Offshoring and Directed Technical Change*

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Abstract

We study the effects of offshoring on wages, welfare and inequality through its impact on technology. To this end, we introduce directed technical change into a model of task-based production and offshoring. A unique final good is produced by combining a skilled and an unskilled intermediate good, each produced from a continuum of tasks. Offshoring takes the form of some of these tasks being transferred from a skill-abundant West to a skill-scarce East. Profit maximization determines both the equilibrium level of offshoring and the rate at which the productivities of skilled and unskilled workers improve. On the one hand, by increasing the price of skill-intensive products, offshoring induces skill-biased technological change. On the other, it also expands the market size for technologies complementing unskilled labor. In the empirically more relevant case, starting from low levels, an increase in offshoring opportunities triggers a transition with falling real wages for unskilled workers in the West, skill-biased technical change and rising skill premia worldwide. When the extent of offshoring becomes sufficiently large, instead, further increases in offshoring induce unskilled-labor biased technical change and reduce the long-run skill premium. Interestingly, wage inequality is highest for relatively low volumes of intermediate good trade. The transitional dynamics also suggest that offshoring and technological change are substitutes in the short run, but complements in the long run. Finally, we provide conditions under which offshoring improves not only the welfare of workers in the East, but also those of both skilled and unskilled workers in the West.

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1 Introduction

The rapid rise of offshoring, which involves many production and service tasks previously produced domestically being sourced from abroad, has been one of the most visible trends in the US labor market over the last three decades. As a result, the share of imported inputs in total intermediate use in the US manufacturing has increased from about 6% in 1980 to over 27% today (Feenstra and Jensen, 2009). The production structure of Apple's video iPod gives a glimpse of these trends. Though designed and engineered in the United States, more than 99% of the production jobs created by this product are located abroad (Linden et al. 2011). Despite its prevalence, the implications of offshoring for wages, skill premia and incomes are still debated.[1] The iPod example illustrates its different potential effects. Though most production jobs are offshored, a significant number of high-skill engineering jobs and lower-skill retail jobs are created in the United States, and more than 50% of the value added of the iPod is captured by domestic companies. With more limited offshoring, some of the production jobs may have stayed in the United States, increasing the demand for the services of lower-skill production workers. But this would have also increased the cost and price of iPods, reducing employment not only in engineering and design occupations but also in retail and other related tasks.

The potential negative effects of offshoring on incomes and/or the wages of lower-skill workers in advanced economies (the "West") have been emphasized by Feenstra and Hanson (1996), Deardorff (2001, 2005), Samuelson (2004) and Hira and Hira (2008). Samuelson, for example, famously pointed out how offshoring could lower Western incomes in a Ricardian trade model if it transfers knowledge to less advanced, lower-wage economies (the "East"), thus eroding the Western technological advantage in a range of tasks. Counteracting this are the efficiency gains due to offshoring, which have been emphasized by several models including Grossman and Rossi-Hansberg (2008) and Rodriguez-Clare (2010).

In this paper, we focus on the effects of offshoring on wages and inequality in the West working through its impact on technology. Returning to the example of Apple products, the variety of iPods may not have been profitable to introduce and develop if labor costs were higher—as they would have been without offshoring. More importantly, iPods and other products may have been designed differently to change their skilled and unskilled labor requirements in the face of these different labor costs.

To provide a framework for studying these issues, we introduce directed technological

[1] Offshoring is not an entirely new phenomenon. The term "production sharing" was coined in 1977 by Peter Drucker to describe activities such as the assembly of electronic equipment in South Korea and Singapore, or cloth processing in Morocco, Malaya and Indonesia. Extensive media coverage of labor market implications of offshoring exploded before the 2004 U.S. presidential election. Current concerns are well captured by what BusinessWeek (2008) defines as one of today's burning questions: Is American tech supremacy thanks to heavy investments in R&D also benefiting U.S. workers? Or are U.S. inventions mainly creating jobs overseas?

change into a benchmark model of task-based production structure and offshoring. The unique final good is produced by combining a skilled and an unskilled intermediate good, each produced from a continuum of tasks. Offshoring takes the form of some of these tasks (of both skilled and unskilled variety) being transferred to the East.[2] Profit-maximizing incentives determine not only how much offshoring will take place in equilibrium, but also the rate at which the productivities of both skilled and unskilled intermediate goods improve.

Though our main contribution concern the implications of an improvement in offshoring possibilities on endogenous technological change, our benchmark (world equilibrium) model of offshoring already highlights a number of effects that have not received sufficient attention in the literature. Our model first enables us to decompose the impact of offshoring on the skill premium into three components. Focusing on the case in which offshoring takes place predominantly in unskilled tasks, these three components are: (i) a labor supply effect, showing that offshoring will increase the effective supply of unskilled labor, thus pushing up the skill premium; (ii) a relative price effect, resulting from the fact that offshoring increases the relative price of the skill-intensive good; and (iii) an efficiency effect, which increases the overall efficiency of the unskilled labor-intensive sector, not only raising incomes on average and unskilled wages, but also potentially reducing the skill premium. Our model and this decomposition highlight that the efficiency effect is most pronounced, and thus offshoring is more likely to benefit low-skill workers, when its extent is limited and the elasticity of substitution between tasks is low. This is because the wage gap between the West and the East is largest in this case—greater offshoring increases wages in the East and closes this gap. Even though the wages of Eastern workers is much lower than the Western unskilled workers against whom they are competing, the paucity of offshoring opportunities implies that the downward pressure they put on unskilled wages in the West is limited, while the efficiency gains from substituting this cheaper form of labor are large. The result that the efficiency effect of offshoring is strongest at first and when tasks are poor substitutes plays an important role in the rest of our analysis.[3]

[2] Acemoglu and Autor (2010) argue that a satisfactory account of recent changes in the US wage structure require both a task-based framework, like the one we propose here, and more than two skill groups (to account for differential changes across decades at the bottom, middle and top of the earnings distribution). Though our framework can be extended to include more than two skill groups, this does not seem to generate additional major insights and we thus opt for simplicity here, focusing on an economy inhabited by skilled and unskilled workers. Our unskilled workers should be interpreted as corresponding to the low and middle skill groups in Acemoglu and Autor's framework.

[3] Our efficiency effect is similar to the efficiency effect that Rodriguez-Clare (2010) obtains in a Ricardian model of offshoring with a single type of labor. It is also related to, though different from, Grossman and Rossi-Hansberg's (2008) productivity effect. In particular, they do not derive the result that this beneficial effect of offshoring tend to be more pronounced when its extent is limited. None of this papers study the role of the elasticity of substitution between tasks, which is instead a key parameter in our model.

Our main results concern the effects of offshoring on technological change. These work through price and market size effects (Acemoglu, 1998, 2002). Offshoring increases the prices of skill-intensive products, inducing skill-biased technological change. Counteracting this, it also expands the market size for technologies complementing unskilled labor, because these can now be offshored and used with the abundant unskilled labor in the East. In the empirically more relevant case in which the elasticity of substitution between tasks is greater than the elasticity of substitution between workers and starting from low levels of offshoring, the price effect dominates and greater offshoring opportunities induce skill-biased technological change. Interestingly, the opposite pattern obtains if the initial level of offshoring is already high, because prices react less when the wage gap between the West and the East is small. In this case, offshoring induces unskilled labor-biased technological change.

The interplay of the direct effects of offshoring and its indirect effects working through technological change generates a nuanced impact on the skill premium and unskilled wages in the West. Though in the absence of directed technological change a greater extent of offshoring tends to monotonically increase the skill premium (and reduces unskilled wages), with directed technological change these relationships are generally inverse U-shaped (focusing again on the case in which tasks are more substitutable than skilled and unskilled workers). For example, starting from very low levels, an increase in offshoring opportunities induces powerful forces towards skill-biased technological change and significantly increases the long-run skill premium. Interestingly, however, when the extent of offshoring becomes sufficiently large, further increases in offshoring reduce the long run skill premium because they induce unskilled labor-biased technological change.

This configuration of effects (with the response of technology factored in) suggests that the rise in offshoring may have contributed to the changes in the college premium and unskilled wages. The first wave of expansion of offshoring in the 1980s was associated with a sharp decline in the real wages of unskilled workers. As offshoring continued to expand in the late 1990s and 2000s, unskilled wages stabilized and began rising (e.g., Acemoglu and Autor, 2011). Though several other factors, including the pace of changes in the domestic supply of skills and institutional variables, have played pivotal roles in these changes, our analysis suggests that the rise in offshoring may have also been an important factor. Moreover, since our results are strongest when the extent of offshoring is limited, the model predicts the most significant wage effects precisely for low volumes of trade in intermediate inputs. Thus, the criticism that trade between developed and developing countries is too small to have a large impact on wages does not apply to our framework.

The dynamics of technology and wages in response to an expansion of offshoring opportunities is also interesting. Immediately after the change in offshoring opportunities, technological change stops for a while because firms first spend resources in order to offshore their existing technologies (tasks). This is followed by a phase of either skill-biased technological change (for levels of offshoring below a critical threshold) or unskilled labor-biased technological change (for levels of offshoring above a critical threshold). This dynamic pattern implies that offshoring and technological change are substitutes in the short run—technological change stops as firms devote resources to offshoring. They are generally complements in the long run, however, and the pace of technological change can be increased because of greater offshoring opportunities. We then show that, if the rate of technological improvements can be increased sufficiently, offshoring improves not only the welfare of workers in the East, but also those of both skilled and unskilled workers in the West. Contrary to static models, however, there is another effect that tend to hurt agents in the West: greater offshoring opportunities reduce the value of existing firms along the transition to the new balanced growth path and therefore leads to fall in the market value of existing assets. Factoring all these effects in, we find that unskilled workers in the West may suffer losses from greater offshoring, skilled workers typically experience moderate gains, while Eastern workers are the big winners.

We then extend our framework to also consider offshoring of skilled tasks. The more general model confirms the main results discussed so far. More interestingly, it naturally yields the new result that offshoring can increase wage inequality both in the West and the East simultaneously—a possibility that is generally precluded in standard trade models (see Wood, 1994). This happens because, despite the presence of complete specialization and technological differences across countries, a zero-profit condition implies a form of conditional factor price equalization: if offshoring costs are identical, the East-West wage gap has to be the same for both skilled and unskilled workers for all domestic and offshore firms to break even.

Our paper is related to three literatures. First, it is a contribution to the growing literature on offshoring, which has already been discussed. Our main point of departure from this literature is the endogeneity of the direction of technological change. Glass and Saggi (2001), Naghavi and Ottaviano (2008), Dinopoulos and Segerstrom (2008), Branstetter and Saggi (2011), and Rodriguez-Clare (2010) endogenize the overall pace of technological change in models with offshoring, but not its direction. All of our main results derive from the endogeneity of the direction of technological change and are thus not shared by these papers or others in this literature.

Second, our paper is a contribution to the large literature on the theoretical determinants of changes in inequality and wages in the United States and other advanced economies. Our model is closely related to task-based approaches, which are discussed in Acemoglu and Autor (2011). Precursors of this approach include Acemoglu and Zilibotti (2001), Autor, Levy and Murnane

(2003) and Costinot and Vogel (2010). Acemoglu and Autor (2011) emphasize the role of technologies replacing tasks previously performed by labor and the similar role of offshoring in this context, but do not model offshoring in detail and do not consider the interplay between offshoring and directed technological change.

Third, our paper builds on and extends models of directed technological change (e.g., Acemoglu, 1998, 2002, 2007, Acemoglu and Zilibotti, 2001, Kiley, 1999, Gancia and Zilibotti, 2008). Within this literature, the set of issues highlighted here are most closely related to works that emphasize the impact of international trade on the direction of technological change, and on the structure of wages through this channel. Acemoglu (2003), Thoenig and Verdier (2003) and Epifani and Gancia (2008) show how international trade can induce technological changes that further increase the demand for skills, thus amplifying its direct impact on the wage structure. This literature has not, to the best of our knowledge, considered offshoring, which has different effects on labor market equilibria and thus on incentives for technological change. Our result that the skill-biased effects on wages are strongest when the extent of trade in task is small is particularly novel and important, given that the rise of offshoring is still a recent phenomenon.

The rest of the paper is organized as follows. Section 2 presents the basic model of task trade and directed technical change. It first isolates all the effects through which offshoring affects wages for a given technology and then studies the interaction between offshoring and technological progress. Section 3 studies welfare effects on all types of workers. Section 4 extends the model to include offshoring of high-skill tasks. Section 5 concludes.

2 Model

The world economy is composed by two countries, West and East, populated by two types of workers, skilled and unskilled, in fixed supply. The West is endowed with L_w units of unskilled workers and H_w units of skilled workers. The East is assumed to be skill scarce. To start with, we restrict the analysis to the case in which the East is endowed with L_e unskilled workers and has no skilled workers. We relax this assumption when we study offshoring of skill services. Besides endowments, the two countries differ in the technological capability of producing existing goods: new technologies are introduced in the West and can be transferred to the East only after paying a fixed offshoring cost. As in models of directed technical change, some technologies complement skilled workers while others complement unskilled workers and the evolution of both is endogenous. Variables with no country index refers to the world economy. For simplicity, there are no barriers to trade on any good.

2.1 Preferences

Our model economy is in continuous time and is populated by infinitely lived households who derive utility from a unique consumption C_t and supply labor inelastically. We assume that the economy admits a representative household with preferences at time $t = 0$ given by

$$U = \int_0^\infty e^{-\rho t} \ln C_t dt,$$

where $\rho > 0$ is the discount rate. This form of preferences is standard and the log utility is imposed to simplify the exposition. The representative household sets a consumption plan to maximize utility, subject to an intertemporal budget constraint and a No-Ponzi game condition. The consumption plan satisfies a standard Euler equation,

$$\dot{C}_t / C_t = r_t - \rho,$$

where r_t is the interest rate. Time indexes will be omitted in what follows as long as this causes no confusion.

2.2 Technology and Market Structure

There is a single final good, used both for consumption and investment, which is given by a CES function of production in two sectors:

$$Y = \left[Y_l^{\frac{\epsilon-1}{\epsilon}} + Y_h^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

where Y_h and Y_l are goods produced with skilled and unskilled labor, respectively, and ϵ is the elasticity of substitution between them. Choosing Y as the numeraire, profit maximization implies the following inverse demand functions:

$$P_h = \left(\frac{Y}{Y_h} \right)^{\frac{1}{\epsilon}} \text{ and } P_l = \left(\frac{Y}{Y_l} \right)^{\frac{1}{\epsilon}}, \tag{2}$$

where P_l and P_h are the prices of Y_l and Y_h, respectively. Note that $P_h/P_l = [Y_l/Y_h]^{\frac{1}{\epsilon}}$.

Production in the two sectors requires intermediate inputs, which in turn are manufactured by workers. In particular,

$$Y_l = E_l \left[\int_0^{A_l} x_{l,i}^\alpha di \right]^{1/\alpha} \quad \text{and} \quad Y_h = E_h \left[\int_0^{A_h} x_{h,i}^\alpha di \right]^{1/\alpha}, \tag{3}$$

where $x_{l,i}$ ($x_{h,i}$) is the quantity of intermediate input $i \in [0, A_l]$ ($i \in [0, A_h]$) and $\sigma = 1/(1-\alpha) > 1$ is the elasticity of substitution between any two variety. A similar notation applies to the other sector. As in models of horizontal innovation, the measure of existing intermediates, A_l and A_h, represents the state of technology in the two sectors and will grow (endogenously) over time. The factors:

$$E_l \equiv (A_l)^{\frac{2\alpha-1}{\alpha}} \quad \text{and} \quad E_h \equiv (A_h)^{\frac{2\alpha-1}{\alpha}} \tag{4}$$

are conveniently chosen so as to make the return from new intermediates consistent with balanced growth for any σ.[4]

Profit maximization implies the following inverse demand function:

$$p_{l,i} = P_l E_l^{\alpha} Y_l^{1-\alpha} x_{l,i}^{\alpha-1} \quad \text{and} \quad p_{h,i} = P_h E_h^{\alpha} Y_h^{1-\alpha} x_{h,i}^{\alpha-1} \tag{5}$$

where $p_{l,i}$ ($p_{h,i}$) is the price of intermediate $i \in [0, A_l]$ ($i \in [0, A_h]$). Each intermediate is produced by a single monopolist with a linear technology requiring one unit of labor per unit of output:

$$x_{l,i} = l_i \quad \text{and} \quad x_{h,i} = Z h_i.$$

where l_i (h_i) is the quantity of unskilled (skilled) labor employed and $Z \geq 1$. Given that demand has a constant price elasticity equal to $\sigma = 1/(1-\alpha)$, profit maximization yields prices that are $1/\alpha$ times the marginal cost: $w_{h,w}/Z$ for high-skill intermediates, $w_{l,w}$ for low-skill intermediates produced in the West and $w_{l,e}$ for low-skill intermediates produced in the East. Profits are therefore a fraction $(1-\alpha)$ of revenues:

$$\pi_{l,i} = (1-\alpha) p_{l,i} x_{l,i} \quad \text{and} \quad \pi_{h,i} = (1-\alpha) p_{h,i} x_{h,i}. \tag{6}$$

2.3 Exogenous Offshoring

In this preliminary section, we consider the wage effects of a given allocation of production between the West and the East, for a given level of technology (A_l, A_h). More precisely, we assume that the West can manufacture the entire measure of existing intermediates, while the East can only produce a fraction $\kappa < \bar{\kappa} \equiv L_e/(L_e + L_w)$. The restriction $\kappa < \bar{\kappa}$ guarantees that wages are lower in the East, so that offshoring production is profitable for Western firms, but it is restricted due to an exogenous – e.g., technological – constraint. It also implies that the a firm who can offshore will not produce in the West. Thus, a measure κA_l of firms produce in the East and the remaining measure $(1-\kappa) A_l$ in the West. Imposing labor market clearing and using the fact that all firms of a given type are identical, we can solve for the quantity produced of any intermediate in the West and the East:

$$x_{l,w} = \frac{L_w}{(1-\kappa) A_l} \quad \text{and} \quad x_{l,e} = \frac{L_e}{\kappa A_l}. \tag{7}$$

[4] See Gancia and Zilibotti (2009) and Acemoglu, Gancia and Zilibotti (2011) for further discussion of this formulation.

The West/East wage gap is:

$$\frac{w_{l,w}}{w_{l,e}} = \frac{p_{l,w}}{p_{l,e}} = \left(\frac{x_{l,w}}{x_{l,e}}\right)^{\alpha-1} = \left(\frac{L_e}{L_w}\frac{1-\kappa}{\kappa}\right)^{1-\alpha}, \tag{8}$$

which is higher than one because $\kappa < \bar{\kappa}$. As production moves to the East (higher κ), demand for unskilled workers falls in the West and increases in the East thereby compressing the wage gap. Note also that, for a given distribution of world production (κ), the elasticity of substitution between unskilled workers in the West and East is $\sigma = 1/(1-\alpha)$.

Substituting (7) into (3) and using (4), we can express total production in the L-sector as:

$$Y_l = A_l \hat{L}, \tag{9}$$

where

$$\hat{L} \equiv \left[\kappa^{1-\alpha} L_e^{\alpha} + (1-\kappa)^{1-\alpha} L_w^{\alpha}\right]^{1/\alpha} \tag{10}$$

is a weighted average of the world endowment of unskilled workers, with weighs that depend on the extent of offshoring. As in model of horizontal innovation, production increases linearly in the number of existing varieties, A_l. More interestingly, for a given number of varieties, production increases also with the extent of offshoring:

$$\frac{\partial \hat{L}}{\partial \kappa} = \frac{1-\alpha}{\alpha}\hat{L}^{1-\alpha}\left[\left(\frac{L_e}{\kappa}\right)^{\alpha} - \left(\frac{L_w}{1-\kappa}\right)^{\alpha}\right] > 0$$

with $\lim_{\kappa\to 0} d\hat{L}/d\kappa = \infty$ and $\lim_{\kappa\to\bar{\kappa}} d\hat{L}/d\kappa = 0$. This is the *efficiency effect* of offshoring: an increase in κ allows an efficiency-enhancing reallocation of production towards countries where wages are lower. In terms of production of Y_l, this is equivalent to an increase in factor endowment, from $\hat{L} = L_w$ when $\kappa \to 0$, to $\hat{L} = L_w + L_e$ when $\kappa \to \bar{\kappa}$. Importantly, the effect is stronger when wages in the East are lower, i.e., when there is little offshoring (low κ), the East has a large endowment of unskilled workers (high L_e/L_w) and the elasticity of substitution between varieties (σ) is low.

Production in the skill-intensive sector is instead:

$$x_{h,w} = \frac{ZH_w}{A_h},$$

where H_w is the Western endowment of skilled workers, Z their productivity and A_h the number of firms. Substituting this into (3) obtains:

$$Y_h = A_h Z H_w. \tag{11}$$

Using, (6), (5), (7), (4) and (9), we can rewrite profits as:

$$\begin{aligned} \pi_{l,w} &= (1-\alpha) P_l \hat{L}^{1-\alpha}\left(\frac{L_w}{1-\kappa}\right)^{\alpha} \\ \pi_{l,e} &= (1-\alpha) P_l \hat{L}^{1-\alpha}\left(\frac{L_e}{\kappa}\right)^{\alpha}. \end{aligned}$$

Similarly, we obtain $\pi_{h,w} = (1-\alpha) P_h H_w$.

We are now in the position to study the effect of the level of offshoring on wages. Consider first the skill premium in the West, denoted for convenience $\omega_w \equiv w_{h,w}/w_{l,w}$. Markup pricing implies $\omega_w = Z \frac{p_{h,w}}{p_{l,w}}$. Then, using (2), (5) and (7), ω_w can be found as:

$E_h \equiv (A_h)^{\frac{2\alpha-1}{\alpha}}$

$$\omega_w = Z \left(\frac{E_h}{E_l}\right)^{\alpha} \frac{P_h}{P_l} \left(\frac{Y_h}{Y_l}\right)^{1-\alpha} \left(\frac{x_{h,w}}{x_{l,w}}\right)^{\alpha-1}.$$

Keeping technology constant, the skill premium is increasing in the relative price (P_h/P_l) and the relative aggregate demand (Y_h/Y_l) for skill-intensive products, and decreasing in relative firm size $(x_{h,w}/x_{l,w})$. To study the effect of offshoring, substitute (2), (4), (9), (11) and (7):

$$\omega_w = \left(\frac{ZA_h}{A_l}\right)^{1-1/\epsilon} \left(\frac{L_w}{1-\kappa}\right)^{1-\alpha} \left(\frac{H_w}{\hat{L}}\right)^{-1/\epsilon} \frac{1}{\hat{L}^{1-\alpha}}. \quad (12)$$

Similarly to Grossman and Rossi-Hansberg (2008), the impact of changes in offshoring on the skill premium can be decomposed into a *labor-supply* effect, $\left(\frac{L_w}{1-\kappa}\right)^{1-\alpha}$, a *relative-price* effect, $\left(H_w/\hat{L}\right)^{-\frac{1}{\epsilon}}$, and an *efficiency* effect, $\hat{L}^{\alpha-1}$. While the first two effects tend to increase the skill premium, the last one tends to reduce it. We now discuss each of them in more details.

First, offshoring displace some Western unskilled workers who need to be rehired by the remaining domestic firms. Holding prices (P_h/P_l) constant, this effect is similar to an increase in the supply of unskilled workers in the West and tends to produce a higher skill premium. Second, offshoring increases production in the L-sector, thereby raising the relative price of the skill-intensive good (P_h/P_l). This *relative-price* effect tends to increase the skill premium too. Third, offshoring increases the overall *efficiency* of the L-sector, thereby reducing Y_h/Y_l. The increase in the relative demand for the unskilled product raises the demand for the offshored factor in the West. The effect is stronger the higher the complementarity between unskilled workers in West and East (low α) and the lower the level of offshoring.

From (12), it is immediate to see that the efficiency effect is always dominated by the price effect when $1-\alpha < 1/\epsilon$ (i.e., $\sigma > \epsilon$). That is, when the elasticity of substitution between tasks performed in the East and in the West (or between unskilled workers in the East and in the West) is higher than the elasticity of substitution between high- and low-skill workers, then offshoring necessarily increases the skill premium in the West.

If instead intermediates are more complementary than high- and low-skill workers ($1-\alpha > 1/\epsilon$ or $\sigma < \epsilon$), then the efficiency effect dominates the price effect. Whether it also dominates the labor-supply effect, depends on the level of offshoring. Since $\lim_{\kappa\to 0} d\hat{L}/d\kappa = \infty$, offshoring at first raises the relative reward of the offshored factor. For high offshoring, however, only the labor-supply effect remains (recall, $\lim_{\kappa\to\bar{\kappa}} d\hat{L}/d\kappa = 0$). The relationship between ω_w and κ in the two cases is depicted in Figure 1.

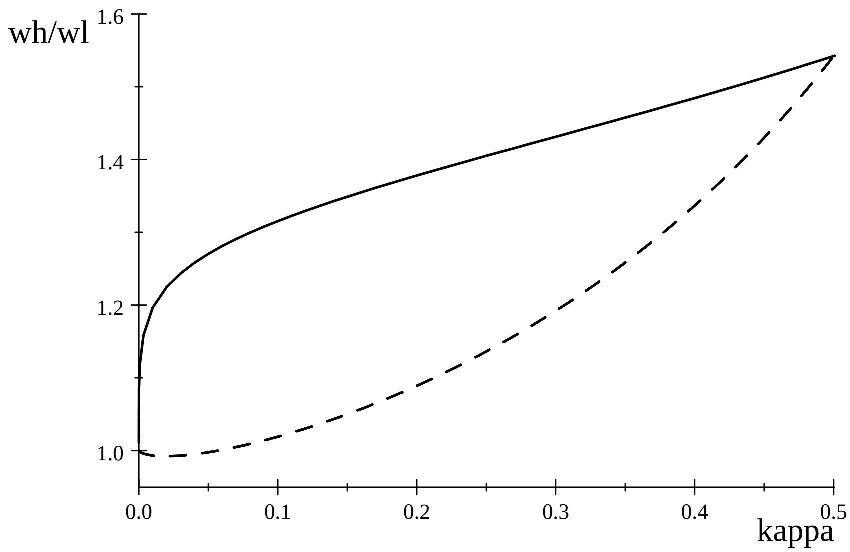

Figure 1: Offshoring and the Skill Premium, $\epsilon = 1.6$, solid: $\sigma = 5$, dashed: $\sigma = 1.25$

The efficiency effect is similar to the productivity effect of trade in tasks in Grossman and Rossi-Hansberg (2008). Yet, it differs in three important respects.[5] First, by taking into account the general equilibrium adjustment of Eastern wages to the higher demand for their services, our model suggests that the efficiency effect will become endogenously weaker as more offshoring takes place and will eventually vanish once wages have converged worldwide. Second, differently from Grossman and Rossi-Hansberg (2008), our model allows for substitutability between intermediates and shows that this elasticity of substitution is one of the key determinant of the strength of the efficiency/productivity effect. Our results warn that assuming no task substitutability, as in several existing models of offshoring, might lead to misleading conclusions, at least quantitatively. For example, while a strong complementarity may be appropriate for specific tasks in some sectors, conventional estimates of the elasticity of substitution between traded intermediates are significantly higher than estimates of the elasticity of substitution between skilled and unskilled labor. In this case, our model predicts the efficiency effect to be always dominated by the price effect. Third, Grossman and Rossi-Hansberg (2008) emphasize the beneficial effect of a reduction in the unit cost of offshoring on all offshored tasks (intensive margin), while we focus on the benefit of offshoring additional tasks (the extensive margin). In both cases, the efficiency/productivity effect exists, but its determinants are different.

[5] For given technology, our static model can be interpreted as a special case of Grossman and Rossi-Hansberg where the cost of offshoring is either zero or prohibitive.

Consider next the effect of offshoring on the level of wages, while keeping constant all other parameters. When $\kappa = 0$ the West is in autarky. When $\kappa = \bar{\kappa}$ the wage of low-skill workers is equalized in East and in the West. For a low range of κ an increase of offshoring may increases the real wage of low-skill workers in the West, due to the efficiency effect. However, for a high range of κ an increase of exogenous offshoring always reduces the real wage of low-skill workers in the West. More formally, the low-skill wage is given by

$$\begin{aligned} w_{l,w} &= \alpha P_l E_l^{\alpha} Y_l^{1-\alpha} x_{l,i}^{\alpha-1} \\ &= \alpha \left(1 + \left(\frac{A_h}{A_l} \frac{Z H_w}{\hat{L}} \right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{1}{\epsilon-1}} A_l \hat{L}^{1-\alpha} \left(\frac{1-\kappa}{L_w} \right)^{1-\alpha}. \end{aligned}$$

Again, the relationship between $w_{l,w}$ and κ can be decomposed into a *price effect*, an *efficiency effect* and a *labor supply* effect.

The price and labor supply effects make $w_{l,w}$ decrease with offshoring, whereas the efficiency effect makes $w_{l,w}$ increase with offshoring. Note that

$$\frac{d \ln w_{l,w}}{d\kappa} = \eta \frac{d\hat{L}}{d\kappa} - \frac{1-\alpha}{1-\kappa} \tag{13}$$

where $\eta = \frac{1-\alpha+(1-\alpha-1/\epsilon)\left(A_h Z H_w / A_l \hat{L}\right)^{\frac{\epsilon-1}{\epsilon}}}{\hat{L}+\hat{L}\left(A_h Z H_w / A_l \hat{L}\right)^{\frac{\epsilon-1}{\epsilon}}}$. As $\kappa \to \bar{\kappa}$, $\frac{d\hat{L}}{d\kappa} \to 0$, hence, both the price and the efficiency effect vanish. Thus, $w_{l,w}$ decreases unambiguously with offshoring at high level of κ. As $\kappa \to 0$, $\frac{d\hat{L}}{d\kappa} \to \infty$, hence, the labor supply effect becomes unimportant in determining the sign of the total effect. Thus, sign of the effect depends on the sign of η. Note that η is necessarily positive if $1-\alpha > 1/\epsilon$ (i.e., $\sigma < \epsilon$). However, if $1-\alpha < 1/\epsilon$ (i.e., $\sigma > \epsilon$), then η is negative if:

$$\frac{A_h Z H_w}{A_l \hat{L}} > \left(\frac{(1-\alpha)\epsilon}{1-\epsilon(1-\alpha)} \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{14}$$

Noting that $\lim_{\kappa \to 0} \hat{L} = L_w$ and using (12) and $\sigma = 1/(1-\alpha)$, this condition can be rewritten as:

$$\omega_w \frac{H_w}{L_w} > \frac{\epsilon}{\sigma - \epsilon}.$$

This condition is plausible. For example, in the US economy the ratio of college to high-school graduates is greater than one and the skill premium greater than 1.5. Imposing $\omega_w \frac{H_w}{L_w} = 1.5$ and $\epsilon = 1.6$, offshoring lowers the real wage of unskilled workers in the West whenever the elasticity of substitution across varieties is larger than 2.66, a value below the empirical estimates in the trade literature.[6]

[6] Once technology is endogenized, the condition will become:

$$\frac{H_w}{L_w} > \left(\frac{(1-\alpha)\epsilon}{1-\epsilon(1-\alpha)} \right)^{\frac{1}{\epsilon-1}}$$

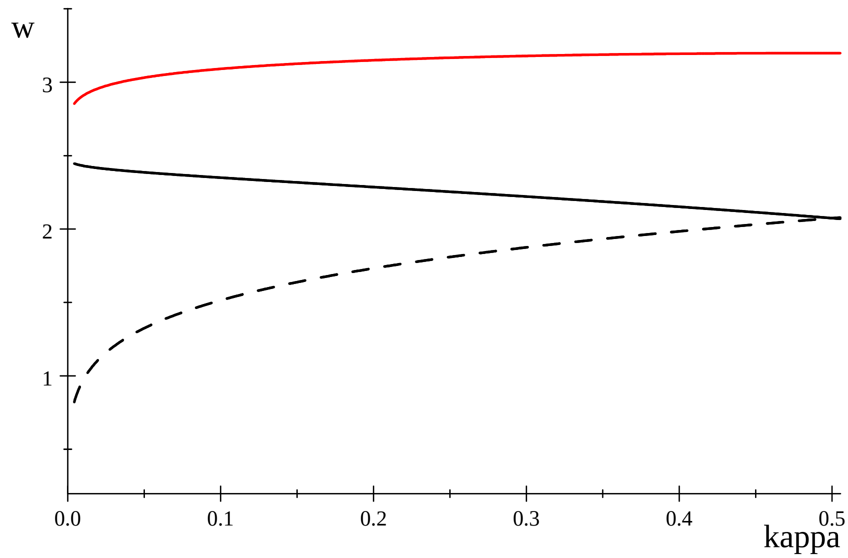

Figure 2: Offshoring and Wages. Red: $w_{h,w}$, Dashed: East, $\epsilon = 1.6$, $\sigma = 5$

The real wages of all other workers unambiguously increase with offshoring:

$$\begin{aligned} w_{h,w} &= \alpha Z P_h E_h^{\alpha} Y_h^{1-\alpha} \left(\frac{Z H_w}{A_h} \right)^{\alpha-1} = \alpha \left(\frac{Y}{A_h Z H_w} \right)^{\frac{1}{\epsilon}} Z A_h \\ w_{l,e} &= \alpha P_l E_l^{\alpha} Y_l^{1-\alpha} x_{l,e}^{\alpha-1} = \alpha \left(\frac{Y}{A_l} \right)^{\frac{1}{\epsilon}} A_l \hat{L}^{1-\alpha-\frac{1}{\epsilon}} \left(\frac{\kappa}{L_e} \right)^{1-\alpha}, \end{aligned}$$

where standard algebra can be used to show that $\hat{L}^{1-\alpha-\frac{1}{\epsilon}} \kappa^{1-\alpha}$ increases with κ. The effect of offshoring on the level of wages for the empirically more relevant case is depicted in Figure 2.

2.4 Endogenous Offshoring and Technology

In this section, we endogenize both technological change and offshoring. We assume that the cost of any innovation (i.e., $\dot{A}_l$ or $\dot{A}_h$) is equal to μ units of the numeraire. By paying an additional setup cost, f, a firm has the option to offshore the production of its variety to an Eastern partner. However, in this case the Western firm only appropriates a share $\tilde{\lambda} \leq 1$ of the profit flow. Different microfoundations are possible. One possibility is weak enforcement of international contracts in the East. For example, we may assume that due to weak institutions, at each point in time international contracts are enforced only with probability $\tilde{\lambda}$. If a contract is not enforced, the Eastern firm can hold up the Western firm, make a take it or leave it offer,

Again, this inequality is likley to hold for realistic parameters. For example, in the US, the ratio of college to high-school graduates is about 1.3. If we set $\epsilon \in [1.5, 2]$ then the condition is satisfied when the elasticity of substitution across varieties is larger than 3.5.

and get all of the surplus. In this case, $\tilde{\lambda}$ could be interpreted as the quality of contracting institutions, particularly as concerns foreigners. Other forms of profit sharing with a local partner would deliver the same result.

Denote by V_l^o the value of a firm that has paid the offshoring cost. The following Hamilton-Jacobi-Bellman (HJB) equation must hold:

$$rV_l^o = \max\{\pi_{l,w}, \tilde{\lambda}\pi_{l,e}\} + \dot{V}_l^o$$

that is, the return from owning the firm is equal to the profit rate plus and capital gain or loss. The max operator takes into account the fact that the firm will produce in the most profitable location. Next, consider a firm that produces a good in the West with the option to offshore it at the cost f. The HJB equation yields:

$$rV_l = \max\left\{\pi_{l,w} + \dot{V}_l,\ r\left(V_l^o - f\right)\right\},$$

where the max operator takes into account the fact that the firm has the option to pay the cost f and become offshore thereby changing its value to V_l^o. Finally, the value of a firm in the skill-intensive sector, with no option to offshore, must satisfy the usual HJB equation:

$$rV_h = \pi_{h,w} + \dot{V}_h.$$

Free-entry implies that the value of introducing a new intermediate and the value of offshoring the production of an existing intermediate cannot exceed the respective cost:

$$\begin{aligned} V_l^o - V_l &\leq f \\ V_l &\leq \mu \\ V_h &\leq \mu \end{aligned}$$

In a Balanced Growth Path (BGP) with positive innovation and offshoring all the free-entry condition must hold as equalities. This implies $V_l = V_h = \mu$, $V_l^o = f + \mu$ and $\dot{V}_l = \dot{V}_h = \dot{V}_l^o = 0$. Notably, offshoring does not affect the value of innovation directly, only through its effects on the profit flow. These equalities together with the HJB equations pin down the BGP interest rate:

$$r = \frac{\tilde{\lambda}\pi_{l,e} - \pi_{l,w}}{f} = \frac{\pi_{l,w}}{\mu} = \frac{\pi_{h,w}}{\mu}. \tag{15}$$

This equation, requiring the instantaneous return to any type of innovation and to offshoring to be equal to the interest rate, uniquely determine both the level of offshoring (κ) and the skill-bias of technology (A_h/A_l) along the BGP.

Consider offshoring first and define $\lambda \equiv \frac{\tilde{\lambda}}{f/\mu+1}$. This parameter varies from zero to one and is an index of offshorability (ranging from prohibitive to zero offshoring costs). From (15), offshoring will continue until $\lambda\pi_{l,e} = \pi_{l,w}$.[7] Substituting profits, the BGP level of offshoring (κ) can be found as a function of λ:

$$\lambda = \left(\frac{\kappa}{1-\kappa}\frac{L_w}{L_e}\right)^{\alpha} \tag{16}$$

$$\kappa = \left(1+\lambda^{-1/\alpha}L_w/L_e\right)^{-1} \tag{17}$$

Intuitively, better contract enforcement, lower offshoring costs and a larger supply of labor in the East make offshoring more attractive. Note also that the condition $\lambda\pi_{L,e} = \pi_{L,w}$ is independent of the equilibrium in the skill-intensive sector.

Substituting (17) into (8) yields West/East wage gap as a function of λ:

$$\frac{w_{l,w}}{w_{l,e}} = \left(\frac{L_e}{L_w}\frac{1-\kappa}{\kappa}\right)^{1-\alpha} = \lambda^{\frac{\alpha-1}{\alpha}}$$

Also, substituting (17) into (10) yields :

$$\hat{L} = \left[\frac{\lambda^{(1-\alpha)/\alpha}L_e + L_w}{\left(L_w+\lambda^{1/\alpha}L_e\right)^{1-\alpha}}\right]^{1/\alpha} \tag{18}$$

To find the direction of technical change, consider the relative BGP value of new innovations in the two sectors:

$$\frac{V_h}{V_l} = \frac{\pi_h}{\pi_l} = \frac{P_h}{P_l}\frac{ZH_w}{\hat{L}^{1-\alpha}\left(\frac{L_w}{1-\kappa}\right)^{\alpha}} \tag{19}$$

As in the canonical model of directed technical change, the relative value of new innovations depends on a price effect and a market size effect. As usual, the price effect encourages the development of innovations used in the sector producing relatively more expensive goods. The market size effect takes a new form, however. While the market size for skill-complement innovations is simply H_w, the effective market size in the other sector depends positively on L_w, L_e and the extent of offshoring. Imposing the BGP technology market clearing condition $V_l = V_h$ into (19), we can solve for the BGP ratio of technologies:

$$\frac{A_h}{A_l} = (ZH_w)^{\epsilon-1}\,\hat{L}^{1-\epsilon+\epsilon\alpha}\left(\frac{1-\kappa}{L_w}\right)^{\epsilon\alpha} = (ZH_w)^{\epsilon-1}\,\hat{L}^{1-\epsilon+\epsilon\alpha}\left(L_w+\lambda^{1/\alpha}L_e\right)^{-\epsilon\alpha} \tag{20}$$

This equation implies that when $\epsilon > 1$, an increase in the world supply of H_w, or $L_w + L_e$ (holding L_w/L_e constant, so as to keep κ unaffected) induces technological change complementing the factor that is becoming more abundant. More importantly, the direction of technological progress also depends on offshorability, in two ways. First, offshorability increases the overall

[7] Note that in a BGP, whenever a firm pays a positive offshoring cost, it must strictly prefer to produce in the East.

production of Y_l, due to the efficiency effect. If $\epsilon < \sigma$, (i.e., $1 - \epsilon + \epsilon\alpha > 0$) the price effect is strong enough that the resulting increase in P_h/P_l encourages Skill-Biased Technical Change (SBTC). If, on the other hand, $\epsilon > \sigma$ $(1 - \epsilon + \epsilon\alpha < 0)$ the relative price does not react enough and offshoring induces the development of innovations for the now more efficient L sector. Second, offshorability increases unambiguously the effective market size for unskilled technologies, because it makes it less costly to employ Eastern unskilled workers. This tends to induce innovation complementing unskilled workers. Given that the efficiency effect vanishes for high values of offshoring, more offshorability may initially lead to skill-biased technological change (if $\epsilon < \sigma$), but will eventually induce the development of technologies favoring unskilled labor.

More formally:

$$\frac{d\ln\left(A_h/A_l\right)}{d\lambda} = \left[\left(1-\epsilon+\epsilon\alpha\right)\frac{d\ln\hat{L}}{d\kappa} - \frac{\epsilon\alpha}{1-\kappa}\right]\frac{d\kappa}{d\lambda}. \tag{21}$$

When $\epsilon < \sigma$, this derivative is positive for small values of κ and turns negative for high values of κ. Since κ is unambiguously increasing in λ, then we can define, for future reference, the threshold value $\hat{\lambda}$ such that the above derivative is equal to zero. This is defined by the implicit function

$$\hat{\lambda} = \phi^{-1}\left(\frac{\epsilon\alpha^2}{\left(1-\epsilon+\epsilon\alpha\right)\left(1-\alpha\right)}\right) \tag{22}$$

where

$$\phi\left(\lambda\right) = \left(\hat{L}\left(\lambda\right)\right)^{-\alpha}\left(\left(\kappa\left(\lambda\right)\right)^{-\alpha}L_e^{\alpha} - \left(1-\kappa\left(\lambda\right)\right)^{-\alpha}L_w^{\alpha}\right)\left(1-\kappa\left(\lambda\right)\right)$$

which can be proven to be monotonically decreasing in λ.[8] In words, $\hat{\lambda}$ is the threshold value of λ such that, if $\lambda < \hat{\lambda}$ $(\lambda \geq \hat{\lambda})$, then $d\left(A_h/A_l\right)/d\lambda > 0$ $\left(d\left(A_h/A_l\right)/d\lambda \leq 0\right)$. Note that a lower ϵ strengthens the price effect, which tends to induce SBTC, and thus implies a higher $\hat{\lambda}$. The relationship between the BGP level of A_h/A_l and κ for the case $\epsilon < \sigma$ is represented in Figure 3.

Before proceeding, it may be useful to compare the results just derived with those obtained in models studying the impact of trade on the direction of technological progress, such as Acemoglu (2003) and Gancia, Muller and Zilibotti (2011). In those models, the equation for the relative profitability of skill-complement innovations, (19), simplifies to:

$$\frac{V_h}{V_l} = \frac{\pi_h}{\pi_l} = \frac{P_h}{P_l}\frac{ZH}{L}$$

where H and L are the relevant endowments. This corresponds to the autarky equilibrium in the present model. The effect of trade integration of the profitability of alternative innovations

[8] To drive this expression, note that:

$$\frac{d\ln\left(A_h/A_l\right)}{d\lambda} = \left[\left(1-\epsilon+\epsilon\alpha\right)\frac{1-\alpha}{\alpha}\hat{L}^{-\alpha}\left(\kappa^{-\alpha}L_e^{\alpha} - \left(1-\kappa\right)^{-\alpha}L_w^{\alpha}\right) - \frac{\epsilon\alpha}{1-\kappa}\right]\frac{d\kappa}{d\lambda}$$

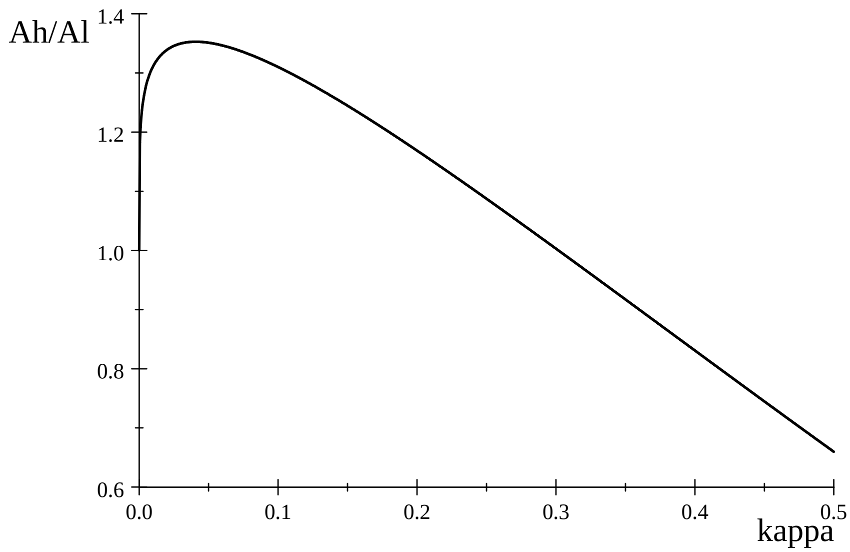

Figure 3: Offshoring and Directed Technical Change, $\epsilon = 1.6$, $\sigma = 5$

then depends on assumptions about the skill endowment in the free-trade equilibrium and the extent of international protection protection of Intellectual Property Rights (IPR). Consider the opening to trade of a large and skill-scarce country, such as China. If international IPR are not protected, the market size for new technologies will not change. Then, the only effect will be an increase in the world price of skill-intensive products (P_h/P_l), which will induce SBTC. If IPR are protected globally, instead, the market size will dominate the price effect and the larger endowment of unskilled workers in the world economy will make it profitable to invest in UBTC. Our model of offshoring nests these two extreme scenarios and predicts an endogenous switch from SBTC to UBTC, as international integration increases. Moreover, this result holds for any level of protection of IPR in the West.

What are the implications for the skill premium? Substituting (20) into (12), we can find the BGP skill premium as:

$$\omega_w = Z^{\epsilon-1} H_w^{\epsilon-2} \hat{L}^{1-\epsilon+\epsilon\alpha} \left(\frac{L_w}{1-\kappa} \right)^{1-\epsilon\alpha} = Z^{\epsilon-1} H_w^{\epsilon-2} \hat{L}^{1-\epsilon+\epsilon\alpha} \left(L_w + \lambda^{1/\alpha} L_e \right)^{1-\epsilon\alpha}. \qquad (23)$$

In the extreme cases of prohibitive and zero offshoring costs, the skill premium is a function of the relative endowment of skilled labor in the West and in the entire world, respectively:

$$\begin{aligned} \lambda &= 0 : \omega_w = Z^{\epsilon-1} \left(\frac{H_w}{L_w} \right)^{\epsilon-2} \\ \lambda &= 1 : \omega_w = Z^{\epsilon-1} \left(\frac{H}{L} \right)^{\epsilon-2}. \end{aligned}$$

As in standard models of directed technical change, the relationship between the skill premium and the relative supply of skill is increasing whenever $\epsilon > 2$. In the more interesting intermediate cases, $\lambda \in (0, 1)$, the effect of offshoring on the skill premium is likely to be non-monotonic and this crucially depends on ϵ and α. This can be seen by differentiating (23) with respect to λ:

$$\frac{\partial \ln \omega_w}{\partial \lambda} = \left[(1 - \epsilon + \epsilon\alpha) \frac{\partial \ln \hat{L}}{\partial \kappa} - \frac{1 - \epsilon\alpha}{1 - \kappa} \right] \frac{\partial \kappa}{\partial \lambda}. \tag{24}$$

For low levels of offshoring, the efficiency effect ($\hat{L}$) is the dominating force. Focusing again on the case $\epsilon < \sigma = 1/(1 - \alpha)$, the skill premium will then increase with λ for two reason: the static effects discussed in the previous section and SBTC. For high levels of offshoring ($\lambda \to 1$), instead, the efficiency effect disappears ($\frac{\partial \ln \hat{L}}{\partial \kappa} = 0$) and UBTC tends to lower wage inequality. Equation (24) then shows that the skill premium will fall with λ whenever $\epsilon > \frac{1}{\alpha}$. Thus, the long-run relationship between ω_w and λ is inverted-U shaped if $\frac{1}{\alpha} < \epsilon < \frac{1}{1-\alpha}$. Note that this outcome is more likely the higher the substitutability between L-complement intermediates (high α). If the elasticity of substitution between varieties, $\sigma = 1/(1 - \alpha)$, is equal to 5, which is a conventional value in the trade literature, the hump shape holds for $\epsilon \in (1.25, 5)$, which is well within consensus estimates. Even with a conservative value of $\sigma = 3$, the implied range for ϵ remains highly plausible, $\epsilon \in (1.5, 2.9)$.[9]

Figure 4 shows the relationship between the skill premium and offshoring for a conventional parametrization $\epsilon = 1.6$ and $\sigma = 5$. The red line holds technology constant at the autarky level, while the black line represents the skill premium once technology has converged to the new BGP equilibrium. As the figure makes it clear, the endogenous reaction of technology provides a strong amplification of the impact of offshoring on the skill premium for low levels of integration, while this pattern is reverted for high levels of offshoring. Thus, the combination of offshoring together with directed technical change can explain a large surge in the skill premium even for low levels of trade between the West and East.

Finally, we can find the BGP growth rate of the world economy as:

$$g = r - \rho \tag{25}$$

where the interest rate can be obtained from the free-entry condition for innovation:

$$\begin{aligned} r &= \pi_h/\mu = (1 - \alpha) P_h Z H_w/\mu \\ &= \frac{1 - \alpha}{\mu} \left\{ \left[\hat{L}^{1-\alpha} \left(L_w + \lambda^{1/\alpha} L_e \right)^{\alpha} \right]^{\epsilon - 1} + (Z H_w)^{\epsilon - 1} \right\}^{\frac{1}{\epsilon - 1}}. \end{aligned} \tag{26}$$

[9] If α is instead lower than 0.5 ($\sigma < 2$), the opposite case may also arise: if $\frac{1}{\alpha} > \epsilon > \frac{1}{1-\alpha}$, then the relationship between ω_w and λ is U shaped. However, the parameter condition for this to be the case does not seem very realistic.

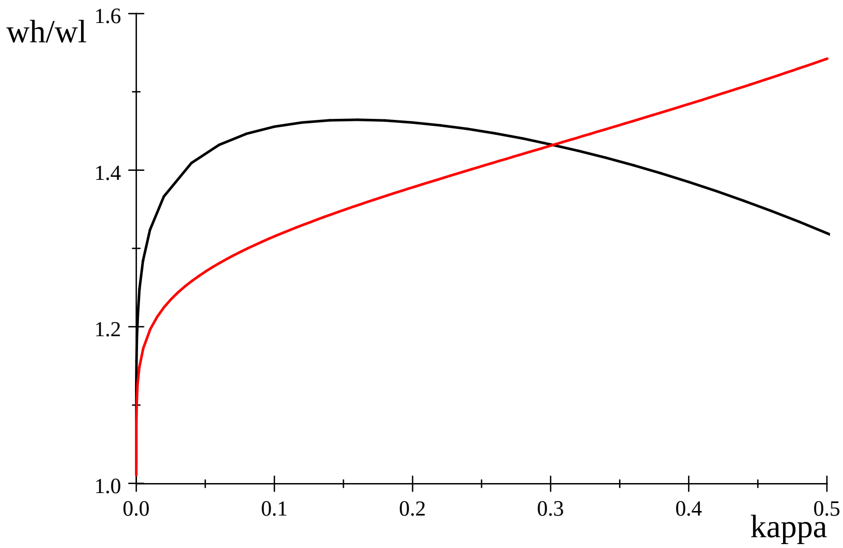

Figure 4: Offshoring and the Skill Premium. Red: constant (A_h, A_l); $\epsilon = 1.6$, $\sigma = 5$

Offshoring increases the growth rate of the world economy. Consumption also grows at the rate g, and can be written as

$$\begin{aligned} C &= \left[\frac{Y}{A_l} - \mu g\left(\frac{A_h}{A_l} + 1\right)\right] A_l \\ &= \left[\left(\hat{L}^{\frac{\epsilon-1}{\epsilon}} + \left(\frac{A_h}{A_l} Z H_w\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} - \mu\left(r-\rho\right)\left(1 + \frac{A_h}{A_l}\right)\right] A_l \equiv \xi A_l. \end{aligned}$$

3 Transitional Dynamics

In this section, we consider the positive implications of an unexpected increase in offshoring opportunities, parameterized by an unexpected decrease in f. Before the shock, the economy is assumed to be in a BGP with positive growth. We established above that the sign of the impact effect of offshoring on wages is in general ambiguous. In particular, there may exist a low-offshoring range such that all wages in the West increase as offshoring increases. However, when condition (14) is satisfied, offshoring unambiguously decreases the low-skill wage at all initial level of offshoring. To avoid a taxonomic presentation, we focus on this case, that appears to be empirically relevant.

We start by stating the following useful results:

Lemma 1 *If at $t = s$ we have $V_z = \mu$, with $z = \{h, l\}$, then $V_z = \mu$ for all $t > s$. Similarly, if at $t = s$ we have $V_l^o - V_l = f$, then $V_l^o - V_l = f$ for all $t > s$.*

Intuitively, if a free-entry condition holds as an equality, then it cannot be expected to became a strict inequality in the future or else agents would realize a capital loss.

Lemma 2 *If $V_l, = V_h = \mu$ and $V_l^o - V_l = f$, then the economy is in the BGP.*

To see this, note that there is only one value of κ consistent $V_h = \mu$ and $V_l^o - V_l = f$. Given that value of κ, there is only one value of A_h/A_l consistent with $V_h = \mu$.

Recall here that the new steady state offshoring is given by:

$$\kappa = \left(1 + \lambda^{-1/\alpha} L_w / L_e\right)^{-1},$$

where $\lambda = \frac{\tilde{\lambda}}{f/\mu+1}$. A decrease in f corresponds to an increase in λ which implies a higher κ in the new BGP. During the transition, standard properties of the models of directed technical change apply: there is innovation in one sector only (either skilled or unskilled), which one depending on the value of λ. However, it is possible that κ changes during the transition simultaneously with one type of innovation.

Proposition 1 *For all $\lambda < \hat{\lambda}$, a (small) increase in offshoring triggers skill-biased technical change (case SBTC), whereas, for all $\lambda \geq \hat{\lambda}$, any increase triggers low-skill-biased technical change (case UBTC), where $\hat{\lambda}$ is defined in (22).*

This proposition follows immediately from the Corollaries above and the effect of κ on the BGP skill-bias of technology (21).

Recall:

$$\begin{aligned} r &= \max\{r_{off}, r_h, r_l\} \\ r_{off} &= \frac{\tilde{\lambda}\pi_{l,e} - \pi_{l,w}}{f}, \; r_h = \frac{\pi_{h,w}}{\mu}, \; r_l = \frac{\pi_{l,w}}{\mu}. \end{aligned}$$

In the initial BGP we have $r_{off} = r_h = r_l$. Suddenly and unexpectedly, f falls. Given that $\pi_{l,e}$, $\pi_{l,w}$ and $\pi_{h,w}$ are unchanged, it must be that

$$r = \frac{\tilde{\lambda}\pi_{l,e} - \pi_{l,w}}{f} > r_h = r_l. \tag{27}$$

Thus, after the shock, there is an initial stage in which innovation stops and there is only offshoring. Namely, the decrease in the cost of offshoring triggers offshoring investments that causes a jump in the interest rate, which in turn makes any innovation unattractive. In this first stage:

$$\begin{aligned} V_l^o - V_l &= f \\ \max\{V_l, V_h\} &< \mu. \end{aligned}$$

As κ increases, $\tilde{\lambda}\pi_{l,e} - \pi_{l,w}$ falls: the wage differential between the West and the East falls, causing a reduction in the profit gap. Consequently, r must fall over time. Meanwhile, P_l falls and P_h grows, due to the increasing efficiency of the low-skill sector. The wages of the low-skill workers in the West fall. The wages of Eastern workers and skilled workers in the West increase. Whether the offshoring only phase is followed by SBTC or UBTC depends on how $\frac{\pi_{h,w}}{\pi_{l,w}}$ varies with κ, which depends on the level of κ. From (21) we can see that, in the case of interest ($\epsilon < \sigma$), offshoring will trigger SBTC (UBTC) if κ is sufficiently low (high). Therefore there are two cases:

1. For $\lambda < \hat{\lambda}$, at some point, the strict inequality $V_h < \mu$ ceases to hold and we start having skill-bias innovation. In the second stage (SBTC+offshoring), we have $V_l^o - V_l = f$, $V_h = \mu$ and $V_l < \mu$. The interest rate is given by:

$$r = \frac{\tilde{\lambda}\pi_{l,e} - \pi_{l,w}}{f} = \frac{\pi_{h,w}}{\mu}.$$

   The condition:

$$\frac{\tilde{\lambda}\pi_{l,e} - \pi_{l,w}}{\pi_{h,w}} = \frac{P_l}{P_h}\frac{\hat{L}^{1-\alpha}\left(\tilde{\lambda}\left(\frac{L_e}{\kappa}\right)^{\alpha} - \left(\frac{L_w}{1-\kappa}\right)^{\alpha}\right)}{ZH_w} = \frac{f}{\mu},$$

   pins down the critical value of κ where the second stage starts, and also the ratio κ/A_h afterward. Solving it for A_h, it yields:

$$A_h = \frac{A_l\hat{L}^{1-\epsilon+\epsilon\alpha}}{(ZH_w)^{1-\epsilon}}\left[\tilde{\lambda}\left(\frac{L_e}{\kappa}\right)^{\alpha} - \left(\frac{L_w}{1-\kappa}\right)^{\alpha}\right]^{-\epsilon}\left(\frac{f}{\mu}\right)^{\epsilon},$$

   which is an increasing function of κ. Thus, in this phase we have $\dot{\kappa} > 0$ and $\dot{A}_h > 0$. Arbitrage guarantees that κ and A_h will grow so as to keep both offshoring and SBTC equally profitable. Note that, during this transition stage, P_h falls and P_l grows, as the expression above shows (note that κ continues to grow). The wage of skilled workers in the West and that of Chinese workers continues to grow, while the sign of the change of the low-skill workers is potentially ambiguous. Yet, it is clear that the skill premium increases further. Eventually, the price adjustment leads to the new balanced growth equilibrium, where there are both types of innovations, and κ remains constant at the new higher level. All wages grow at a rate which is higher than in the initial level.

2. For $\lambda \geq \hat{\lambda}$, at some point, the strict inequality $V_l < \mu$ ceases to hold, due to the fall of the wages of skilled workers in the West. Then, we start having low-skill-bias innovation. In the second stage (UBTC+offshoring), we have $V_l^o - V_l = f$, $V_l = \mu$ and $V_h < \mu$. The

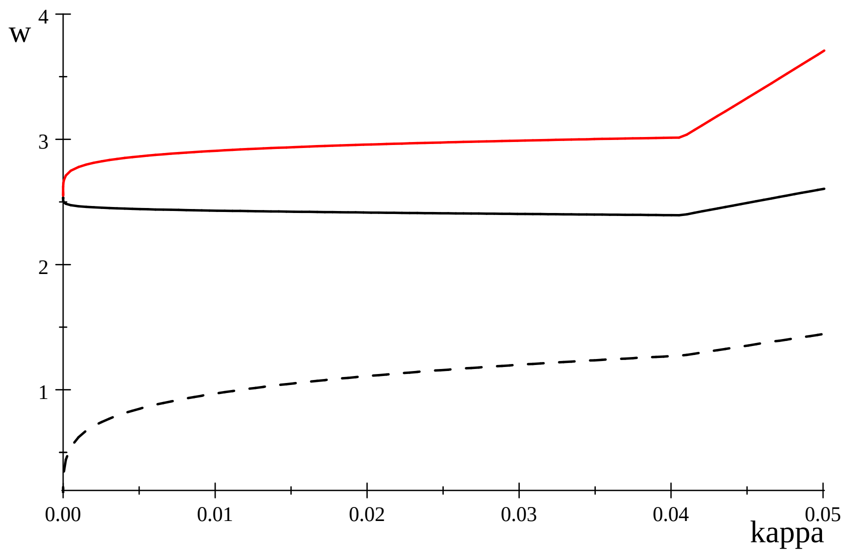

Figure 5: Red: $w_{h,w}$, Dashed: East, $\epsilon = 1.6$, $\sigma = 5$

interest rate is given by:

$$r = \frac{\tilde{\lambda}\pi_{l,e} - \pi_{l,w}}{f} = \frac{\pi_{l,w}}{\mu}$$

$$\Rightarrow \frac{\tilde{\lambda}\pi_{l,e} - \pi_{l,w}}{\pi_{l,w}} = \tilde{\lambda}\left(\frac{L_e}{L_w}\frac{1-\kappa}{\kappa}\right)^{\alpha} - 1 = \frac{f}{\mu}.$$

Note that in this stage there is offshoring, but κ remains constant. In this stage, the wages of all workers increase and the skill premium falls. Eventually, the price adjustment (P_h/P_l grows) leads to the new balanced growth equilibrium, where there are both types of innovations, and κ remains constant at the new higher level. All wages grow at a rate which is higher than in the initial level. The skill premium in the West may be higher or lower than in the initial steady state depending on parameters.

Finally, note that, for $\lambda < \hat{\lambda}$, large increases in λ have an ambiguous effect on the direction of technical change. If the change is sufficiently large, then we may observe UBTC instead of SBTC. This is due to the inverse-U shaped relationship between λ and A_h/A_l (see equation (20)).

4 Welfare Analysis (INCOMPLETE)

We now assess the welfare effect of an unexpected fall in the cost of offshoring for different types of workers. This exercise requires computing the discounted utility of agents both along the transitional dynamics and in the new BGP. Since we do not have closed form solutions for the time path of wages and interest rates along the transition, we can only perform this exercise numerically. Before discussing the simulations, it is nonetheless useful to derive some further analytic results.

Using preferences and the Euler equation, utility of an agent i at time zero is:

$$U_{i,0} = \int_0^\infty e^{-\rho t} \ln C_{i,t} dt = \frac{\ln C_{i,0}}{\rho} + \int_0^\infty e^{-\rho t} \left(\int_0^t r_s ds - \rho t \right) dt. \tag{28}$$

Initial consumption at time zero, $C_{i,0}$, can be found combining the Euler equation and the lifetime budget constraint:

$$C_{i,0} = \rho \left[\int_0^\infty w_{i,t} \exp \left(- \int_0^t r_s ds \right) dt + a_{i,0} \right]$$

where $w_{i,t}$ is the wage of agent i and $a_{i,0}$ is the value of asset holdings at $t = 0$. The welfare effect of a shock can be decomposed into an impact effect, i.e., the instantaneous jump in the level of consumption $C_{i,0}$, and a growth effect, which includes the growth rate along the transition and the higher growth once in the new BGP. A shock to offshorability affects the initial level of consumption both by changing the present discounted value of wages and the initial asset position. To understand the latter effect, note that the only assets in the economy are firms, whose value is given by:

$$V_{j,0} = \int_0^\infty \exp \left[- \int_0^t r_s ds \right] \pi_{j,t} dt.$$

Along a BGP, $V_{j,0} = \mu$. Yet, during the first phase of the transition (offshoring only) we must have $V_{j,0} < \mu$. Thus, after the shock hits the economy at $t = 0$ the owners of firms must incur in a capital loss.

In the absence of the shock, utility at $t = 0$ along the old BGP would instead be given by (28) after imposing:

$$\begin{aligned} \int_0^t r_s ds &= (g^* + \rho) t \\ w_{i,t} &= w_{i,0} e^{g^* t} \end{aligned}$$

where a star denote BGP values. This yields:

$$U_{i,0}^* = \frac{\ln C_{i,0}^*}{\rho} + \frac{g_0^*}{\rho^2}$$

where

$$C_{i,0}^* = w_{i,t}^* + \rho a_{i,0}^*.$$

For the simulation, we set the following parameter values:

$$H_w = L_w = L_e, \ \epsilon = 1.6, \ Z = 1.8, \ \rho = 0.04$$

The elasticity of substitution between high- and low-skill workers, ϵ, and the discount rate, ρ, are taken among the conventional values in the literature. Z is chosen so as to obtain a skill premium in autarky around 1.5. We then consider two possible values for the elasticity of substitution between intermediates, $\alpha = 0.8$ ($\sigma = 5$) and $\alpha = 0.7$ ($\sigma = 3.33$). The offshoring shock is chosen so as to generate a transition from $\kappa_0 = 0.02$ to $\kappa_T = 0.05$. Although this is a fairly small shock, its impact on the skill premium is significant, around +4.6%. Finally, the cost of innovation is chosen so as to obtain an annual BGP growth rate around 1% in a low-growth scenario and around 2% in a high-growth scenario.

We attribute to each group of workers (skilled and unskilled) an initial share of the assets in the economy proportional to their PDV of wages in the initial BGP. We then compute the welfare effect of the offshoring shock for skilled and unskilled workers expressed as the equivalent change in their level of consumption in the old BGP, Δc_h^* and Δc_l^*. The results are reported in the following table.

	$\alpha = 0.8$		$\alpha = 0.7$	
	$\Delta\omega = +4.6\%$		$\Delta\omega = +4.7\%$	
$g = 1\%$	$\Delta c_l^* = -1\%$;	$\Delta c_h^* = +2\%$	$\Delta c_l^* = -0.27\%$,	$\Delta c_h^* = +3\%$
$g = 2\%$	$\Delta c_l^* = -0.19\%$;	$\Delta c_h^* = +3\%$	$\Delta c_l^* = +0.6\%$,	$\Delta c_h^* = +4\%$

While the world representative consumer is always better off, there are important distributional effects of offshoring. The Chinese gain the most (not reported), and the high-skill workers in the West also experience positive welfare gains. However, low-skill workers in the West may lose. Whether this is the case or not hinges crucially on the size of the dynamic effects. For instance, if technology does not respond, or respond little, there are little dynamic gains and distributional effects dominate. In general, whether we are in times of large or small growth opportunities is crucial. Moreover, gains are in general larger when intermediates are more complementary (low α).

5 H-Offshoring

The model can be extended to study offshorability of skill-intensive intermediates. To do so, we now assume that the East is endowed with H_e units of skilled labor, but that the West is skill abundant: $H_w/L_w > H_e/L_e$. For simplicity, we restrict the analysis to the BGP.

It is immediate to verify that, for given technology (A_h, A_l) and offshoring (κ_h, κ_l), the skill premium in the West and in the East is:

$$\omega_w = \left(\frac{ZA_h}{A_l}\right)^{1-1/\epsilon}\left(\frac{\hat{H}}{\hat{L}}\right)^{-1/\epsilon}\left(\frac{\hat{H}}{\hat{L}}\right)^{1-\alpha}\left(\frac{L_w}{H_w}\frac{1-\kappa_h}{1-\kappa_l}\right)^{1-\alpha}$$

$$\omega_e = \left(\frac{ZA_h}{A_l}\right)^{1-1/\epsilon}\left(\frac{\hat{H}}{\hat{L}}\right)^{-1/\epsilon}\left(\frac{\hat{H}}{\hat{L}}\right)^{1-\alpha}\left(\frac{L_e}{H_e}\frac{\kappa_h}{\kappa_l}\right)^{1-\alpha}.$$

where $\hat{H} \equiv \left[\kappa^{1-\alpha}H_e^{\alpha} + (1-\kappa)^{1-\alpha}H_w^{\alpha}\right]^{1/\alpha}$. Comparative statics to changes in (κ_h, κ_l) follow directly from the baseline case.

More interesting results arise when offshoring is endogenous. Assuming as a benchmark that offshoring costs are the same in the two sectors, equilibrium offshoring is pin down by the conditions:

$$\lambda\pi_{l,e} = \pi_{l,w}$$

$$\lambda\pi_{h,e} = \pi_{h,w}.$$

Substituting profits, these yield:

$$\kappa_l = \left(1+\lambda^{-1/\alpha}L_w/L_e\right)^{-1}$$

$$\kappa_h = \left(1+\lambda^{-1/\alpha}H_w/H_e\right)^{-1}.$$

Since the East is skill-scarce, it is easy to see that the relative extent of offshoring, $\frac{\kappa_l}{\kappa_h}$, declines monotonically from $\frac{H_w/H_e}{L_w/L_e}$ to $\frac{1+H_w/H_e}{1+L_w/L_e}$. Interestingly, offshoring is endogenously more prevalent in the L-sector, to take advantage of the large supply of L workers in the East. As λ increases, however, offshoring increases relatively more in the "lagging" H sector. This pattern, represented graphically in Figure , accords well with the available evidence.

Next, note that the indifference conditions between domestic and offshore production imply that the international wage gap is a constant function of λ:

$$\frac{w_{l,w}}{w_{l,e}} = \frac{w_{h,w}}{w_{h,e}} = \lambda^{\frac{\alpha-1}{\alpha}}.$$

It follow immediately that the skill premium is the same in both countries. This is a remarkable result: offshoring generates conditional factor price equalization even if the two countries are fully specialized and have different technology capabilities.

The BGP skill premium is now:

$$\omega_w = \omega_e = Z^{\epsilon-1}\left(\frac{\hat{L}}{\hat{H}}\right)^{1-\epsilon+\epsilon\alpha}\left(\frac{L_w+\lambda^{1/\alpha}L_e}{H_w+\lambda^{1/\alpha}H_e}\right)^{1-\epsilon\alpha}.$$

Note that an increase in λ raises both terms in parenthesis, because East is skill-scarce. Intuitively, offshoring has a larger impact in the L sector, because the East has a large endowment of L workers. As a result, comparative statics to changes in λ are similar to the baseline case (in which all the terms now at the denominator were constant). In particular, the relationship between λ and ω_w is still likely to be non-monotonic. In this case, for sufficiently low levels of offshoring, the extended model is consistent with the evidence of offshoring raising skill premia both in the origin and in the destination country.

6 Conclusions

Offshoring of jobs to low-wage countries and skill-biased technical change are among the most visible and debated trends in the US labor markets. In these paper, we have argued that these two phenomena are deeply interlinked and that the interaction between the two is important in explaining the dynamics of wages. By building a model where both offshoring and the skill-bias of technology are endogenous and depend on each others, our analysis has uncovered a number of new results.

First, we have shown that a decline in the cost of offshoring tasks has in general ambiguous effects on both the level of wages, the skill premium and the direction of technological progress. Despite this complexity, our framework is tractable enough to identify the contrasting effects and the parameters governing the alternative outcomes. In our preferred parametrization, we have found that starting from an equilibrium with a low volume of offshoring a decline in offshoring costs triggers a transition characterized initially by falling real wages for unskilled workers in the West and followed by skill-biased technical change. These predictions accord well with the available evidence from the US labor market. Once the volume of offshoring reaches a critical level, however, the model suggests that further offshoring will be followed by unskilled-biased technical change and a lower skill premium.

Contrary to the conventional wisdom that low volumes of trade are unlikely to have significant effects on wages, our model implies that the globalization of production should have the largest impact on the skill premium precisely for moderate levels of integration. Moreover, once offshoring of both skilled and unskilled tasks is introduced, our model yields a form of conditional factor price equalization so that offshoring has the same effect on skill premia worldwide. This result contributes to explaining why, contrary to the standard Stolper-Samuelson theorem, globalization seems to be associated to higher skill premia even in skill-scarce countries.

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A Appendix

A.1 Full Dynamics (Incomplete)

In this section, we analyze the transitional dynamics of the system after a shock - unexpected decrease in offshoring cost f. Recall that we assume that we start on a balanced growth path characterized by a higher f, such that

$$\begin{aligned}
\kappa_0 &= \left(1+\lambda_0^{-1/\alpha} L_w/L_e\right)^{-1} \\
\frac{A_{h,0}}{A_{l.0}} &= (ZH_w)^{\epsilon-1}\, \hat{L}_0^{1-\epsilon+\epsilon\alpha} \left(L_w+\lambda_0^{1/\alpha} L_e\right)^{-\epsilon\alpha} \\
&= (ZH_w)^{\epsilon-1} \left[\kappa_0^{1-\alpha} L_e^{\alpha} + (1-\kappa_0)^{1-\alpha} L_w^{\alpha}\right]^{\frac{1-\epsilon+\epsilon\alpha}{\alpha}} \left(L_w+\lambda_0^{1/\alpha} L_e\right)^{-\epsilon\alpha}
\end{aligned}$$

A.1.1 Case 1: $\lambda < \hat{\lambda}$

In this case (for small shocks), we know from Lemmas 1 and 2 that there are three stages

1. only offshoring: κ grows and both A_l and A_h remain constant;
2. offshoring +SBTC: : κ and A_h grow whereas A_l remains constant;
3. BGP: κ is constant (but there is offshoring) and both A_l and A_h grow at a constant rate.

During stages 1 and 2, we have the following system of dynamic equations:

$$\begin{aligned}
\frac{\dot{C}_t}{C_t} &= r\left(A_{h,t}, A_{l,0}, \kappa_t\right) - \rho \\
\mu \dot{A}_{h,t} + f A_{l,0} \dot{\kappa}_t &= Y\left(A_{h,t}, A_{l,0}, \kappa_t\right) - C_t
\end{aligned}$$

where κ_0 is given by the pre-shock steady-state condition, $\kappa_0 = \left(1+\lambda_0^{-1/\alpha} L_w/L_e\right)^{-1}$, and

$$\begin{aligned}
r\left(A_{h,t}, A_{l,t}, \kappa_t\right) &= \left(\frac{Y\left(A_{h,t}, A_{l,t}, \kappa_t\right)}{A_{l,0}\hat{L}\left(\kappa_t\right)}\right)^{\frac{1}{\epsilon}} \frac{(1-\alpha)\left(\hat{L}\left(\kappa_t\right)\right)^{1-\alpha}\left(\tilde{\lambda}\left(\frac{L_e}{\kappa_t}\right)^{\alpha} - \left(\frac{L_w}{1-\kappa_t}\right)^{\alpha}\right)}{f} \\
Y\left(A_{h,t}, A_{l,t}, \kappa_t\right) &= \left(1+\left(\frac{A_{l,t}\hat{L}\left(\kappa_t\right)}{A_{h,t} Z H_w}\right)^{\frac{\epsilon-1}{\varepsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} A_{h,t} Z H_w \\
\hat{L}\left(\kappa_t\right) &= \left[\kappa_t^{1-\alpha} L_e^{\alpha} + (1-\kappa_t)^{1-\alpha} L_w^{\alpha}\right]^{1/\alpha}
\end{aligned}$$

Clearly, in stage 1, $A_{h,t} = A_{h,0}$ and $\dot{A}_{h,t} = 0$. During stage 2, we have:

$$A_h\left(\kappa_t\right) = \frac{A_{l,0}\left(\hat{L}\left(\kappa_t\right)\right)^{1-\epsilon+\epsilon\alpha}}{(ZH_w)^{1-\epsilon}} \left(\tilde{\lambda}\left(\frac{L_e}{\kappa_t}\right)^{\alpha} - \left(\frac{L_w}{1-\kappa_t}\right)^{\alpha}\right)^{-\epsilon} \left(\frac{f}{\mu}\right)^{\epsilon}.$$

Thus, we can rewrite the dynamic system of stage 1 as:

$$\begin{aligned}
\frac{\dot{C}_t}{C_t} &= r\left(A_{h,0}, A_{l,0}, \kappa_t\right) - \rho, \\
f A_{l,0} \dot{\kappa}_t &= Y\left(A_{h,0}, A_{l,0}, \kappa_t\right) - C_t.
\end{aligned} \tag{29}$$

and that of stage 2 as:

$$\begin{aligned}\frac{\dot{C}_t}{C_t} &= r\left(A_h\left(\kappa_t\right), A_{l,0}, \kappa_t\right) - \rho, \qquad (30)\\ \left(\mu \frac{d}{d\kappa_t} A_h\left(\kappa_t\right) + f A_{l,0}\right)\dot{\kappa}_t &= Y\left(A_h\left(\kappa_t\right), A_{l,0}, \kappa_t\right) - C_t.\end{aligned}$$

It is now useful to define two thresholds, κ_1 and κ^*. The first is the level of κ such that one transits from stage 1 to stage 2, i.e., such that SBTC becomes positive. The second is the balanced growth threshold (such that even UBTC becomes profitable). Then, κ_1 and κ^* are defined by the following conditions

$$\begin{aligned} r_{off}\left(A_{h,0}, A_{l,0}, \kappa_1\right) &= r_h\left(A_{h,0}, A_{l,0}, \kappa_1\right)\\ r_{off}\left(A_h\left(\kappa^*\right), A_{l,0}, \kappa^*\right) &= r_h\left(A_h\left(\kappa^*\right), A_{l,0}, \kappa^*\right)\end{aligned}$$

where

$$\begin{aligned} r_{off}\left(A_{h,t}, A_{l,t}, \kappa_t\right) &= \left(\frac{Y\left(A_{h,t}, A_{l,t}, \kappa_t\right)}{A_{l,t}\hat{L}\left(\kappa_t\right)}\right)^{\frac{1}{\epsilon}} \frac{(1-\alpha)\left(\hat{L}\left(\kappa_t\right)\right)^{1-\alpha}\left(\tilde{\lambda}\left(\frac{L_e}{\kappa_t}\right)^{\alpha} - \left(\frac{L_w}{1-\kappa_t}\right)^{\alpha}\right)}{f},\\ r_l\left(A_{h,t}, A_{l,t}, \kappa_t\right) &= \left(\frac{Y\left(A_{h,t}, A_{l,t}, \kappa_t\right)}{A_{l,t}\hat{L}_t}\right)^{\frac{1}{\epsilon}} \frac{(1-\alpha)\left(\hat{L}\left(\kappa_t\right)\right)^{1-\alpha}\left(\frac{L_w}{1-\kappa_t}\right)^{\alpha}}{\mu},\\ r_h\left(A_{h,t}, A_{l,t}, \kappa_t\right) &= \left(\frac{Y\left(A_{h,t}, A_{l,t}, \kappa_t\right)}{A_{h,t} Z H_w}\right)^{\frac{1}{\epsilon}} \frac{(1-\alpha) Z H_w}{\mu}.\end{aligned}$$

Using the expressions above, we can show that κ_1 is unique (under the maintained assumption that $1-\alpha < 1/\epsilon$). In particular, it is the unique solution to the equation

$$\left(\hat{L}\left(\kappa_1\right)\right)^{1-\alpha-1/\epsilon}\left(\tilde{\lambda}\left(\frac{L_e}{\kappa_1}\right)^{\alpha} - \left(\frac{L_w}{1-\kappa_1}\right)^{\alpha}\right) = \left(\frac{A_{l,0}}{A_{h,0}}\right)^{\frac{1}{\epsilon}} \frac{f\left(ZH_w\right)^{1-1/\epsilon}}{\mu}, \qquad (31)$$

where uniqueness is guaranteed by the fact that the LHS is decreasing in κ_1. κ^* admits a closed-form expression, given by

$$\kappa^* = \left(1 + \lambda^{-1/\alpha} L_w / L_e\right)^{-1} > \kappa_1. \qquad (32)$$

DOUBLE CHECK THIS INEQUALITY

In stage 3 (BGP), we know that C is proportional to Y, A_l and A_h. Let g denotes the steady-state growth rate. Then:

$$\begin{aligned}\frac{C}{A_l} &= \left(\frac{Y\left(A_{h,t}, A_{l,t}, \kappa^*\right)}{A_{l,t}} - \mu g\left(a\left(\kappa^*\right)+1\right)\right)\\ &= \left[\left(\left(\hat{L}\left(\kappa^*\right)\right)^{\frac{\epsilon-1}{\epsilon}} + \left(a\left(\kappa^*\right) H_w\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} - \mu\left(r\left(\kappa^*\right) - \rho\right)\left(1 + a\left(\kappa^*\right)\right)\right] \equiv \xi_l\left(\kappa^*\right).\end{aligned}$$

where $r\left(\kappa^*\right)$ is the steady-state interest rate and $a\left(\kappa^*\right)$ denotes the steady-state skill bias:

$$a\left(\kappa^*\right) \equiv \frac{A_h}{A_l} = \left(ZH_w\right)^{\epsilon-1}\left[\left(\kappa^*\right)^{1-\alpha} L_e^{\alpha} + \left(1-\kappa^*\right)^{1-\alpha} L_w^{\alpha}\right]^{\frac{1-\epsilon+\epsilon\alpha}{\alpha}}\left(L_w + \lambda^{1/\alpha} L_e\right)^{-\epsilon\alpha}.$$

Thus, if a trajectory converges to the BGP at time $t = T$, it must feature

$$C_T = \xi_l(\kappa^*) \cdot A_{l,0}.$$

This is the "terminal condition" for the transition in stages 1 and 2 (we will later show that any trajectory not converging to BGP cannot be optimal).

To summarize, we have

- an initial condition κ_0;
- an autonomous system of differential equation given by (29) if $\kappa \in [\kappa_0, \kappa_1]$ and (30) if $\kappa \in [\kappa_1, \kappa^*]$
- a terminal condition, $C_T = \xi_l(\kappa^*) \cdot A_{l,0}$.

Note that C_T is not a fixed point of the dynamic system, since consumption is strictly increasing at that point. So, T will be finite. By the standard properties of vector fields there is one and only one path linking the initial and the terminal condition. Moreover, we know that along such path $r(A_{h,t}, A_{l,0}, \kappa_t) > \rho$ and $Y(A_{h,t}, A_{l,0}, \kappa_t) > C_t$. Thus, convergence is monotonic: in the first stage, C and κ grow, and in the second stage C, κ and A_h all grow. In conclusion, there exists one unique path converging to the BGP. Standard arguments show that the transversality condition hold along that BGP.

Next, we must rule out that there are other trajectories in the vector field that are equilibria. There are two types of trajectories to be considered. First, there are trajectories that reach $\kappa_t = \kappa^*$ in finite time. For such trajectories standard arguments in models of DTC rule out that anything but the BGP is an equilibrium. In particular, recall that at $\kappa_t = \kappa^*$ the three arbitrage conditions hold simultaneously (irrespective of the level of consumption), thus κ_t becomes constant and A_h and A_l grow at a common rate. Suppose the system reaches κ^* at a consumption level higher than $C_T = \xi_l(\kappa^*) \cdot A_{l,0}$. Then, the C/Y ratio will grow and in finite time $C = Y$ implying consumption growth must be zero, violating the Euler equation. Suppose the system reaches κ^* at a consumption level lower than $C_T = \xi_l(\kappa^*) \cdot A_{l,0}$. Then, the C/Y ratio will fall over time and tend to zero asymptotically, violating the transversality equation. Second, there are trajectories that never cross the $\kappa_t = \kappa^*$ locus. It is also straightforward to prove that no such trajectory is optimal. They all involve high consumption growth attaining the $C = Y$ boundary in finite time, where the Euler condition is necessarily violated.

ADD A FIGURE WITH DYNAMIC SYSTEM IN C/A_l AND κ.

A.1.2 Case 2: $\lambda \geq \hat{\lambda}$

In this case, we know from Lemmas 1 and 2 that there are three stages

1. only offshoring: κ grows and both A_l and A_h remain constant;
2. offshoring +UBTC: : A_h grows whereas κ (but there is offshoring) and A_l remains constant;
3. BGP: κ is constant (but there is offshoring) and both A_l and A_h grow at a constant rate.

During stages 1 and 2, we have the following system of dynamic equations:

$$\begin{aligned}\frac{\dot{C}_t}{C_t} &= r\left(A_{h,0}, A_{l,t}, \kappa_t\right) - \rho \\ \left(\mu + f\kappa_t\right)\dot{A}_{l,t} + fA_{l,t}\dot{\kappa}_t &= Y\left(A_{h,0}, A_{l,t}, \kappa_t\right) - C_t\end{aligned}$$

where κ_0 is given by the pre-shock steady-state condition, $\kappa_0 = \left(1 + \lambda_0^{-1/\alpha} L_w/L_e\right)^{-1}$, and r, Y and $\hat{L}$ are defined as above.

Clearly, in stage 1, $A_{l,t} = A_{h,0}$ and $\dot{A}_{l,t} = 0$. In this stage, κ_t grows. Thus, the dynamic system simplifies to

$$\begin{aligned}\frac{\dot{C}_t}{C_t} &= r\left(A_{h,0}, A_{l,0}, \kappa_t\right) - \rho \\ fA_{l,0}\dot{\kappa}_t &= Y\left(A_{h,0}, A_{l,0}, \kappa_t\right) - C_t\end{aligned} \tag{33}$$

that is a system of autonomous differential equations in C_t and κ_t.

Stage 1 ends when $\kappa_t = \kappa^*$. Thereafter, $\dot{\kappa}_t = 0$, and the system simplifies to

$$\begin{aligned}\frac{\dot{C}_t}{C_t} &= r\left(A_{h,0}, A_{l,t}, \kappa^*\right) - \rho, \\ \left(\mu + f\kappa^*\right)\dot{A}_{l,t} &= Y\left(A_{h,0}, A_{l,t}, \kappa^*\right) - C_t.\end{aligned} \tag{34}$$

that is a system of autonomous differential equations in C_t and $A_{l,t}$.

Next, we must determine the condition when skilled innovation is also restored. This is triggered as the level of $A_{l,t}$ reaches

$$\begin{aligned} r_l\left(A_{h,0}, A_{l,t}, \kappa^*\right) &= r_h\left(A_{h,0}, A_{l,t}, \kappa^*\right) \\ &\Leftrightarrow \quad A_{l,t} = A_{h,0}\left(\left(\hat{L}\left(\kappa^*\right)\right)^{1-\alpha-1/\epsilon}\left(\frac{L_w}{1-\kappa^*}\right)^{\alpha}\left(ZH_w\right)^{1/\epsilon-1}\right)^{\epsilon}.\end{aligned}$$

Note that this is the standard dynamics of DTC models.

To find the solution trajectory, we proceed backwards. First, find the steady-state consumption ratio such that

$$\begin{aligned}\frac{C}{A_h} &= \left(\frac{Y\left(A_{h,t}, A_{l,t}, \kappa^*\right)}{A_{h,t}} - \mu g\left(\left(a\left(\kappa^*\right)\right)^{-1} + 1\right)\right) \\ &= \left[\left(\left(\frac{\hat{L}\left(\kappa^*\right)}{a\left(\kappa^*\right)}\right)^{\frac{\epsilon-1}{\epsilon}} + \left(ZH_w\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} - \mu\left(r\left(\kappa^*\right) - \rho\right)\left(1 + \left(a\left(\kappa^*\right)\right)^{-1}\right)\right] \equiv \xi_h\left(\kappa^*\right).\end{aligned}$$

where $r\left(\kappa^*\right)$ is the steady-state interest rate and $a\left(\kappa^*\right)$ denotes, as above, the steady-state skill bias. Thus, if a trajectory converges to the BGP at time $t = T$, it must feature

$$C_T = \xi_h\left(\kappa^*\right) \cdot A_{h,0}.$$

This yields the terminal condition of the system (34). The initial condition of that system is $A_{l,t} = A_{l,0}$, corresponding to the starting point of the process of UBTC. Standard properties of the dynamic system ensure that there exists one and only one trajectory linking the initial and the terminal condition. Moreover, along this trajectory bot C and A_l grow over time. Define as $C_1(A_{l,0})$ the level of consumption that is found integrating backwards the system in correspondence of the initial level of A_l.

Next, we can use $C_1(A_{l,0})$ as the terminal condition of the system (33). Recall that this is a system in the endogenous variables C_t and κ_t, where κ_0 is predetermined by the pre-shock steady-state condition. Again, we have an initial and a terminal condition, plus monotonicity. This ensures that there exists a unique trajectory linking the two points, and hence a unique solution. This concludes the proof that the trajectory converging to the BGP exists and is unique. We then appeal to standard results showing that such trajectory satisfies the transversality condition.

Ruling out trajectories that do not converge to the BGP follows the same logic of the other case.

ADD A 2 PANEL FIGURE WITH DYNAMIC SYSTEM IN C/A_h AND κ (panel A) and C/A_h AND A_l (panel A)