$

The good is produced by a representative firm that behaves competitively. Each member of the household, that supplies a differentiated labor variety, behaves under monopolistic competition. They set wages as in Calvo (1983), with probability of being able to revise the wage $1 - \alpha^w$. The households that are not able to set wages in period 0, all share the same wage w_{-1} . Other prices are taken as given. There is a complete set of state contingent assets. We consider an additional tax, a payroll tax on the wage bill paid by firms, τ_t^p . As long as an investment tax credit is used, the capital income tax is redundant, so we set it to zero, $\tau_t^k = 0$.

Efficiency. The feasible allocation that now maximizes the utility of the representative household must have

$$n_{ht} = N_t, \text{ for all } h, \text{ and } t \quad (50)$$

These conditions equalize marginal rates of substitution to marginal rates of transformations across varieties of labor. Furthermore, the same efficiency conditions for the aggregates, as above, (40), (41), (42), must hold.

Equilibria with sticky wages. The representative firm minimizes $\int_0^1 w_{ht} n_{ht} dh$, where w_{ht} is the wage of the h -labor, for a given aggregate N_t , subject to (48). The demand of n_{ht} is

$$n_{ht} = \left(\frac{w_{ht}}{W_t} \right)^{-\theta^w} N_t, \quad (51)$$

where W_t is the aggregate wage level, given by

$$W_t = \left[\int_0^1 w_{ht}^{1-\theta^w} dh \right]^{\frac{1}{1-\theta^w}}. \quad (52)$$

It follows that $\int_0^1 w_{ht} n_{ht} dh = W_t N_t$.

The firm maximizes profits, subject to the production function (49), taking prices of both output, P_t , and inputs, W_t and U_t , as given. This implies

$$\frac{U_t}{(1 + \tau_t^p) W_t} = \frac{F_k \left(\frac{K_t}{N_t} \right)}{F_n \left(\frac{K_t}{N_t} \right)} \quad (53)$$

and

$$P_t = \frac{(1 + \tau_t^p) W_t}{A_t F_n \left(\frac{K_t}{N_t} \right)}. \quad (54)$$

The period-by-period budget constraints for the household are (29). We assume that wages are set before labor income taxes¹⁶. The optimal wage setting conditions by the monopolistic competitive workers are now

$$w_t = \frac{\theta^w}{(\theta^w - 1)} E_t \sum_{j=0}^{\infty} \eta_{t,j}^w \frac{u_L(t+j)}{u_C(t+j)} \frac{(1 + \tau_{t+j}^c) P_{t+j}}{(1 - \tau_{t+j}^n)}, \quad (55)$$

¹⁶This assumption is important. It is analogous to the assumption that prices are set before consumption taxes.

and

$$\eta_{t,j}^w = \frac{(1 - \tau_{t+j}^n) (\alpha^w \beta)^j \frac{u_C(t+j)}{(1+\tau_{t+j}^c)P_{t+j}} (W_{t+j})^{\theta^w} N_{t+j}}{E_t \sum_{j=0}^{\infty} (1 - \tau_{t+j}^n) (\alpha^w \beta)^j \frac{u_C(t+j)}{(1+\tau_{t+j}^c)P_{t+j}} (W_{t+j})^{\theta^w} N_{t+j}}. \quad (56)$$

The wage level (52) can be written as

$$W_t = \left[(1 - \alpha^w) w_t^{1-\theta^w} + \alpha^w W_{t-1}^{1-\theta^w} \right]^{\frac{1}{1-\theta^w}}. \quad (57)$$

The marginal condition for the accumulation of capital is (30) with $\tau_t^k = 0$, the intertemporal condition for noncontingent bonds is (12), while market clearing for the final good is given by (49).

From (51), we can write

$$N_t = \left[\int_0^1 \left(\frac{w_{ht}}{W_t} \right)^{-\theta^w} dh \right]^{-1} (1 - L_t). \quad (58)$$

From the expression for the wage level, (52), it follows that $\int_0^1 \left(\frac{w_{ht}}{W_t} \right)^{-\theta^w} dh$ is greater than or equal to one. It is strictly greater than one when there is wage dispersion, $\frac{w_{ht}}{W_t} \neq 1$ for a set of positive measure of workers. This means that for a given level of total time working $1 - L_t$, the resources available for production are maximized when there is no wage dispersion. Using (58), we can write the resources constraint as

$$C_t + G_t + K_{t+1} - (1 - \delta) K_t = A_t F \left(K_t, \left[\int_0^1 \left(\frac{w_{ht}}{W_t} \right)^{-\theta^w} dh \right]^{-1} (1 - L_t) \right), \quad (59)$$

which is, for sticky wages, the analog to the resource constraint with a cost of price dispersion under sticky prices, (39).

An equilibrium for $\{C_t, L_t, N_t, K_t\}$, $\{P_t, w_t, W_t, U_t\}$, $\{i_t \geq 0, \tau_t^c, \tau_t^n, \tau_t^p, s_t^I\}$, is characterized by conditions (12), (45) with $\tau_t^k = 0$, (55) and (56), (57), (53), (54), (59), and

$$1 - L_t = N_t \sum_{j=0}^{t+1} \varpi_j^w \left(\frac{w_{t-j}}{W_t} \right)^{-\theta},$$

where ϖ_j^w is the share of household members that have set wages j periods before, $\varpi_j^w = (\alpha^w)^j (1 - \alpha^w)$, $j = 0, 2, \dots, t$, and $\varpi_{t+1}^w = (\alpha^w)^{t+1}$, which is the share of workers that have never set wages. They all charge an exogenous wage w_{-1} . As before, we do not need to keep track of the budget constraints, since lump sum taxes adjust to satisfy the budget.

We can now show that it is possible to use labor income and payroll taxes to implement the first best allocation when the zero bound constraint binds.

Implementing the first best when wages are sticky. Efficiency requires that labor be the same across varieties. In order for this to be an equilibrium outcome it must be that wages be constant over time, $w_t = w_{-1} = W$. It is useful to write the wage setting condition, (55), in recursive form, as

$$w_t = \eta_{t,0}^w \frac{\theta^w}{\theta^w - 1} \frac{u_L(t)}{u_C(t)} \frac{(1 + \tau_t^c) P_t}{(1 - \tau_t^n)} + (1 - \eta_{t,0}^w) E_t w_{t+1}. \quad (60)$$

It follows that for $w_t = w_{-1}$, the wage setting condition is the same as if wages were flexible

The conditions above that define an equilibrium for $\{C_t, N_t, K_t\}, \{P_t, W = w_{-1}, U_t\}, \{i_t \geq 0, \tau_t^c, \tau_t^n, \tau_t^p, s_t^I\}$ become

$$\frac{u_C(C_t, N_t, \xi_t)}{(1 + \tau_t^c) P_t} = (1 + i_t) E_t \left[\frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{(1 + \tau_{t+1}^c) P_{t+1}} \right], \quad (61)$$

$$A_t F_n \left(\frac{K_t}{N_t} \right) = \frac{\theta^w}{\theta^w - 1} \frac{(1 + \tau_t^c)(1 + \tau_t^p)}{(1 - \tau_t^n)} \frac{u_L(C_t, N_t, \xi_t)}{u_C(C_t, N_t, \xi_t)}, \quad (62)$$

$$\frac{u_C(C_t, N_t, \xi_t)}{(1 + \tau_t^c)} = E_t \left\{ \frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{(1 + \tau_{t+1}^c)} \left[\frac{(1 - s_{t+1}^I)}{(1 - s_t^I)} (1 - \delta) + \frac{1}{(1 - s_t^I)} A_{t+1} F_k \left(\frac{K_{t+1}}{N_{t+1}} \right) \right] \right\}, \quad (63)$$

together with the firms' conditions (54) and (53), and the market clearing condition (49).

Away from the zero bound, the first best can be implemented with a state-contingent nominal interest rate and a constant subsidy, that eliminates the markup distortion. The constant subsidy can be given with any of the taxes, on consumption, labor income, or the payroll tax. The investment tax credit must be set to zero. The price level, P_t , must move with the shocks to satisfy (54).

When nominal interest rates are zero, it is possible to use unconventional fiscal policy to implement the first best exactly as in the case of sticky prices and flexible wages that we analyzed before—an interesting robustness property of unconventional fiscal policy. The same taxes that were used above when prices were sticky, can be used when wages are sticky. To see this, set the payroll tax to zero $\tau_t^p = 0$, then given τ_t^c , (61) can be satisfied by an appropriate choice of τ_{t+1}^c , (62) by an appropriate choice of τ_t^n , (63) by an appropriate choice of s_{t+1}^I , and (54) by P_t .

With sticky wages and flexible prices, it is also possible to use a different combination of taxes to implement the first best. Indeed, it is possible to use only labor income and payroll taxes. Let the nominal interest rate be some sequence with $i_t \geq 0$, and let $\tau_t^c = s_t^I = 0$, for all s^t . Then (61) would be satisfied by P_{t+1} , (54) is satisfied by τ_t^p , (62) is satisfied by τ_t^n , and (53) by U_t . The only restrictions on the equilibrium allocations are the resource constraints (49) together with (63) once $\tau_t^c = s_t^I = 0$. These are all conditions that are satisfied at the first best. When the nominal interest rate is zero, the real interest rate can still be negative as long as there is consumer price inflation. As before, that can be accomplished by consumption taxes that increase over time. But with flexible prices, it can also be accomplished by costless producer price inflation (equation (61)). However, because flexible-price firms set the price as a mark up over marginal cost (equation (54)), there is price inflation if there is inflation in wages gross of payroll taxes. Inflation in payroll taxes avoids the costs of wage dispersion. The labor income tax is used to compensate for the distortion in the intratemporal margin created by the payroll tax, as is clear from (62). In order to eliminate the mark up distortion in this margin, it must be that $\frac{(\theta^w - 1)(1 + \tau_t^p)}{\theta^w(1 - \tau_t^n)} = 1$, so that the increasing payroll tax is compensated by a decreasing labor income tax.

Both sticky prices and wages. The case of both sticky prices and wages is of a different nature. It is still possible to use unconventional fiscal policy to replicate the effects of negative nominal interest rates

and hence circumvent the zero bound constraint. This can be done through the same combination of time-varying consumption and labor taxes, and of a time varying investment tax credit, that we considered above. However, replicating the effects of negative nominal interest rate is not enough to implement the first best solution. Indeed, even away from the zero bound, monetary policy alone cannot undo the effects of the real rigidities caused by both sticky prices and wages. It would still be possible to achieve the first best, but it would be necessary to use state-contingent fiscal policy also when the zero bound constraint is not binding. When both prices and wages are rigid, it must be that both producer prices and wages before labor taxes are constant. If tax policy was not state contingent, equations (62) and (54) would imply that marginal rates of substitution and transformation could not be state contingent, which is obviously not the case in general for the first best allocation.

When there are both sticky prices and wages, the first best is described by the same equations above, with two differences. The first is that the markup in the goods prices must be taken into account. This means that equation (62) is replaced by

$$\frac{u_L(C_t, L_t, \xi_t)}{u_C(C_t, L_t, \xi_t)} = \frac{(\theta - 1)(\theta^w - 1)}{\theta} \frac{(1 - \tau_t^n)}{\theta^w} \frac{A_t F_n \left(\frac{K_t}{N_t} \right)}{(1 + \tau_t^p)(1 + \tau_t^c)}, \quad (64)$$

(63) is replaced by (47), and equation (54), by

$$P = \frac{\theta}{(\theta - 1)} \frac{(1 + \tau_t^p) W}{A_t F_n \left(\frac{K_t}{N_t} \right)}. \quad (65)$$

The second difference is that the price level in (61) must be constant over time, $P_t = P = p_{-1}$, as it is in (65).

Away from the zero bound, in order to implement the first best, it is enough to use payroll and labor income taxes. Payroll taxes satisfy condition (65); labor income taxes satisfy condition (64) with $\frac{(\theta-1)(\theta^w-1)}{\theta} \frac{(1-\tau_t^n)}{\theta^w} = 1$, so that (64) coincides with the intratemporal efficiency condition (40). There must also be a constant investment subsidy, $s^I = \frac{1}{\theta}$, so that (47) becomes the intertemporal efficiency condition (41). The nominal interest rate must be equal to the real, $1 + i_t = \frac{u_C(C_t, N_t, \xi_t)}{E_t[\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})]}$, to satisfy (61).

At the zero bound, real interest rates can be negative when producer prices are stable, if consumption taxes grow over time (equation (61)). When both prices and wages are stable, the necessary adjustments in the real wage faced by the firms are achieved with the payroll tax, so that for example, wages gross of taxes paid by the firm may be lowered when marginal labor productivity is low (equation (54)). The other taxes must compensate for the distortions created by the consumption taxes and the payroll taxes, so that the efficiency conditions (40) and (41) may be verified. Condition (64) can be satisfied with a choice of τ_t^n so that (40) holds. The joint taxes τ_t^c , τ_t^n , and τ_t^p , must be such that production is subsidized and the joint markup distortions from the production and labor markets are eliminated. The intertemporal condition (47) is satisfied by the investment subsidy s_{t+1}^I , so that (41) holds. The resource constraint is the only remaining restriction on the optimal policy problem. It follows that the resulting allocation is the one corresponding to the first best.

5 Second best

We have shown that tax policy can be used to achieve full efficiency, when nominal interest rates are at the zero bound. But in order for that to be the case, the government must be able to raise lump sum taxes to pay for government spending and the subsidies that eliminate the mark up distortion. In this section we impose the restrictions on taxes in the second best Ramsey literature, assuming away lump sum taxes, so that $T_t = 0$ for $t \geq 0$. We show that the zero bound does not restrict the second best problem. While the optimal allocation and relative prices are in general different from before¹⁷, we show that the way taxes are used to overcome the zero bound is essentially the same.

Equilibria. We study the model with capital but, for simplicity, we allow for flexible wages, so that payroll taxes are redundant. Thus, we only consider taxes on consumption, labor income, capital income, investment, and profits. This is the model in Section 3, with the only difference that $T_t = 0$ for $t \geq 0$. The household budget constraint is (29) with $T_t = 0$, together with a terminal condition $\lim_{T \rightarrow \infty} Q_{0,T+1} \bar{B}_T + E_0 Q_{0,T+1} B_{T,T+1} \geq 0$. The present value budget constraint for the household as of period zero, with equality, is

$$E_0 \sum_{t=0}^{\infty} Q_{0,t} [(1 + \tau_t^c) P_t C_t - (1 - \tau_t^n) W_t N_t] = E_0 \sum_{t=0}^{\infty} Q_{0,t} (1 - \tau_t^d) \Pi_t$$

$$\bar{B}_0 + B_{-1,0} + (1 - \delta) (1 - s_0^I) P_0 K_0 + (1 - \tau_0^k) U_0 K_0 + \tau_0^k \delta P_0 K_0.$$

Using the marginal conditions of the household (10), as well as (11) which implies

$$Q_{0,t} \frac{(1 + \tau_t^c) P_t}{(1 + \tau_0^c) P_0} = \frac{\beta^t u_C(C_t, N_t, \xi_t)}{u_C(C_0, N_0, \xi_0)},$$

we obtain

$$E_0 \sum_{t=0}^{\infty} \beta^t [u_C(C_t, N_t, \xi_t) C_t - u_L(C_t, N_t, \xi_t) N_t] = E_0 \sum_{t=0}^{\infty} \beta^t u_C(C_t, N_t, \xi_t) \frac{(1 - \tau_t^d) \Pi_t}{(1 + \tau_t^c) P_t} +$$

$$u_C(C_0, N_0, \xi_0) \frac{\bar{B}_0 + B_{-1,0}}{(1 + \tau_0^c) P_0} + u_C(C_0, N_0, \xi_0) \left[(1 - \delta) \frac{(1 - s_0^I) K_0}{(1 + \tau_0^c)} + \frac{(1 - \tau_0^k) \frac{U_0}{P_0} + \tau_0^k \delta}{(1 + \tau_0^c)} K_0 \right], \quad (66)$$

where

$$\Pi_t = \int_0^1 \left(p_{it} - \frac{W_t}{A_t F_n(t)} \right) \left(\frac{p_{it}}{P_t} \right)^{-\theta} Y_t di.$$

Circumventing the zero bound constraint at the second best. We start by showing that it is optimal for the Ramsey planner to fully tax profits in every state. In this model, current firms profits are pure rents, so that they should be taxed fully. However, future profit taxes are distortionary, since, as equation (14) makes clear, they affect the weights in the price setting decision of firms. In this second best environment, it could be optimal for the Ramsey planner to abstain from fully taxing profits in order to obtain a particular effect on the prices set by firms. It turns out, however, that it is optimal to tax all profits at the limiting

¹⁷Even if $T_t = 0$, for all t , under certain conditions the first best may be attained.

rate of 100%. The reason is that whatever is the effect that profit taxes have on the prices set by the firms, it can alternatively be accomplished with state contingent consumption taxes.

The price setting condition is affected by the profit taxes through the weights defined in (14). Those weights can also be written recursively as

$$\frac{1}{\eta_{t,0}} = 1 + \alpha E_t \left[\frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{u_C(C_t, N_t, \xi_t)} \frac{P_t}{P_{t+1}} \frac{(1 - \tau_{t+1}^d)(1 + \tau_t^c)}{(1 - \tau_t^d)(1 + \tau_{t+1}^c)} \left(\frac{P_{t+1}}{P_t}\right)^\theta \frac{Y_{t+1}}{Y_t} \frac{1}{\eta_{t+1,0}} \right]. \quad (67)$$

Using the intertemporal condition (12), the zero bound constraint can be written as

$$E_t \left[\frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{u_C(C_t, N_t, \xi_t)} \frac{P_t}{P_{t+1}} \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right] \leq 1, \quad (68)$$

This and (67) are the only equilibrium conditions that depend on $\{\tau_t^d\}$ and on $\{\tau_t^c\}$, other than the implementability constraint (66). This is so, because the intratemporal condition

$$\frac{u_C(C_t, N_t, \xi_t)}{u_L(C_t, N_t, \xi_t)} = \frac{(1 + \tau_t^c) P_t}{(1 - \tau_t^n) W_t}, \quad (69)$$

does not restrict either the prices or the allocation since there is always a path $\{\tau_t^n\}$ that satisfies it.

Let us now take an arbitrary sequence $\left\{ \frac{1 - \tau_{t+1}^d}{1 - \tau_t^d} \right\}_{t=0}^\infty$. For any sequence of prices and allocation, the conditions (67) and (68) can both be satisfied by a choice of the sequence of consumption taxes $\{\tau_t^c\}_{t=0}^\infty$. To be more specific, consider a particular history s^t and assume that τ_t^c has been chosen, with $\tau_0^c > -1$. The system (67) and (68) with equality can always be solved for τ_{t+1}^c , as long as the two equations are not colinear. A necessary and sufficient condition is that $\frac{1 - \tau_{t+1}^d}{1 - \tau_t^d} \left(\frac{P_{t+1}}{P_t}\right)^\theta \frac{Y_{t+1}}{Y_t} \frac{1}{\eta_{t+1,0}}$ is not constant across the possible continuation histories, or in other words that this date $t + 1$ random variable is not perfectly predictable at time t . This is true in general in a stochastic economy. We have just shown that the only equilibrium condition restricting $\{\tau_t^d\}$ is (66).

For any sequence $\left\{ \frac{1 - \tau_{t+1}^d}{1 - \tau_t^d} \right\}_{t=0}^\infty$, we can pick $\tau_0^d \rightarrow 1$, implying that $\tau_t^d \rightarrow 1$, for all t . This makes the value of after tax profits go to zero, which is optimal as long as aggregate Π_t is positive. We assume this to be the case, since otherwise the firms that do not set prices would be having very negative profits because of the price setting restrictions¹⁸.

Notice that when we claim that $\{\tau_t^c\}$ can be picked to satisfy the equilibrium condition (68) for any sequence of prices and allocation, we are also showing that the zero bound constraint is not a restriction on the set of equilibrium allocations and prices.

We now show that it is optimal to eliminate price dispersion at the second best solution. In order for all prices to be equal it must be that $p_{it} = P_t = p_{-1}$. With no price dispersion, the price setting condition (34) becomes

$$p_{-1} = \frac{\theta}{\theta - 1} \frac{W_t}{A_t F_n}, \quad (70)$$

¹⁸The losses would have to be larger than the positive profits of the firms that set prices.

and the resource constraints are

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t = A_t F(N_t, K_t). \quad (71)$$

The implementability conditions for the set of equilibrium variables $\{C_t, N_t, K_t\}$, W_0 , and $\{\tau_0^c, \tau_0^k, s_0^I\}$ are

$$E_0 \sum_{t=0}^{\infty} \beta^t [u_C(C_t, N_t, \xi_t) C_t - u_L(C_t, N_t, \xi_t) N_t] = \quad (72)$$

$$u_C(C_0, N_0, \xi_0) \left[\frac{\bar{B}_0 + B_{-1,0}}{(1 + \tau_0^c) P_0} + \frac{(1 - \delta)(1 - s_0^I)K_0}{(1 + \tau_0^c)} + \frac{(1 - \tau_0^k) \frac{F_k W_0}{F_n P_0} K_0 + \tau_0^k \delta K_0}{(1 + \tau_0^c)} \right],$$

with $P_0 = p_{-1}$ and W_0 satisfying (70) for $t = 0$, together with the resource constraints (71). The other equilibrium conditions are satisfied by other equilibrium prices or taxes. In particular, the marginal condition for capital accumulation (38) is satisfied by τ_{t+1}^k , or s_{t+1}^I ; and the price setting condition (70) for $t \geq 1$ is satisfied by W_t .

With price dispersion, (72) is still an implementability condition, but P_0 is not the exogenous initial price p_{-1} , and W_0 is not given by (70) for $t = 0$. Furthermore, the resource constraint has the additional term corresponding to the cost of price dispersion in (37), repeated here

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di \right]^{-1} A_t F(N_t, K_t).$$

In this case, the implementable set cannot be summarized by a small set of constraints. There are other implementability constraints. It is straightforward to show that price dispersion is not part of the second best solution. To see this notice, first, that for any P_0 and W_0 , the initial taxes τ_0^c , τ_0^k , and s_0^I , can be chosen so that the value of the right hand side of the implementability condition (72) remains unchanged. Thus, this condition is the same with or without price dispersion. But the resource constraint with price dispersion will have $\left[\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di \right]^{-1} < 1$, which reduces resources available for consumption and leisure. It follows that it is optimal to eliminate price dispersion, even if the solution is a second best. The reason for this is that, when final goods can be taxed, it is not optimal to distort production, as in Diamond and Mirrlees (1971).

We have shown that second best policy keeps the price level stable over time, just as in the first best. The unconventional fiscal policy that overcomes the zero bound is described in exactly the same way. When the real interest rate, that in general will be different from the one in the first best, is positive, the nominal interest rate will be equal to the real interest rate and consumption taxes will not have to be used for this purpose. This would be second best policy in normal times. But when real interest rates at the second best become negative, consumption taxes have to be used to overcome the zero bound constraint on the nominal rate and, as before, other taxes will be used to compensate for the undesired distortions caused on other margins. In addition to overcoming the zero bound, taxes in this second best solution will in general be moving with the shocks¹⁹.

¹⁹Notice that the initial taxes, τ_0^c , s_0^I and τ_0^k , are lump sum taxes, and that under certain conditions it may be possible to achieve the first best.

Time-inconsistency. When the policy instruments include lump-sum taxes, there is (unconventional) fiscal policy that fully neutralizes the effects of the zero bound constraint, allows to achieve the first best, and removes the only source of time inconsistency. Instead, when lump sum taxes are not admissible, as we assumed in this section, the first best may not be achieved and, in general, optimal policy is time inconsistent. However, as we have just shown, the zero bound is not a restriction to the second best problem. Just observing the implementability condition (72), it is clear that the initial taxation of capital is a source of time inconsistency, as is the initial depletion of nominal debt. However, the time inconsistency of policy is unrelated to the zero bound constraint.

Revenue-neutrality There is a sense in which unconventional tax policy at the zero bound is revenue neutral: The implementability condition obtained from the budget constraint is the same regardless of real interest rates being positive or negative, regardless of policy being conducted with interest rates or with taxes.

6 More general results

So far we have shown that tax policy can be used to overcome the zero bound constraint and implement either the first best or a second best solution where price dispersion is not optimal. But the irrelevance of the zero bound constraint is a far more general result. It holds for the whole set of implementable allocations, regardless of how distorted they may be, whether prices are stable or not. To make this point, we now consider a more general environment where it is neither feasible, nor optimal to eliminate price dispersion. Specifically, we analyze the model of Section 3, but allow for idiosyncratic technology shocks and a nondegenerate initial distribution of prices. We also rule out lump sum taxes. We consider taxes on consumption, investment, labor and capital income, and profits.

The production function of each good i , now, uses labor, n_{it} , and capital, k_{it} , according to

$$c_{it} + i_{it} + g_{it} = y_{it} = A_t A_{it} F(k_{it}, n_{it}), \quad (73)$$

where A_{it} is a firm specific productivity shock that is *i.i.d.* across firms.

Let $\gamma_{it} \in \{0, 1\}$ be *i.i.d.* random variables, such that, if $\gamma_{it} = 1$, the firm can revise the price. Marginal cost is now firm specific, $MC_i(U_t, W_t) = \frac{U_t}{A_t A_{it} F_k(t)} = \frac{W_t}{A_t A_{it} F_n(t)}$. The firms that are able to change prices choose the price p_{it}^* to maximize profits where output y_{it+j} must satisfy the technology constraint (73), and the demand function, obtained from (7), (8), and (28) where $Y_t = C_t + I_t + G_t$. The optimal price set by these firms is

$$p_{it}^* = \frac{\theta}{(\theta - 1)} E_t \sum_{j=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{A_{t+j} A_{it+j} F_n(t+j)}, \quad (74)$$

where the weights are given by (14). The price of firm i is $p_{it} = p_{it}^*$ if $\gamma_{it} = 1$, and $p_{it} = p_{it-1}$, otherwise.

The present value budget constraint for the household as of period zero, with equality, is equation (66), where

$$\Pi_t = \int_0^1 \left(p_{it} - \frac{W_t}{A_t A_{it} F_n(t)} \right) \left(\frac{p_{it}}{P_t} \right)^{-\theta} Y_t di. \quad (75)$$

Using the demand functions (8), (28) and (7), and (73), it follows that

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} A_{it}^{-1} di \right]^{-1} A_t F(N_t, K_t). \quad (76)$$

In this economy production efficiency cannot be achieved. With idiosyncratic shocks, if prices were flexible, then firms would set

$$p_{it} = \frac{\theta}{\theta - 1} \frac{W_t}{A_t A_{it} F_n(t)}.$$

The resource constraints would be written as

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t = \left[\int_0^1 (A_{it})^{\theta-1} di \right]^{\frac{1}{\theta-1}} A_t F(N_t, K_t), \quad (77)$$

to be compared with (76). With sticky prices, it is impossible to replicate the volatile pattern of relative prices needed to have production efficiency²⁰.

Equilibria. An equilibrium for $\{C_t, N_t, K_t\}$, $\{p_{it}, p_{it}^*, P_t, W_t, U_t\}$, and $\{i_t, \tau_t^c, \tau_t^n, \tau_t^k, s_t^I, \tau_t^d\}$ is characterized by the household marginal conditions (10), (12), with $i_t \geq 0$, (38), the firms marginal condition for the choice of factors (33), the price setting constraint (74), where $Y_t = C_t + K_{t+1} - (1 - \delta)K_t + G_t$, the condition for the price level (6), the budget constraint (66), and, finally, the resource constraints (76), with $p_{it} = p_{it}^*$ if $\gamma_{it} = 1$, and $p_{it} = p_{it-1}$, otherwise. If, at time zero, firm i cannot optimally choose the price, because $\gamma_{i0} = 0$, then $p_{i0} = p_{i,-1}$.

Circumventing the zero bound constraint. In order to show that the zero bound does not restrict the set of allocations, consider an allocation and a sequence of prices $\{p_{it}, p_{it}^*, P_t, W_t, U_t, C_t, N_t, K_t\}$ that satisfy the equilibrium conditions above, but do not necessarily satisfy the zero bound constraint. Let $\{\tau_t^c, \tau_t^n, \tau_t^k, s_t^I, \tau_t^d, i_t\}$ be a sequence of taxes and nominal interest rates that supports those prices and allocation. The same allocation and process for prices can be implemented with another sequence for taxes and interest rates $\{\tilde{\tau}_t^c, \tilde{\tau}_t^n, \tilde{\tau}_t^k, \tilde{s}_t^I, \tilde{\tau}_t^d, \tilde{i}_t\}$, in such a way that the zero bound constraint is satisfied. We now explain how to construct consumption, profit, and labor and capital income taxes that implement the original allocation with the new interest rate $\tilde{i}_t = \max\{i_t, 0\}$.

In order to perform this construction recursively, consider a history s^t , and assume that $\tilde{\tau}_t^c$ has been chosen, with $\tilde{\tau}_0^c = \tau_0^c$. We construct $\tilde{\tau}_{t+1}^c$ in such a way that

$$\frac{1}{1 + \tilde{i}_t} = E_t \left[\frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{u_C(C_t, N_t, \xi_t)} \frac{P_t}{P_{t+1}} \frac{1 + \tilde{\tau}_t^c}{1 + \tilde{\tau}_{t+1}^c} \right]$$

²⁰This volatility would require a richer set of instruments, i.e. consumption and labor taxes specific to each firm in the economy.

holds. The ratio $\frac{1+\tilde{\tau}_{t+1}^c}{1+\tilde{\tau}_t^c}/\frac{1+\tau_{t+1}^c}{1+\tau_t^c}$ can be predetermined, which we assume is the case. Given a path for the consumption tax $\{\tilde{\tau}_t^c\}$, it is possible to find a path for the profit tax $\{\tilde{\tau}_t^d\}$ such that

$$\frac{1-\tilde{\tau}_t^d}{1+\tilde{\tau}_t^c} = \frac{1-\tau_t^d}{1+\tau_t^c}.$$

This keeps the weights in the price setting equation unchanged (14), and, therefore, also satisfies (74). It also verifies the budget constraint (66). We then set labor taxes as

$$\frac{1-\tilde{\tau}_t^n}{1+\tilde{\tau}_t^c} = \frac{1-\tau_t^n}{1+\tau_t^c}. \quad (78)$$

that satisfies the household's intratemporal condition (10). Finally, we set $\tilde{\tau}_t^k = \tau_t^k$, for $t \geq 0$, and $\tilde{s}_0^I = s_0^I$, and construct \tilde{s}_{t+1}^I , given \tilde{s}_t^I , to satisfy

$$\frac{u_C(C_t, N_t, \xi_t)}{(1+\tilde{\tau}_t^c)} = E_t \left\{ \frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{(1+\tilde{\tau}_{t+1}^c)} \left[\frac{(1-\tilde{s}_{t+1}^I)}{(1-\tilde{s}_t^I)} (1-\delta) + \frac{(1-\tau_{t+1}^k) \frac{U_{t+1}}{P_{t+1}} + \tau_{t+1}^k \delta}{(1-\tilde{s}_t^I)} \right] \right\} \quad (79)$$

Since $\tilde{\tau}_0^k = \tau_0^k$ and $\tilde{s}_0^I = s_0^I$, the budget constraint (66) is satisfied. These new paths for the tax rates and interest rates $\{\tilde{\tau}_t^c, \tilde{\tau}_t^n, \tilde{\tau}_t^k, \tilde{s}_t^I, \tilde{\tau}_t^d, \tilde{i}_t\}$ satisfy all the equilibrium conditions, leaving the variables $\{p_{it}, p_{it}^*, P_t, W_t, U_t, C_t, N_t, K_t\}$ unchanged, and satisfying the zero bound constraint.

Here, we have not imposed restrictions on the tax rates, but there is one natural restriction on the profit tax that may apply, that it cannot be greater than one. We also don't allow for the profit tax to be one, since in that case, firms would always have zero profits, regardless of their choices. It is easy to see that as $\tau_t^d < 1$ for all t , then $\tilde{\tau}_t^d < 1$ for all t . Notice that whenever $i_t < 0$, and therefore $\tilde{i}_t = 0$, then $\frac{1+\tilde{\tau}_{t+1}^c}{1+\tilde{\tau}_t^c} > \frac{1+\tau_{t+1}^c}{1+\tau_t^c}$, and when $i_t \geq 0$, then $\frac{1+\tilde{\tau}_{t+1}^c}{1+\tilde{\tau}_t^c} = \frac{1+\tau_{t+1}^c}{1+\tau_t^c}$. It follows that $\tilde{\tau}_t^c \geq \tau_t^c$, so that (78) implies that $\tilde{\tau}_t^d \leq \tau_t^d < 1$.

Revenue neutrality. The budget constraint (66), with aggregate profits described in (75) is a function of the allocation for consumption and labor, initial taxes τ_0^c , and τ_0^k , the initial price level initial P_0 , and capital K_0 , and debt levels $\bar{B}_0 + B_{-1,0}$. Indirectly through profits, it is also depends on $\{p_{it}, P_t W_t, K_t, N_t, Y_t\}$ and on the ratio $\frac{1-\tilde{\tau}_t^d}{1+\tilde{\tau}_t^c}$. In the sense that the same government expenditures path and initial debt is still financed with the alternative (unconventional tax policy), the policy with negative nominal interest rates or unconventional tax policy raise the same revenues. In this sense the tax policy that overcomes the zero bound constraint is revenue neutral.

7 Numerical illustration

For our numerical exploration, we analyze a variant of the model in Section 5 which features capital, and allows for both sticky prices and sticky wages. We modify the model so as to incorporate adjustment costs to capital. The accumulation equation for capital is

$$K_{t+1} = I_t + (1-\delta)K_t - \frac{\sigma_I}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t$$

where σ_I controls the magnitude of adjustment costs to capital.

With this modification, the model becomes very similar to one analyzed by Christiano et al. (2011); the only difference being that we allow for distortionary taxes. So as to facilitate comparison, the functional forms we use follow Christiano et al. (2011). Thus, we specify that all production functions are Cobb-Douglas

$$y_{it} = A k_{it}^{\alpha_F} n_{it}^{1-\alpha_F},$$

we take utility to be of the form

$$\xi_t \frac{[C_t^\gamma L_t^{1-\gamma}]^{1-\sigma} - 1}{1-\sigma}$$

and we assume that monetary policy follows a Taylor rule truncated at zero

$$i_t = \max(Z_t, 0)$$

where

$$Z_t = \beta^{-1} (1 + \pi_t)^{\phi_1(1-\rho_R)} (Y_t/Y)^{\phi_2(1-\rho_R)} [\beta (1 + i_{t-1})]^{\rho_R} - 1.$$

We imagine that the economy is initially at a steady state with constant consumption taxes $\bar{\tau}^c$, capital taxes $\bar{\tau}^k$, and labor income taxes $\bar{\tau}^n$. The revenues from these taxes is used to finance a constant level of government expenditures $\bar{G} = g\bar{Y}$. The rest is rebated to households in the form of a lump-sum transfer/tax. We set all the other taxes to zero in steady state.

Parameter values. For taxes, we follow Drautzburg and Uhlig (2011) and set $\bar{\tau}^c = 0.05$, $\bar{\tau}^k = 0.36$, and $\bar{\tau}^n = 0.28$.

The rest of our calibration follows Christiano et al. (2011). We take a period to be a quarter. We take the parameters in the utility function to be $\gamma = 0.29$, $\sigma = 2$, and we set the discount factor to be $\beta = 0.99$. We pick $\alpha_F = 0.33$, $\delta = 0.02$ and choose $\sigma_I = 17$. We choose $g = 0.2$. For monetary policy, we set $\rho_R = 0$, $\phi_1 = 1.5$ and $\phi_2 = 0$. Finally, for the stickiness parameters, we set $\alpha = \alpha^w = 0.85$ (except when we explicitly consider flexible prices or flexible wages).

Shock. The shock that we consider is the same as that of Christiano et al. (2011). At time zero, the economy is in its non-stochastic steady state. At time one, agents learn that the discount factor will be $\hat{\beta} > \beta$ for T periods (requiring $\beta \xi_{t+1}/\xi_t = \hat{\beta}$ for T periods), and then return to its steady state value (so that ξ_t is constant for $t \geq T + 1$). The shock is sufficiently large so that the zero bound on the nominal interest rate binds between two time periods t_1 and t_2 (determined endogenously) with $1 \leq t_1 \leq t_2 \leq T$. We take $\hat{\beta} = 1.01$ and $T = 10$.

Results. Figure 1 displays the path of the economy when taxes are kept constant at their steady-state values (inflation rates and interest rates are annualized). To facilitate comparison with Christiano et al.

(2011), we start by assuming that prices are sticky but that wages are flexible. We will analyze other configurations below.

The zero bound binds in periods one through six. The shock leads to a substantial decline in investment, hours worked, output and consumption. Given that the nominal interest rate is zero, the large associated deflation implies a sharp rise in the real interest rate, which rationalizes the responses of consumption and investment. The intuition for this outcome is explained in further details in Christiano et al. (2011).

In our numerical exercise, in the spirit of Christiano et al. (2011), we simply seek to replicate the allocation that would occur, were a non-truncated Taylor rule to be used. This Taylor rule does not replicate the flexible price, flexible wage allocation. We view the Taylor rule as describing the behavior of policy in normal times, and we simply seek to remove the constraint on this behavior imposed by the zero lower bound by using unconventional fiscal policy. We could alternatively seek to replicate the flexible price, flexible wage allocation. Indeed we examine this case briefly below at the end of our description of the results.²¹

Figure 2 displays the path of the economy when unconventional fiscal policy is used to replicate the allocation that would arise if monetary policy followed a non-truncated Taylor rule allowing for negative nominal interest rates $i_t = Z_t$, when in fact monetary policy follows a truncated Taylor rule $i_t = \max(Z_t, 0)$ that disallows negative nominal interest rates.²² Compared with the allocation that arises with constant taxes, the drops in consumption and output are greatly mitigated, and investment actually booms. This outcome is intuitive. Households become more patient. As a result, they reduce consumption and increase savings. The increase in savings leads to an investment boom. Output and hours drop together with consumption because the shock initially increases price markups compared to their desired steady-state level. This causes deflation in producer prices $P_{t+1}/P_t - 1 < 0$.²³ Increasing consumption taxes over time overturn this effect, resulting in inflation in consumer prices $(P_{t+1}(1 + \tau_{t+1}^c))/(P_t(1 + \tau_t^c)) - 1 > 0$. Since the zero bound initially binds, $i_t = 0$ and the real interest rate $(1 + i_t)(P_t(1 + \tau_t^c))/(P_{t+1}(1 + \tau_{t+1}^c))$ decreases. However the decrease in the real interest rate (inclusive of consumption taxes) is smaller than the increase in the discount factor, explaining the increasing path of consumption.

Figure 3 displays the corresponding path for consumption taxes, labor taxes and the investment tax credit. As explained in the main text, unconventional fiscal policy requires an increasing path of consumption taxes, a decreasing path of labor taxes, and positive investment tax credit (we keep capital taxes constant). In the

²¹Note however that in this environment with exogenous taxes, the flexible price, flexible wage allocation is not necessarily optimal.

²²In our numerical exercise, in the spirit of Christiano et al. (2011), we simply seek to replicate the allocation that would occur, were a non-truncated Taylor rule to be used. This Taylor rule does not replicate the flexible price, flexible wage allocation. We view the Taylor rule as describing the behavior of policy in normal times, and we simply seek to remove the constraint on this behavior imposed by the zero lower bound by using unconventional fiscal policy. We could alternatively have sought to replicate the flexible price, flexible wage allocation. Indeed we examine this case below briefly. Note however that in this environment with taxes, the flexible price, flexible wage allocation is not necessarily optimal.

²³This is a manifestation of the aforementioned fact that the non-truncated Taylor rule does not replicate the flexible price, flexible wage allocation.

interest of space, the path of profit taxes τ_t^d is not displayed: it is the mirror image of consumption taxes—it is such that $(1 - \tau_t^d)/(1 + \tau_t^c)$ is constant over time. In this baseline simulation, consumption taxes increase gradually from 0.05, to 0.14, labor taxes decrease gradually from 0.28 to 0.21, and the investment tax credit (which is zero in steady state) first jumps in the first period to 0.09 and then decreases gradually towards 0. All taxes stabilize in period six when Z_t turns positive.²⁴

We then consider the alternative case where prices are flexible but wages are sticky. As explained in Section 5, there are two ways to engineer an unconventional fiscal policy that replicates the allocation that would arise if a non-truncated Taylor rule were used when in fact a truncated Taylor rule is used: (i) using consumption taxes, labor taxes, and an investment tax credit as above; (ii) using labor taxes and payroll taxes. For brevity, we only present the results for (ii). Figure 4 displays the corresponding path of labor income taxes and payroll taxes. The payroll tax increases gradually over time and the labor tax decreases gradually over time. The total adjustment is 0.17 for payroll taxes and 0.12 for income taxes.

For completeness, we also consider the cases where both prices and wages are sticky. Figure 5 displays the path for consumption taxes, labor income taxes, and the investment tax credit, necessary to replicate the allocation that would arise if a non-truncated Taylor rule were used when in fact a truncated Taylor rule is used.

Finally, we briefly analyze the flexible price, flexible wage economy and how unconventional taxes can be used to implement the corresponding allocation. This also helps understand why the non-truncated Taylor rule leads to a different outcome. Figure 6 displays the path of the economy for the flexible price, flexible wage economy. It is interesting to compare this economy to the economy with sticky prices and flexible wages where unconventional taxes are used to replicate the non-truncated Taylor rule (Figure 2). The drop in consumption is significantly lower, the increase in investment is much higher, and now hours and output increase. Note that the real interest actually starts higher than in the economy where unconventional taxes are used to replicate the non-truncated Taylor rule. However, the real interest rate stays negative for the whole duration of the shock (10 periods), whereas in Figure 2, it becomes positive again in period 6. As a result consumption is now above steady state from period 6 on (in Figure 2, consumption is always below steady state). It then reverts slowly towards its steady-state value—the long transition occurs because the capital stock increased substantially because of the shock; it then slowly decreases towards its steady-state value. Unconventional taxes can be used to replicate this allocation when prices are sticky, with zero price inflation. It suffices to implement an increasing path of consumption taxes, a decreasing path for labor income taxes, and an investment tax credit as long as the real interest rate of the flexible price, flexible wage allocation is negative. Then consumption and labor income taxes are kept constant and the nominal interest rate is set equal to the real interest rate of the flexible price, flexible wage economy. These taxes

²⁴We could have alternatively chosen to decrease the capital tax instead of introducing an investment tax credit. However, the necessary reduction in the capital tax would have been large—much larger than the required adjustment in the investment tax credit. The reason is that the capital tax is a tax on the net returns to capital, while the investment tax credit acts as a subsidy on the gross return to capital.

are displayed in Figure 7. Note that the overall adjustments in these taxes are smaller, but last longer (10 periods instead of 6), than when unconventional fiscal policy is used to replicate the non-truncated Taylor rule (Figure 3).

Welfare. In order to compare welfare under different scenarios, we need a welfare metric and a reference allocation. We choose to use the steady-state allocation as our reference allocation. This reference allocation is arbitrary—we could have chosen any other allocation. Table 1 computes the welfare losses associated with different scenarios compared with the welfare of the steady-state allocation. The metric that we use how much (in percentages) steady-state consumption has to be decreased for T^W periods in order to yield the same date-0 utility as the scenario under consideration. Formally, the table reports the value of λ that solves

$$\sum_{t=0}^{\infty} \beta^t U(C_t, L_t, \xi_t) = \sum_{t=0}^{T^W} \beta^t U((1 - \lambda)C_{ss}, L_{ss}, \xi_t) + \sum_{t=T^W+1}^{\infty} \beta^t U(C_{ss}, L_{ss}, \xi_t).$$

The higher is T^W , the lower (in absolute value) are the resulting welfare effects. In the limit where $T^W = \infty$, the welfare effects can be interpreted as a percentage of lifetime consumption. Even though the shock that we consider leads to a very large recession, this recession is not very long-lived, and it occurs only once in our experiment economy. As a result, the corresponding welfare effects are small when computed as a fraction of lifetime consumption. We report the numbers for $T^W = 10$ and $T^W = \infty$. We think that these different normalizations help appreciate the severity of the problems arising from the zero bound.

We see that the welfare gains from using unconventional fiscal policy are substantial. With sticky prices and flexible wages, these benefits are equivalent to permanent 0.4% permanent increase in consumption ($T^W = \infty$) or equivalently a 2.2% increase in consumption for 10 quarters ($T^W = 10$). With flexible prices and sticky wages, these gains are 0.8% ($T^W = \infty$) and 4.3% ($T^W = 10$). With both sticky prices and sticky wages, the gains become 0.2% ($T^W = \infty$) and 1.0% ($T^W = 10$).²⁵

Finally, the last line of Table 1 reports the welfare for the flexible price, flexible wage allocation. Note that this allocation features negative welfare losses (welfare gains) over the steady-state allocation of -3.7% ($T^W = 10$) and -0.6% ($T^W = \infty$). As explained above, one way to replicate this allocation when either prices, wages are sticky, by using a different monetary policy—setting the nominal interest equal to the real interest rate of the flexible price, flexible wage economy when it is positive, and equal to zero otherwise—combined with unconventional fiscal policy—when the real interest rate of the flexible price, flexible wage economy is negative.²⁶

²⁵Table 1 shows that the welfare losses of the sticky price, sticky wage economy, and the gains from unconventional fiscal policy, are lower than under the other two scenarios (sticky prices, flexible wages, and flexible prices, sticky wages). This somehow surprising result is a manifestation of the following observation, discussed in Chiristiano et al. (2011). When the zero bound binds, more flexible prices or wages can make the outcome worse by leading to more deflation, higher real interest rates, higher markups, less consumption and less output—an effect consistent with the potentially destabilizing effects of marginal increases in price flexibility discussed in Delong and Summers (1986).

²⁶As explained in Section 5, when both prices and wages are sticky, this allocation can still be replicated but it now requires

Avoiding negative taxes: building a buffer in good times. Consider for simplicity our baseline calibration with only sticky prices when unconventional taxes are used to replicate the non-truncated Taylor rule. At the end of the episode, unconventional fiscal policy leads to permanently higher consumption taxes and permanently lower labor income taxes. To the extent that one considers that labor income subsidies are hard to implement, one potential concern is that if the episode were to repeat itself (potentially several times) in the future, there would be less room to conduct unconventional fiscal policy without running into negative labor income taxes.

One way to alleviate this concern is to build a buffer of taxes in good times. We have in mind the following. At $t = 10$, we impose a zero nominal interest rate and instead replicate the effects of a positive interest rate Z_t by applying our unconventional fiscal policy in reverse, using a decreasing path for consumption taxes, and increasing path for labor taxes, and an investment tax (a negative investment tax credit).²⁷ We can do this until consumption and labor taxes are back at their steady-state values. After that, taxes are kept constant, and monetary policy is set according to the Taylor rule $i_t = Z_t$. The corresponding path for taxes is displayed in Figure 8.

8 Conclusions

In this paper we study the liquidity trap in the context of a standard New Keynesian model. We characterize jointly optimal monetary and fiscal policy. Our main result is to demonstrate how distortionary taxes can be used to replicate the effects of negative nominal interest rates and completely circumvent the zero bound problem. We label this scheme *unconventional fiscal policy*. It involves engineering over time an increasing path for consumption taxes, a decreasing path for labor taxes, coupled with a temporary investment tax credit or a temporary cut in capital income taxes. Moreover, numerical calibrations indicate that the magnitude of the tax changes are reasonable. Overall, the zero bound constraint on nominal interest rates is not a relevant restriction on policy when both fiscal and monetary policy are flexible.

Our results speak to the recent literature on using inefficient commitments to low future interest rates or government spending policies to circumvent the zero bound constraint in the New-Keynesian model. Unconventional fiscal policy can be used to perfectly stabilize the economy at zero cost, in a time-consistent and revenue-neutral manner. In richer or more constrained environments where lump sum taxes are unavailable, or in the presence of idiosyncratic shocks, efficient allocations cannot be implemented. However unconventional fiscal policy can still perfectly replicate the effects of negative nominal interest rates. The corresponding solution is not time-consistent, but for reasons that are unrelated to the zero bound constraint.

of time-varying labor income or payroll taxes even when the real interest rate of the flexible price, flexible wage economy is positive.

²⁷That we choose to start building the buffer at $t = 10$ is somewhat arbitrary. We could have chosen to start implementing this policy at any date after $t = 6$ when Z_t becomes positive again.

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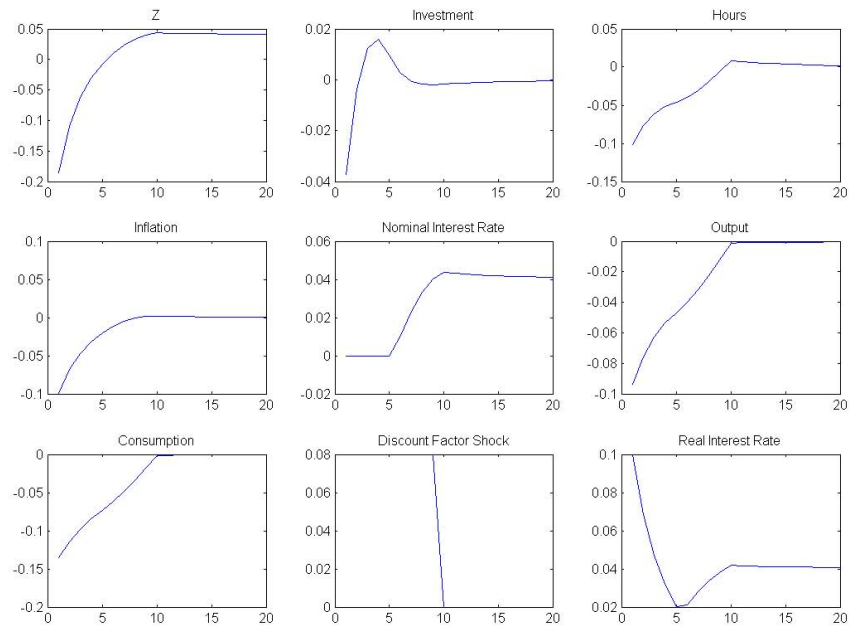


Figure 1: Dynamic path for the economy, sticky prices, flexible wages, constant taxes.

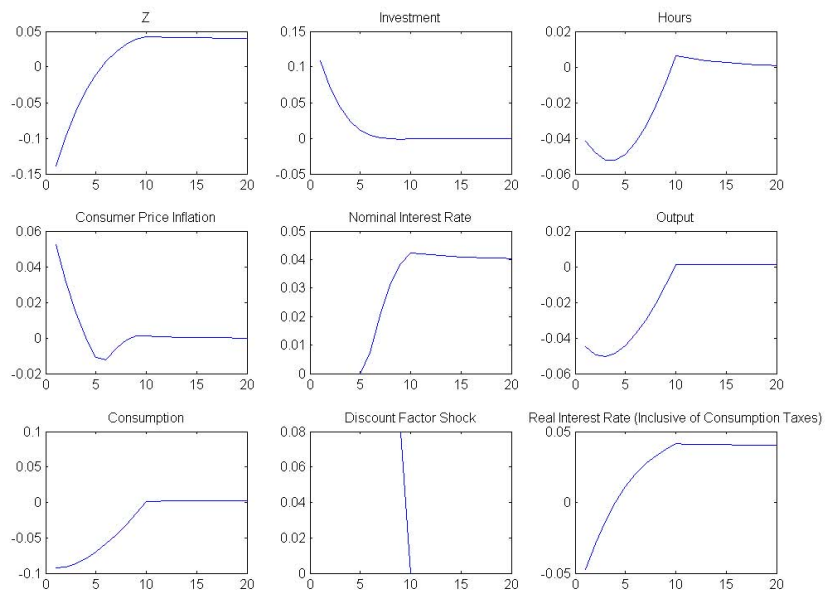


Figure 2: Dynamic path for the economy, sticky prices, flexible wages, unconventional taxes.

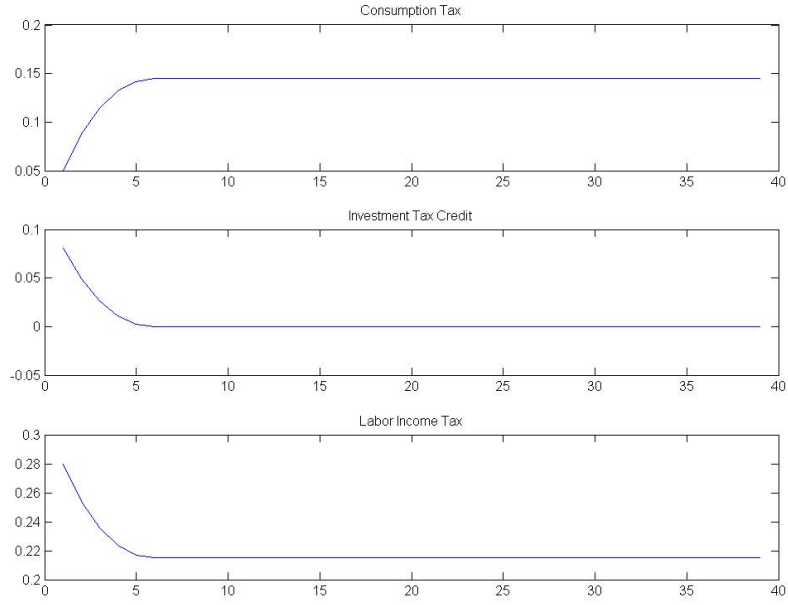


Figure 3: Dynamic path for unconventional taxes, sticky prices, flexible wages.

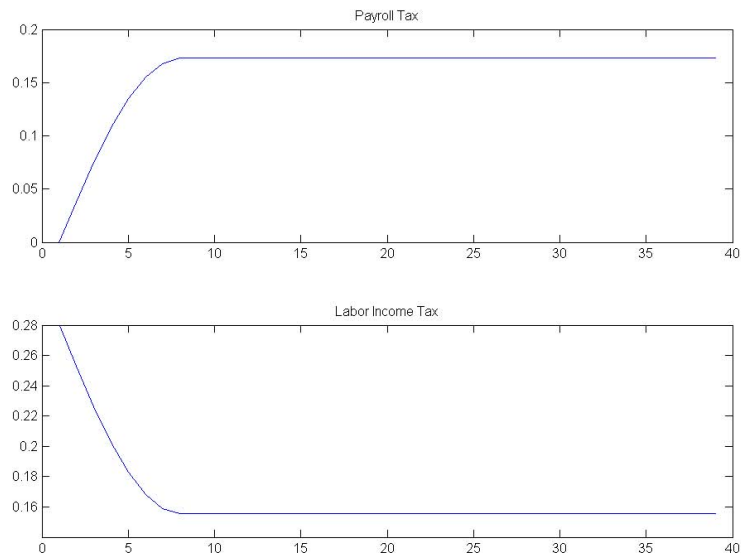


Figure 4: Dynamic path for the alternative set of unconventional taxes, flexible prices, sticky wages.

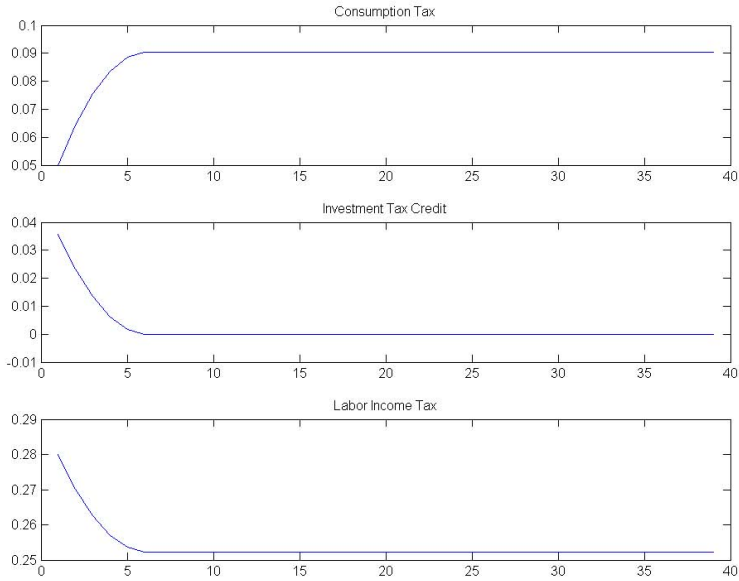


Figure 5: Dynamic path for unconventional taxes, sticky prices, sticky wages.

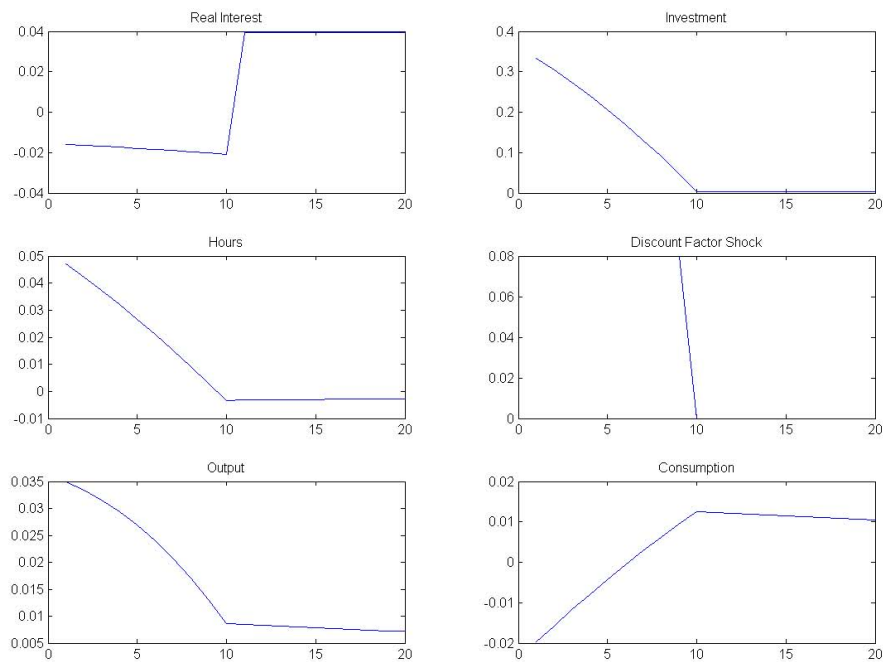


Figure 6: Flexible price allocation.

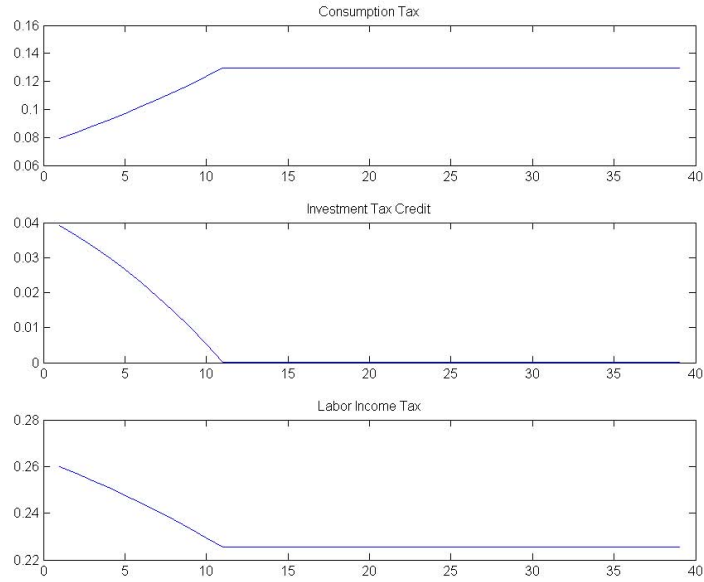


Figure 7: Dynamic path for unconventional taxes to replicate the flexible price, flexible wage allocation: sticky prices, flexible wages.

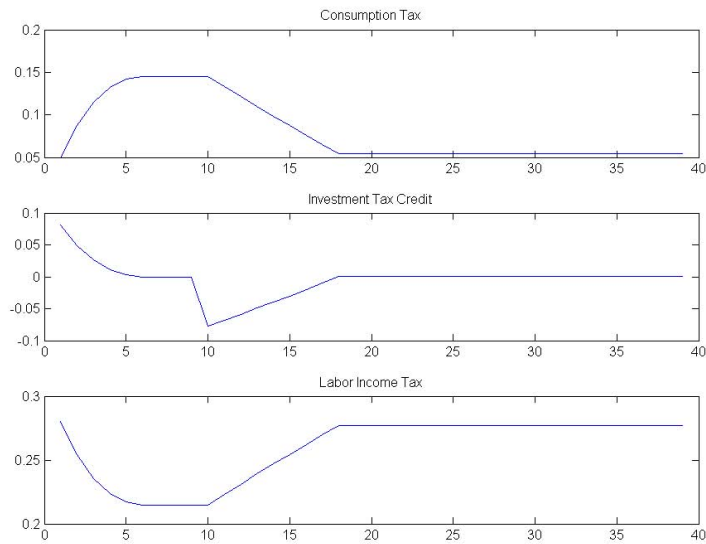


Figure 8: Building a buffer in good times: dynamic path for unconventional taxes, sticky prices, flexible wages.

	$T^W = 10$	$T^W = \infty$
Constant taxes, sticky prices, flexible wages	6.5%	1.2%
Unconventional taxes, sticky prices, flexible wages	4.3%	0.8%
Constant taxes, flexible prices, sticky wages	5.4%	1.0%
Unconventional taxes, flexible prices, sticky wages	1.1%	0.2%
Constant taxes, sticky prices, sticky wages	3.5%	0.6%
Unconventional taxes, sticky prices, sticky wages	2.5%	0.4%
Constant taxes, flexible prices, flexible wages	-3.7%	-0.6%

Table 1: Welfare losses relative to steady state. Constant taxes vs. unconventional taxes. Different values of T^W refer to different welfare metrics (proportional decrease of steady-state consumption for the first T^W periods).