URBAN POPULATION AND AMENITIES

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Abstract

We use a frictionless neoclassical general-equilibrium model to explain cross-metro variation in population density based on three broad amenity types: quality of life, productivity in tradables, and productivity in non-tradables. Analytically, we demonstrate the dependence of quantities on amenities through substitution possibilities in consumption and production. Our calibrated model predicts large elasticities, consistent with variation in U.S. data, and estimates of local labor supply and demand. From only differences in wages and housing costs, we explain half of the variation in density, especially through quality of life amenities. We also show that density information can provide or refine measures of land value and local productivity. Our approach can be used to study a wide variety of urban quantities.

Keywords: Population density, productivity, quality of life.

JEL Numbers: H2, H4, J3, Q5, R1

1 Introduction

In the United States, population densities vary across space far more than the prices of labor and housing. At the metropolitan level, the average residential density of New York is almost 50 times that of Texarkana. Meanwhile, wage levels in the highest-paying metro are not even double that of the lowest, and housing costs in the most expensive metro average only four times that of the lowest. In this paper, we examine whether small differences in prices are compatible with large differences in quantities, like population density, in the neoclassical model of Rosen (1979) and Roback (1982), with mobile households and firms, and both tradable and non-tradable sectors.

In the neoclassical model, differences in prices and quantities across metro areas stem from local amenities, which work through three different channels: quality of life, tradable-sector productivity, and non-tradable-sector productivity. The first two channels determine the extent to which people follow jobs or jobs follow people, a topic long debated (Blanco 1963, Borts and Stein 1964).[1] The third channel determines whether both jobs and people follow availabile housing, a subject that has received more recent attention (Glaeser and Gyourko 2006, Glaeser, Gyourko, and Saks 2006, Saks 2008). Although researchers regularly use the neoclassical model to examine the relationship between prices and amenities, they rarely do so to examine the relationship between quantities and amenities. Existing work often imposes strong restrictions on the model or alters its structure, particularly in the non-tradable sector, and provides only numerical results, which offer limited intuition (e.g., Haughwout and Inman 2001, Rappaport 2008a, 2008b, Desmet and Rossi Hansberg forthcoming, Moretti forthcoming).[2]

Here we consider the relationships between amenities and quantities analytically, using the canonical neoclassical model with few restrictions. We show how quantity differences depend on

[1] See Hoogstra, Florax, and Dijk (2005) for an interesting meta-analysis of this literature.

[2] Haughwout and Inman (2001) reduce the non-tradable sector to a fixed land market. Rappaport (2008a, 2008b) constrains productivity in the non-tradable sector to be the same as in the tradable sector, and assumes the elasticity of substitution between factors in tradable production is one. Glaeser, Gyourko, and Saks (2006) and Moretti (forthcoming) use an ad-hoc partial equilibrium supply function, thereby excluding labor from the non-tradable sector, and force households to consume a fixed amount of housing. Desmet and Rossi-Hansberg (forthcoming) constrain elasticities of substitution in consumption and tradable production to be one, and model the non-tradable sector using a monocentric city, where households consume a single unit of housing. Only Rappaport's work is useful for studying population density, although his work is done numerically, and is not linked to data in a close manner.

cost and expenditure shares, tax rates, and separate margins of substitution in consumption and in both types of production. The substitution margins reflect three separate behavioral responses that lead to higher densities, including the willingness of households to crowd into existing housing, shifts in production away from land-intensive goods, and the construction of housing at greater heights. Urban quantities depend on these substitution possibilities in a first-order manner, while prices do not. Using a pre-set calibration of the United States economy from Albouy (2009), our results suggest that substitution possibilities in the non-tradable sector, including housing, are particularly important.

The analytical exercise maps reduced-form elasticities often estimated in the literature, e.g., of local labor or housing supply, to more elementary structural parameters. This mapping reframes partial-equilibrium shifts in supply and demand as general-equilibrium responses to amenity changes, e.g., an increase in labor demand is mapped to an increase in tradable-sector productivity. The calibrated model implies that quantities respond much more than prices to differences in amenities over the long run. The model produces large (positive) labor-supply elasticities that are remarkably consistent with estimates found in Bartik (1991) and Notowidigo (2012), and even larger (negative) labor-demand elasticities consistent with estimates in Card (2001). Moreover, our numbers are consistent with the stylized fact that population density varies by an order of magnitude more than wages and housing costs across metro areas.

Our research complements work on agglomeration economies, which examines the reverse relationship of how population affects amenities, especially productivity. For example, we model how areas with higher quality of life become denser, thereby making them more productive through agglomeration. Agglomeration then creates a multiplier effect, whereby higher density increases productivity, bringing forth even higher density and productivity. We also consider the possibility of greater density reducing quality of life through congestion. Under our calibration, we find that these multiplier effects are potentially important, magnifying or dampening long-run behavioral responses up to 15-percent.

We apply the model empirically by using it to relate observable prices to population density in 276 American metropolitan areas using Census data. The pre-set calibration does remarkably well, explaining half of the variation in population density through quality of life and tradable-sector productivity predicted by two simple measures of wages and housing costs. Our calibration fits the data better than those that ignore substitution possibilities, e.g., in consumption or non-tradable production, or assume that they are all unit elastic, as in a Cobb-Douglas economy.

If the calibration produces accurate elasticity values, variation in population density not explained by quality of life may substitute for missing data on land prices and help to identify productivity in the tradable and non-tradable sectors. This approach suggests that metro areas such as New York, Chicago, and Houston have rather productive non-tradable sectors, at least historically. On the other hand, metros such as San Francisco and Seattle have far less productive non-tradable sectors, despite having very productive tradable sectors.

Our last exercise explores the relative importance of different amenities in explaining where people live. A variance decomposition suggests that quality of life explains a greater fraction of population density than does tradable-sector productivity, even though the latter varies more in value and affects wage and housing costs more. This conclusion is reinforced if population density increases tradable-sector productivity or reduces quality of life. Productivity in non-tradables explains density more than the other types of amenities, although this may have much to do with how it is measured. We also simulate how population density might change if federal taxes were made geographically neutral. This tends to increase the influence of tradable-sector productivity and multipliers from agglomeration feedback.

The general-equilibrium model of location, with homogenous agents, provides a different point of view than dynamic partial-equilibrium models of location with heterogenous agents (e.g., Kennan and Walker 2011). Partial-equilibrium approaches typically do not consider how wages and housing costs depend on population sizes. Moreover, the focus of such work is to explain migration decisions over short and medium run horizons, while we examine population differences over the very long run.

The rest of the paper is organized as follows: Section 2 introduces the model and discusses analytical results. Section 3 provides results from the calibrated model. Section 4 estimates long run elasticities of labor and housing demand and supply. Section 5 provides new productivity estimates and decomposes the determinants of population density. Section 6 concludes.

2 Locational Equilibrium of Quantities, Prices, and Amenities

2.1 Set-up

To explain how prices and quantities vary with amenity levels across cities, we use the model of Albouy (2009a), which adds federal taxes to the general-equilibrium three-equation Roback (1982) model. The national economy contains many cities, indexed by j, which trade with each other and share a homogenous population of mobile households. Households supply a single unit of labor in their city of residence; they consume a numeraire traded good x and a non-traded "home" good y with local price p^j.[3] All input and output markets are perfectly competitive, and all prices and quantities are homogenous within cities, though they vary across cities.

Cities differ exogenously in three general attributes, each of which is an index meant to summarize the value of amenities to households and firms: (i) quality of life Q^j raises household utility, (ii) trade-productivity A_X^j lowers costs in the tradable sector, and (iii) home-productivity A_Y^j lowers costs in the non-tradable sector.[4]

Firms produce traded and home goods out of land, capital, and labor. Within a city, factors receive the same payment in either sector. Land L is heterogenous across cities, immobile, and receives a city-specific price r^j. Each city's land supply $L^j(r^j)$ may depend positively on r^j.

Capital K is fully mobile across cities and receives the price $\bar{\imath}$ everywhere. The supply of capital in each city K^j is perfectly elastic at this price. The national level of capital may be fixed

[3] In application, the price of the home good is equated with the cost of housing services. Non-housing goods are considered to be a composite commodity of traded goods and non-housing home goods.

[4] All of these attributes depend on a vector of natural and artificial city amenities, $\mathbf{Z}^j = (Z_1^j, ..., Z_K^j)$, through functional relationships $Q^j = \widetilde{Q}(\mathbf{Z}^j)$, $A_X^j = \widetilde{A_X}(\mathbf{Z}^j)$, and $A_Y^j = \widetilde{A_Y}(\mathbf{Z}^j)$. For a consumption amenity, e.g., clement weather, $\partial\widetilde{Q}/\partial Z_k > 0$; for a trade-production amenity, e.g., navigable water, $\partial\widetilde{A_X}/\partial Z_k > 0$; for a home-production amenity, e.g., flat geography, $\partial\widetilde{A_Y}/\partial Z_k > 0$. It is possible that a single amenity affects more than one attribute or affects an attribute negatively.

or depend on $\bar{\imath}$. Households N are fully mobile, have identical tastes and endowments, and each supplies a single unit of labor. Household size is fixed. Wages w^j vary across cities because households care about local prices and quality of life. The total number of households is $N^{TOT} = \sum_j N^j$, which may be fixed or determined by international migration.

Households own identical diversified portfolios of land and capital, which pay an income $R = (N_{TOT})^{-1} \sum_j r^j L^j$ from land and $I = (N_{TOT})^{-1} \sum_j \bar{\imath} K^j$ from capital. Total income $m^j = w^j + R + I$ varies across cities only as wages vary. Out of this income households pay a linear federal income tax τm^j, which is redistributed in uniform lump-sum payments T. Household preferences are modeled by a utility function $U(x, y; Q^j)$ which is quasi-concave over x, y, and Q^j. The expenditure function for a household in city j is

$$e(p^j, u; Q^j) \equiv \min_{x,y} \{x + p^j y : U(x, y; Q^j) \geq u\}.$$

Assume Q enters neutrally into the utility function and is normalized so that $e(p^j, u; Q^j) = e(p^j, u)/Q^j$, where $e(p^j, u) \equiv e(p^j, u; 1)$.[5]

Operating under perfect competition, firms produce traded and home goods according to the functions $X^j = A_X^j F_X(L_X^j, N_X^j, K_X^j)$ and $Y^j = A_Y^j F_Y(L_Y^j, N_Y^j, K_Y^j)$, where F_X and F_Y are concave and exhibit constant returns to scale. We assume Hicks-Neutral productivity. Unit cost in the traded-good sector of city j is

$$c_X(r^j, w^j, \bar{\imath}; A_X^j) \equiv \min_{L,N,K} \{r^j L + w^j N + \bar{\imath} K : A_X^j F(L, N, K) = 1\}.$$

Similar to the relationship between quality of life and the expenditure function, let $c_X(r^j, w^j, \bar{\imath}; A_X^j) = c_X(r^j, w^j, \bar{\imath})/A_X^j$, where $c_X(r^j, w^j, \bar{\imath}) \equiv c_X(r^j, w^j, \bar{\imath}; 1)$. A symmetric definition holds for the unit cost in the home-good sector c_Y.

[5]The model generalizes to a case with heterogenous workers that supply different fixed amounts of labor if these workers are perfect substitutes in production, have identical homothetic preferences, and earn equal shares of income from labor.

2.2 Equilibrium Conditions

Each city can be described by a system of sixteen equations in sixteen endogenous variables: three prices p^j, w^j, r^j, and thirteen quantities $x^j, y^j, X^j, Y^j, N^j, N^j_X, N^j_Y, L^j, L^j_X, L^j_Y, K^j, K^j_X, K^j_Y$. We begin by having these depend on three exogenous attributes Q^j, A^j_X, A^j_Y and a land supply function $L^j(r^j)$. In this scenario, the system of equations has a block-recursive structure, allowing us to first determine prices, where most researchers stop, then determine per-capita consumption quantities, and finally, production quantities, including total population. This block-recursive structure is broken if amenities are made endogenous to quantities, e.g., if $A^j_X = A^j_{X0}(N^j)^\alpha$ where A^j_{X0} is trade-productivity due to fixed natural advantages, and $(N^j)^\alpha$ is due to agglomeration economies. Endogenizing amenities is more important for comparative statics, e.g., increasing A^j_{X0} changes N^j, than measurement, where N^j may be treated as fixed so long as we are satisfied in measuring the composition $A^j_{X0}(N^j)^\alpha$. Throughout, we adopt a "small open city" assumption and take nationally determined variables $\bar{u}, \bar{\iota}, I, R, T$ as given for any individual city.[6]

2.2.1 Price Conditions

Since households are fully mobile, they must receive the same utility across all inhabited cities. Higher prices or lower quality of life are compensated with greater after-tax income,

$$e(p^j, \bar{u})/Q^j = (1-\tau)(w^j + R + I) + T, \tag{1}$$

where $\bar{u}$ is the level of utility attained nationally by all households.

Firms earn zero profits in equilibrium. For given output prices, firms in more productive cities

[6] In a closed city, we could instead take N^j or K^j as given, and endogenize factor incomes R^j or I^j. In the open city we assume that the federal government's budget is given by $\tau \sum_j N^j m^j + T \sum_j N^j = 0$, so a city with average income receives a transfer which exactly offsets its taxes.

must pay higher rents and wages,

$$c_X(r^j, w^j, \bar{\imath})/A_X^j = 1 \tag{2}$$

$$c_Y(r^j, w^j, \bar{\imath})/A_Y^j = p^j. \tag{3}$$

Equations (1), (2), and (3) simultaneously determine the city-level prices p^j, r^j, and w^j for each city as implicit functions of the three attributes Q^j, A_X^j, and A_Y^j. In equilibrium, these conditions provide a one-to-one mapping between unobserved city attributes and potentially observable prices, obviating the need to examine quantities.

2.2.2 Consumption Conditions

In deciding their consumption quantities x^j and y^j, households face the budget constraint

$$x^j + p^j y^j = (1-\tau)(w^j + R + I)^j + T, \tag{4}$$

where p^j and w^j are determined by the price conditions. Optimal consumption is determined in conjunction with the tangency condition

$$(\partial U/\partial y) \,/\, (\partial U/\partial x) = p^j. \tag{5}$$

As we assume preferences are homothetic, Q^j does not affect the marginal rate of substitution. Thus, in areas where Q^j is higher, but p^j is the same, households consume less of x and y in equal proportions, holding the ratio y/x constant, similar to an income effect. Holding Q^j constant, increases in p^j are compensated by increases in w^j so that households reduce their relative consumption y/x due to a pure substitution effect.

2.2.3 Production Conditions

With prices and per-capita consumption levels accounted for, Levels of output X^j, Y^j, employment N^j, N_X^j, N_Y^j, capital K^j, K_X^j, K_Y^j, and land L^j, L_X^j, L_Y^j are determined by eleven equations describing production and market clearing. The first six express conditional factor demands using Shephard's Lemma. Because of constant returns to scale and Hicks neutrality, the derivative of the uniform unit-cost function equals the ratio of the relevant input, augmented by city-specific productivity, to output:

$$\partial c_X/\partial w = A_X^j N_X^j / X^j \tag{6}$$

$$\partial c_X/\partial r = A_X^j L_X^j / X^j \tag{7}$$

$$\partial c_X/\partial i = A_X^j K_X^j / X^j \tag{8}$$

$$\partial c_Y/\partial w = A_Y^j N_Y^j / Y^j \tag{9}$$

$$\partial c_Y/\partial r = A_Y^j L_Y^j / Y^j \tag{10}$$

$$\partial c_Y/\partial i = A_Y^j K_Y^j / Y^j. \tag{11}$$

The next three conditions express the local resource constraints for labor, land, and capital under the assumption that factors are fully employed.

$$N^j = N_X^j + N_Y^j \tag{12}$$

$$L^j = L_X^j + L_Y^j \tag{13}$$

$$K^j = K_X^j + K_Y^j \tag{14}$$

Equation (13) differs from the others as local land is determined by the supply function,

$$L^j = L^j(r^j), \tag{15}$$

Together, the assumptions of an internally homogenous open city, exogenous amenities, and homogeneity of degree one cost and expenditure functions imply that all of the production quantity predictions increase proportionally with the quantity of land. If land in a city doubles, labor and capital will migrate in to also double, so that all prices and per-capita quantities remain the same. By focusing on density, we can normalize land supply to a single unit, so that $L^j(r^j) = 1$ for all cities.[7]

The last condition requires that the local home-good market clears.

$$Y^j = N^j y^j \tag{16}$$

Walras' Law makes redundant the market clearing equation for tradable output, which includes net per-capita transfers from the federal government $T - \tau m^j$.

2.3 Log-Linearization around National Averages

The system described by conditions (1) to (16) is generally non-linear.[8] To obtain closed-form solutions, we log-linearize these conditions. Hence, we express each city's price and quantity differentials in terms of its amenity differentials, relative to the national average. These differentials are expressed in logarithms so that for any variable z, $\hat{z}^j \equiv \ln z^j - \ln \bar{z} \cong (z^j - \bar{z}) / \bar{z}$ approximates the percent difference in city j of z relative to the national average $\bar{z}$.

The log-linearization requires several economic parameters evaluated at the national average. For households, denote the share of gross expenditures spent on the traded and home good as $s_x \equiv x/m$ and $s_y \equiv py/m$; denote the share of income received from land, labor, and capital income as $s_R \equiv R/m$, $s_w \equiv w/m$, and $s_I \equiv I/m$. For firms, denote the cost share of land, labor,

[7]In principle, land supply can vary on two different margins. At the extensive margin, an increase in land supply corresponds to a growing city boundary. At the intensive margin, an increase in land supply takes the form of employing previously unused land within a city's border. By assuming that all factors are employed, we rule out the possibility of any intensive margin changes.

[8]One exception is when the economy is fully Cobb-Douglas, and there is no income received from land, capital, or government. In Appendix A, we present results from a nonlinear simulation of the model and argue that log-linearization accurately approximates the model.

and capital in the traded-good sector as $\theta_L \equiv rL_X/X$, $\theta_N \equiv wN_X/X$, and $\theta_K \equiv \bar{\imath}K_X/X$; denote equivalent cost shares in the home-good sector as ϕ_L, ϕ_N, and ϕ_K. Finally, denote the share of land, labor, and capital used to produce traded goods as $\lambda_L \equiv L_X/L$, $\lambda_N \equiv N_X/N$, and $\lambda_K \equiv K_X/K$. Assume the home-good is more cost-intensive in land relative to labor than the traded-good, both absolutely, $\phi_L \geq \theta_L$, and relatively, $\phi_L/\phi_N \geq \theta_L/\theta_N$, implying $\lambda_L \leq \lambda_N$.

The first three price conditions are log-linearized as

$$-s_w(1-\tau)\hat{w}^j + s_y\hat{p}^j = \hat{Q}^j \tag{1*}$$

$$\theta_L\hat{r}^j + \theta_N\hat{w}^j = \hat{A}_X^j \tag{2*}$$

$$\phi_L\hat{r}^j + \phi_N\hat{w}^j - \hat{p}^j = \hat{A}_Y^j \tag{3*}$$

These conditions are examined in depth in Albouy (2009b), and so here we just note that these expressions involve only cost and expenditure shares and the marginal tax rate.

The log-linearized conditions describing consumption introduce the elasticity of substitution in consumption, $\sigma_D \equiv -e \cdot (\partial^2 e/\partial^2 p)/[\partial e/\partial p \cdot (e - p \cdot \partial e/\partial p)]$,

$$s_x\hat{x}^j + s_y\left(\hat{p}^j + \hat{y}^j\right) = (1-\tau)s_w\hat{w}^j \tag{4*}$$

$$\hat{x}^j - \hat{y}^j = \sigma_D\hat{p}^j \tag{5*}$$

Substituting equation (1*) into equations (4*) and (5*) produces the solutions $\hat{x}^j = s_y\sigma_D\hat{p}^j - \hat{Q}^j$ and $\hat{y}^j = -s_x\sigma_D\hat{p}^j - \hat{Q}^j$. These describe the substitution and quality of life effects discussed earlier.

Even though our model contains homogenous households, one can think of higher values of σ_D as approximating households with heterogeneous preferences who sort across cities. Households with stronger tastes for y will choose to live in areas with lower prices p. At the equilibrium levels of utility, an envelope of the mobility conditions for each type forms that of a representative household, with greater preference heterogeneity reflected as more flexible substitution.[9]

[9] Roback (1980) discusses this generalization and the generalizations below for σ_X and σ_Y.

The log-linearized conditional factor demands describe how input demands depend on output, productivity, and relative input prices.

$$\hat{N}_X^j = \hat{X}^j - \hat{A}_X^j + \theta_L \sigma_X^{LN} \left(\hat{r}^j - \hat{w}^j\right) - \theta_K \sigma_X^{NK} \hat{w}^j \tag{6*}$$

$$\hat{L}_X^j = \hat{X}^j - \hat{A}_X^j + \theta_N \sigma_X^{LN} (\hat{w}^j - \hat{r}^j) - \theta_K \sigma_X^{KL} \hat{r}^j \tag{7*}$$

$$\hat{K}_X^j = \hat{X}^j - \hat{A}_X^j + \theta_L \sigma_X^{KL} \hat{r}^j + \theta_N \sigma_X^{NK} \hat{w}^j \tag{8*}$$

$$\hat{N}_Y^j = \hat{Y}^j - \hat{A}_Y^j + \phi_L \sigma_Y^{LN} (\hat{r}^j - \hat{w}^j) - \phi_K \sigma_Y^{NK} \hat{w}^j \tag{9*}$$

$$\hat{L}_Y^j = \hat{Y}^j - \hat{A}_Y^j + \phi_N \sigma_Y^{LN} (\hat{w}^j - \hat{r}^j) - \phi_K \sigma_Y^{KL} \hat{r}^j \tag{10*}$$

$$\hat{K}_Y^j = \hat{Y}^j - \hat{A}_Y^j + \phi_L \sigma_Y^{KL} \hat{r}^j + \phi_N \sigma_Y^{NK} \hat{w}^j \tag{11*}$$

The dependence on input prices is determined by three partial (Allen-Uzawa) elasticities of substitution in each sector. These are defined for each pair of factors, e.g., $\sigma_X^{LN} \equiv c_X \cdot (\partial^2 c_X / \partial w \partial r)$ $/ \, (\partial c_X / \partial w \cdot \partial c_X / \partial r)$ is for labor and land in the production of X. These values are taken at the national average because we assume that production technology does not differ across cities. To simplify matters, we also assume that the partial elasticities within each sector are the same, i.e., $\sigma_X^{NK} = \sigma_X^{KL} = \sigma_X^{LN} \equiv \sigma_X$, and similarly for σ_Y, as with a constant elasticity of substitution production function.

A higher value of σ_X corresponds to more flexible production of the traded-good. With a single traded good, firms can vary their production only by changing inputs. In a generalization with multiple traded goods sold at fixed prices, firms could adjust their product mix to specialize in producing goods where their input costs are relatively low. For example, areas with high land costs and low labor costs would produce goods that use labor intensely but not land. A representative zero-profit condition can be drawn as an envelope of the zero-profit conditions for each good, with a greater variety of goods reflected as greater substituion possibilities, i.e., a larger σ_X.

A related argument exists for home goods. For instance, a high value of σ_Y means that housing producers can use labor and capital to build taller buildings in areas where the price of land is high. Buildings can also be subdivided to produce more, but smaller, housing units. If all home goods are perfect substitutes, then an envelope of zero-profit conditions may be used as a representative zero-profit condition. Because housing is durable, arriving at an equilibrium may take a long time. For non-housing home goods, one can imagine retailers using taller shelves and restaurants hiring more labor to move clients through faster.

Log-linearizing the resource constraints for labor, land, and capital yields

$$\hat{N}^j = \lambda_N \hat{N}^j_X + (1 - \lambda_N)\hat{N}^j_Y \tag{12*}$$

$$\hat{L}^j = \lambda_L \hat{L}^j_X + (1 - \lambda_L)\hat{L}^j_Y \tag{13*}$$

$$\hat{K}^j = \lambda_K \hat{K}^j_X + (1 - \lambda_K)\hat{K}^j_Y. \tag{14*}$$

Equations (12*), (13*), and (14*) imply that the sector-specific changes in factors affect overall changes in proportion to the factor share. The condition for land supply uses the elasticity $\varepsilon^j_{L,r} \equiv (\partial L^j/\partial r) \cdot (r/L^j)$.

$$\hat{L}^j = \varepsilon^j_{L,r}\hat{r}^j \tag{15*}$$

As new land is assumed identical to old land, the impact of amenities on population and other production quantities will involve the term $\varepsilon^j_{L,r}\hat{r}^j$ since quantities are proportional to the amount of land. By focusing on density, we assume $\varepsilon^j_{L,r} = 0$ for all cities. Wrapping up, the market clearing condition for home-goods is simply

$$\hat{N}^j + \hat{y}^j = \hat{Y}^j. \tag{16*}$$

2.4 Solving the Model

The solutions for the endogenous variables are expressed in terms of the amenity differentials $\hat{Q}^j$, $\hat{A}^j_X$, and $\hat{A}^j_X$. Because of the block-recursive structure, only equations (1*) to (3*) are needed for the price differentials.

$$\hat{r}^j = \frac{1}{s_R}\frac{\lambda_N}{\lambda_N - \tau\lambda_L}\left[\hat{Q}^j + \left(1 - \frac{1}{\lambda_N}\tau\right)s_x\hat{A}_X^j + s_y\hat{A}_Y^j\right] \tag{17}$$

$$\hat{w}^j = \frac{1}{s_w}\frac{1}{\lambda_N - \tau\lambda_L}\left[-\lambda_L\hat{Q}^j + (1-\lambda_L)s_x\hat{A}_X^j - \lambda_L s_y\hat{A}_Y^j\right] \tag{18}$$

$$\hat{p}^j = \frac{1}{s_y}\frac{1}{\lambda_N - \tau\lambda_L}\left[(\lambda_N - \lambda_L)\hat{Q}^j + (1-\tau)(1-\lambda_L)s_x\hat{A}_X^j - (1-\tau)\lambda_L s_y\hat{A}_Y^j\right] \tag{19}$$

Higher quality of life leads to higher land and home-good prices but lower wages. Higher trade-productivity increases all three prices, while higher home-productivity increases land prices but decreases wages and the home-good price.

Putting solution (19) in equations (4*) and (5*) yields the per-capita consumption differentials

$$\hat{x}^j = \frac{\sigma_D(1-\tau)}{\lambda_N - \tau\lambda_L}\left[\frac{\sigma_D(\lambda_N - \lambda_L) - \lambda_N + \tau\lambda_L}{\sigma_D(1-\tau)}\hat{Q}^j + (1-\lambda_L)s_x\hat{A}_X^j - \lambda_L s_y\hat{A}_Y^j\right] \tag{20}$$

$$\hat{y}^j = -\frac{s_x}{s_y}\frac{\sigma_D(1-\tau)}{\lambda_N - \tau\lambda_L}\left[\frac{s_x\sigma_D(\lambda_N - \lambda_L) + s_y(\lambda_N - \tau\lambda_L)}{s_x\sigma_D(1-\tau)}\hat{Q}^j + (1-\lambda_L)s_x\hat{A}_X^j - \lambda_L s_y\hat{A}_Y^j\right] \tag{21}$$

Households in trade-productive areas substitute towards tradable consumption and away from non-tradable consumption, while households in home-productive areas do the opposite. In nicer areas, households consume fewer home goods; whether they consume fewer tradable goods is ambiguous, as the substitution effect is positive, but the income effect is negative.

Unfortunately, solutions for the other quantities, which also rely on production equations (6*) through (16*) are more complicated and harder to intuit. As a notational short-cut, we express the change in each quantity with respect to amenities using three reduced-form elasticities, each composed of structural parameters. For example, the solution for population density is expressed by

$$\hat{N}^j = \varepsilon_{N,Q}\hat{Q}^j + \varepsilon_{N,A_X}\hat{A}_X^j + \varepsilon_{N,A_Y}\hat{A}_Y^j, \tag{22}$$

where $\varepsilon_{N,Q}$ is the elasticity of population density with respect to quality of life and ε_{N,A_X} and

ε_{N,A_Y} are defined similarly. The first reduced-form elasticity is given by

$$\varepsilon_{N,Q} = \left[\frac{\lambda_N - \lambda_L}{\lambda_N}\right] + \sigma_D \left[\frac{s_x(\lambda_N - \lambda_L)^2}{s_y \lambda_N (\lambda_N - \lambda_L \tau)}\right] + \sigma_X \left[\frac{\lambda_L^2}{s_w(\lambda_N - \lambda_L \tau)} + \frac{\lambda_L \lambda_N}{s_R(\lambda_N - \lambda_L \tau)}\right]$$
$$+ \sigma_Y \left[\frac{\lambda_L^2(1-\lambda_N)}{s_w \lambda_N (\lambda_N - \lambda_L \tau)} + \frac{\lambda_N(1-\lambda_L)}{s_R(\lambda_N - \lambda_L \tau)} - \frac{(\lambda_N - \lambda_L)^2}{s_y \lambda_N (\lambda_N - \lambda_L \tau)}\right]$$
$$+ \varepsilon_{L,r} \left[\frac{\lambda_N}{s_R(\lambda_N - \lambda_L \tau)}\right] \tag{23}$$

We provide similar expressions for ε_{N,A_X} and ε_{N,A_Y} in Appendix B.

After collecting terms by their corresponding structural elasticity, as in (23), one can see how higher quality of life raises population through five behavioral responses. The first term reflects that households are willing to consume fewer goods in nicer areas, similar to an income effect. The second term, with σ_D, captures how households increase density by substituting away from land-intensive goods, e.g., by crowding into existing housing. The third, with σ_X, expresses the ability of firms in the traded-sector to substitute away from land towards labor and capital. The fourth, with σ_Y, reflects how home-goods become less land intensive, e.g., buildings get taller. The fifth, with $\varepsilon_{L,r}$ provides the population gain on the extensive margin, from more land being used.[10]

Each of the reduced-form elasticities between a quantity and a type of amenity may have up to five similar structural effects. The key differences between the price and quantity solutions is that the latter depend directly on substitution elasticities.

2.5 Agglomeration Effects

Given the above set-up, introducing simple forms of endogenous amenities is straightforward. We consider two types we believe to be the most common: positive economies of scale in tradable production and negative economies of scale in quality of life. For simplicity, both are assumed to depend on population density, i.e., $A_X^j = A_{X0}^j (N^j)^\alpha$ and $Q^j = Q_0^j (N^j)^{-\gamma}$, where A_{X0}^j and Q_0^j represent city j's "natural advantages", and $\alpha \geq 0$ and $\gamma \geq 0$ are the reduced-form agglomeration

[10]We include the term associated with the elasticity of land supply in equation (23) for expository purposes. When allowing $\varepsilon_{L,r}$ to be non-zero, $\hat{N}$ corresponds to population, as opposed to population density.

elasticities. Natural advantages could be determined by local geographic features, local policies, or be the result of historical path dependence (Bleakley and Lin 2012). The agglomeration processes for productivity may be due to non-rival input sharing or knowledge spillovers, while the diseconomies in quality of life may be due to congestion or pollution. The main assumption here is that these processes follow a power law.

These agglomeration feedback effects result in the population solution now being

$$\begin{aligned}\hat{N}^j =&\ \varepsilon_{N,Q}(\hat{Q}_0^j - \gamma\hat{N}^j) + \varepsilon_{N,A_X}(\hat{A}_{X0}^j + \alpha\hat{N}^j) + \varepsilon_{N,A_Y}\hat{A}_{Y0}^j \\ =&\ (1 + \gamma\varepsilon_{N,Q} - \alpha\varepsilon_{N,A_X})^{-1}\left(\varepsilon_{N,Q}\hat{Q}_0^j + \varepsilon_{N,A_X}\hat{A}_{X0}^j + \varepsilon_{N,A_Y}\hat{A}_{Y0}^j\right) \\ \equiv&\ \tilde{\varepsilon}_{N,Q}\hat{Q}_0^j + \tilde{\varepsilon}_{N,A_X}\hat{A}_{X0}^j + \tilde{\varepsilon}_{N,A_Y}\hat{A}_{Y0}^j, \end{aligned} \tag{24}$$

taking $A_{Y0}^j = A_Y^j$ as fixed. The multiplier $(1 + \gamma\varepsilon_{N,Q} - \alpha\varepsilon_{N,A_X})^{-1}$ reflects how the impact of natural advantages is magnified through positive economies of scale and dampened by negative ones. The multiplier effect depends as much on the population elasticities $\varepsilon_{N,Q}$ and ε_{N,A_X} as on the agglomeration parameters γ and α. Equation (24) simply re-expresses the reduced-form elasticities in terms of only natural advantages. The modified elasticities may be smaller or larger than the originals, depending on the agglomeration effects, and are appropriate to use in comparative static exercises when the level of a natural advantage changes.

Our framework could also be used to study a variety of more complicated endogenous feedback effects, although these would require more complicated solutions.

2.6 Identification of Production Amenities and Land Values

With accurate data on all price differentials $\hat{r}^j, \hat{w}^j$, and $\hat{p}^j$ and knowledge of national economic parameters, we can estimate amenity differentials $\hat{Q}^j$, $\hat{A}_X^j$, and $\hat{A}_Y^j$ with equations (1*), (2*), and (3*). Reliable land value data comparable across metropolitan areas is not readily available, making it hard to identify trade- and home-productivity using equations (2*) and (3*).[11] Combining these equations to eliminate $\hat{r}^j$ yields

$$\frac{\theta_L}{\phi_L}\hat{p}^j + \left(\theta_N - \phi_N \frac{\theta_L}{\phi_L}\right)\hat{w}^j = \hat{A}_X^j - \frac{\theta_L}{\phi_L}\hat{A}_Y^j. \tag{25}$$

As Albouy (2009b) discusses, one can estimate trade-productivity using the inferred cost formula on the left-hand side of equation (25) if we assume home-productivity is constant across cities $\hat{A}_Y^j = 0$. The resulting trade-productivity estimates are biased downwards, albeit slightly, in home-productive areas.[12]

Without such a restriction, home and trade-productivity cannot be separately identified in equation (25), since higher trade-productivity pushes wages and housing costs upwards in the same proportion that home-productivity pushes them downwards.[13] To solve this identification problem, we use additional information from population density not predicted by quality of life ("excess density"). This comes from combining equations (1*) and (22), yielding

$$\hat{N}^j - \varepsilon_{N,Q}\underbrace{[s_y\hat{p}^j - s_w(1-\tau)\hat{w}^j]}_{\hat{Q}^j} = \varepsilon_{N,A_X}\hat{A}_X^j + \varepsilon_{N,A_Y}\hat{A}_Y^j. \tag{26}$$

Equation (26) shows that excess density, on the left hand side, is explained by either trade or home-productivity, on the right hand side. Because the system containing equations (25) and (26) is exactly identified, our amenity estimates will *perfectly predict* population densities given our parameter choices. Solving this system, we obtain measures of productivity based on the

[11] Albouy and Ehrlich (2012) estimates $\hat{r}^j$ using recent transaction purchase data, which is only available for recent years. Their analysis discusses several conceptual and empirical challenges from this approach.

[12] This point is seen directly in equation (25) after noting that $\theta_L << \phi_L$.

[13] From equation (2*), note that $\hat{A}_X^j$ equals the costs faced by traded-good firms. We define $\hat{A}_X^j - \frac{\theta_L}{\phi_L}\hat{A}_Y^j$ as the costs of traded-good firms relative to home-good firms. The adjustment factor θ_L/ϕ_L arises because we eliminate $\hat{r}^j$.

differentials $\hat{N}^j$, $\hat{w}^j$, and $\hat{p}^j$

$$\hat{A}_X^j = \frac{\theta_L[N^j - \varepsilon_{N,Q}(s_y p^j - s_w(1-\tau)w^j)] + \phi_L \varepsilon_{N,A_Y}[\frac{\theta_L}{\phi_L}p^j + (\theta_N - \phi_N \frac{\theta_L}{\phi_L})w^j]}{\theta_L \varepsilon_{N,A_X} + \phi_L \varepsilon_{N,AY}} \tag{27}$$

$$\hat{A}_Y^j = \frac{\phi_L[N^j - \varepsilon_{N,Q}(s_y p^j - s_w(1-\tau)w^j)] - \phi_L \varepsilon_{N,A_Y}[\frac{\theta_L}{\phi_L}p^j + (\theta_N - \phi_N \frac{\theta_L}{\phi_L})w^j]}{\theta_L \varepsilon_{N,A_X} + \phi_L \varepsilon_{N,AY}} \tag{28}$$

Trade-productiivty is measured by higher excess density and inferred costs. Home-productivity is measured more strongly by higher excess density, and by lower inferred costs.

This strategy identifies land value differences by substituting the solutions into (2*) or (3*):

$$\hat{r}^j = \frac{\hat{N}^j - \varepsilon_{N,Q}(s_y \hat{p}^j - s_w(1-\tau)\hat{w}^j) - \varepsilon_{N,A_X}\theta_N \hat{w}^j - \varepsilon_{N,A_Y}(\hat{p}^j - \phi_N \hat{w}^j)}{\theta_L \varepsilon_{N,A_X} + \phi_L \varepsilon_{N,AY}}$$

This rent measure depends on excess density not predicted by the restricted productivity differentials we would estimate if land values were equal, i.e., if $\hat{r}^j = 0$, then $\hat{A}_X^j = \theta_N \hat{w}^j$ and $\hat{A}_Y^j = \phi_N \hat{w}^j - \hat{p}^j$. Excess density beyond the restricted productivity differentials indicates higher land values.

3 Calibrating the Model and Calculating Elasticities

3.1 Parameter Choices

Calibrating the model to the U.S. economy poses varying degrees of difficulty. Cost and expenditure shares require information on the first moments of data (i.e., means) and may be ascertained with some accuracy. Elasticities of substitution require credible identification involving second moments (i.e., covariances of quantities with prices), and thus are subject to greater uncertainty.

The main calibration we use is shown in Table 1. It follows that of Albouy (2009a). We leave discussion of expenditure and cost shares to Appendix C. We initially use $\sigma_D = \sigma_X = \sigma_Y = 0.667$, as discussed in Albouy (2009a). We provide sensitivity analysis surrounding elasticities of substitution below.

For illustrative purposes, we consider fairly large values for the agglomeration elasticities. Thus, we use $\alpha = 0.06$ for the positive effect of population density on trade-productivity and $\gamma = 0.015$ for the negative effect on quality of life.[14]

A few potential complications deserve special attention. First, incorrect parameter values might bias our estimates. As discussed in Appendix C, the parameters come from a variety of sources and are generally estimated across different years, geographies, and industries. Second, the log-linearized model is most accurate for small deviations from the national average. As shown in Figure 1, population density varies significantly, which could bias our results. We present a non-linear simulation in Appendix A which suggests that our main conclusions are not affected by the linear approximation.

We demonstrate how elasticities of substitution affect reduced form elasticities in Table 2. Our estimates also might contain error due to certain modeling assumptions; we leave for future research the task of examining how estimates vary with the model's key assumptions (e.g., perfect competition, free mobility). Our model most appropriately describes a long run equilibrium, where moving costs or other frictions likely have little impact. Finally, the elasticity of home-good production likely varies across cities (Saiz 2010). For example, home-producers in coastal cities might find it more difficult to substitute away from capital or labor towards land. We do not consider city-specific production elasticities, but Table 2 clearly shows how our reduced-form estimates change with σ_Y, as discussed next.

3.2 Reduced-Form Elasticities

In Table 2, we demonstrate how the reduced-form population elasticities depend on the structural elasticities of substitution by substituting in the values of all of the other parameters. Thus, the five

[14]Ciccone and Hall (1996) estimate an elasticity of labor productivity with respect to population density of 0.06. Rosenthal and Strange (2004) argue that a one-percent increase in population leads to no more than a 0.03-0.08 percent increase in productivity. Our choice of $\alpha = 0.06$ is broadly consistent with estimates reported in Table 2 of Glaeser and Gottlieb (2008).

effects seen in equation (23) are calibrated as

$$\varepsilon_{N,Q} \approx 0.77 + 1.26\sigma_D + 1.95\sigma_X + 8.02\sigma_Y + 11.85\varepsilon_{L,r} \tag{29}$$

Calculating all of the elasticities using the main calibration where $\sigma_D = \sigma_X = \sigma_Y = 0.667$ and $\varepsilon_{L,r} = 0$ yields the population density differential in terms of the three amenity types,

$$\hat{N}^j \approx 8.26\hat{Q}^j + 2.21\hat{A}_X^j + 2.88\hat{A}_Y^j. \tag{30}$$

This expression is potentially misleading since a one-point increase in $\hat{Q}^j$ has the value of a one-point increase in income, while one-point increases in $\hat{A}_X^j$ and $\hat{A}_X^j$ have values of s_x and s_y of income due to the size of their respective sectors. Normalizing the effects so that they are of equal value increases the coefficients on the productivity effects,

$$\begin{aligned}\hat{N}^j \approx & \varepsilon_{N,Q}\hat{Q}^j + \frac{\varepsilon_{N,A_X}}{s_x} s_x \hat{A}_X^j + \frac{\varepsilon_{N,A_Y}}{s_y} s_y \hat{A}_Y^j \\ & = 8.26\hat{Q}^j + 3.45 s_x \hat{A}_X^j + 8.00 s_y \hat{A}_Y^j.\end{aligned} \tag{31}$$

Thus we see that both quality of life and home-productivity have large impacts on local population density, with an increase worth one-percent of income increasing population density by at least eight percentage points; this effect is more than double the effect of trade-productivity. Much of these differences depend on taxes, which discourage workers from being in trade-productive areas and push them towards high quality of life and home-productive areas (Albouy 2009a). Making taxes geographically neutral results in amenities having more similar effects,[15]

$$\hat{N}^j \approx 7.09\hat{Q}^j + 5.81 s_x \hat{A}_X^j + 7.55 s_y \hat{A}_Y^j.$$

[15] As in Albouy (2009a), we define "geographically neutral" taxes as those which do not distort the household location decision, summarized in equation (1*).

The effects are still not equal: when quality of life increases, income effects imply households will pack themselves more into housing, while substitution effects will cause producers to substitute away from land towards labor, making consumption less land intensive. When trade-productivity rises these substitution effects are weaker, and households still demand compensation in terms of land-intensive goods.

The numbers in Table 2 imply that the most important substitution elasticity affecting location decisions is σ_Y. Without it, additional home-good production comes only from increases in home-productivity or land released from the traded-good sector. Letting σ_Y remain a free parameter,

$$\hat{N}^j \approx (2.92 + 8.02\sigma_Y)\hat{Q}^j + (1.32 + 3.21\sigma_Y)s_x\hat{A}_X^j + (3.17 + 7.24\sigma_Y)s_y\hat{A}_Y^j. \quad (32)$$

Setting $\sigma_Y = 0$ yields much lower elasticities: population density cannot increase much when the housing stock cannot be made denser.[16] On the other hand, setting $\sigma_D = 0$ eliminates substitution effects in consumption, but allows for income effects. As can be seen from Table 2, setting $\sigma_D = 0$ means households respond less to quality of life and trade-productivity, but more to home-productivity. In this case, a city's productivity in building housing is more important than the consumption amenities it offers households.

The overall dependence of density on substitution possibilities may be gauged by restricting the elasticities to be equal, $\sigma_D = \sigma_X = \sigma_Y \equiv \sigma$, revealing relatively small constants:

$$\hat{N}^j \approx (0.77 + 11.23\sigma)\hat{Q}^j + (5.17\sigma)s_x\hat{A}_X^j + (0.77 + 8.78\sigma)s_y\hat{A}_Y^j. \quad (33)$$

In a Cobb-Douglas economy, $\sigma = 1$, the implied elasticities are almost 50-percent higher than in the base calibration $\sigma = 0.667$.[17] Assuming a Cobb-Douglas economy seems innocuous for predicting prices, when substitution elasticities have no first-order effect, but it significantly affects results when modeling quantities.

[16]These estimates might be more accurate in predicting population flows to negative shocks in the spirit of Glaeser and Gyourko (2005), who highlight the asymmetric impact of durable housing on population flows.

[17]When $\sigma = 1$, we obtain $\hat{N}^j \approx 12.00\hat{Q}^j + 5.18s_x\hat{A}_X^j + 10.92s_y\hat{A}_Y^j$

The multiplier effect for agglomeration feed back can also be calibrated. Calibrating our fairly large values for positive and negative economies shows that the two could possibly offset each other.

$$\frac{1}{1+\gamma\varepsilon_{N,Q}-\alpha\varepsilon_{N,A_X}} \approx \frac{1}{1+(0.015)(8.26)-(0.06)(2.21)} \approx 1.01$$

Taken individually, the multiplier for positive feedback (through trade-productivity) is 1.15, while for negative feedback (through quality of life) it is 0.89. These calibrated values suggest that the bias from ignoring agglomeration feedback might be modest. Basic agglomeration economies or diseconomies do not seem to dominate other location forces due to natural advantages, historical path dependence, or other local idiosyncrasies.

We have discussed results for only one urban quantity, population density. In Table 3 we list the reduced-form elasticities for all endogenous prices and quantities. Panel A presents results for the baseline tax treatment, while Panel B presents results for geographically neutral federal taxes.

4 General-Equilibrium Elasticities and Empirical Estimates

Our model sheds light on commonly estimated elasticities of local labor demand or housing supply, predicated on partial-equilibrium models that consider labor and housing markets separately. Our general-equilibrium model considers housing and labor markets simultaneously. The adjustments underlying these elasticities might take place over the course of decades, if not generations. For example, our model may account for changes in the durable housing stock or shifts in labor across exportable sectors. As we discuss below, the source of the change in supply or demand may matter a great deal.

4.1 Local Labor Supply and Demand

In our general-equilibrium model, an increase in labor demand is caused by an increase in trade-productivity A_X^j. If tradable goods are heterogenous and the number of cities is large, the increase in trade-productivity could be due to an increase in the world price of the output produced in the city. Holding productivity (and agglomeration) constant, a larger work force pushes down wages, as firms must complement it with ever scarcer and more expensive land.[18]

An increase in labor supply can be brought about by an increase in quality of life Q^j, which reduces the wage workers require. With homogenous workers, the labor supply curve slopes upward because workers must be compensated for rising home-good prices.[19]

According to the calibration, an increase in trade-productivity produces the ratio

$$\frac{\partial \hat{N}}{\partial \hat{w}} = \frac{\partial \hat{N}/\partial \hat{A}_X}{\partial \hat{w}/\partial \hat{A}_X} \approx \frac{2.210}{1.091} \approx 2.02,$$

which may be interpreted as an elasticity of local labor supply. Researchers have frequently tried to estimate this elasticity using a method from Bartik (1991).[20] Estimates seen in Bartik and Nowtowidigo (2012) are generally in the range of 2 to 4. These fairly large values are remarkably close to that predicted by the calibration. In addition, we can use the model to interpret possible issues with the estimates. If increases in demand (i.e., increases in A_X) are positively correlated with increases in supply (i.e., increases in Q), then the elasticity of labor supply will be be biased upwards.[21]

[18]Some models simply assume a fixed factor in production, e.g., land which is only available for the tradable sector. Here, land in the tradable sector must compete with land in the non-tradable sector, causing the price to rise as more households enter and consume non-tradable goods, such as housing.

[19]If workers have heterogeneous tastes, then the slope of the supply curve would rise, as higher wages will attract those with weaker tastes for living in a location.

[20]Bartik uses an instrumental variable which predicts changes in local labor demand based on national changes in industrial composition and a city's past industrial structure. In general, an instrumental variable can identify $(\partial \hat{N}/\partial \hat{A}_X)/(\partial \hat{w}/\partial \hat{A}_X)$ so long as the IV is correlated with changes in trade-productivity and uncorrelated with changes in quality of life or home-productivity. Reconciling the Bartik IV with a long run general-equilibrium model is challenging. If labor and capital are fully mobile, then a city's past industrial structure should have very little predictive power over its current industrial structure. On the other hand, if a city's past industrial structure is correlated with unobservable determinants of current labor demand, then the Bartik instrument is endogenous.

[21]The estimates in Notowdigdo (2012) reveal an increase in housing costs, along with higher wages, that are consistent with a small increase in quality of life.

The slope of the labor demand curve may be identified from an exogenous change in quality of life. According to our calibration, the elasticity of labor demand is large.

$$\frac{\partial \hat{N}}{\partial \hat{w}} = \frac{\partial \hat{N}/\partial \hat{Q}}{\partial \hat{w}/\partial \hat{Q}} \approx \frac{8.261}{-0.359} \approx -23.01$$

To our knowledge, the closest empirical analog to this elasticity comes the immigration literature, which regularly studies how relative wages vary with relative labor supply. A common empirical strategy emphasizes that existing immigrant enclaves are attractive to new immigrants from similar source countries (e.g., Card 2001). Cross-sectional variation in quality of life comes from historic immigration patterns. In general, this literature finds wages at the city level to be fairly unresponsive to increases in labor supply, consistent with the large elasticity above.

The model also highlights an atypical supply and demand increase that could be brought forth through higher housing productivity. In this case, firms may demand more labor to produce more home-goods, while the supply of workers increases because the cost-of-living falls. According to our calibration, the net result is an increase in labor supply and a modest decrease in wages.

$$\frac{\partial \hat{N}}{\partial \hat{w}} = \frac{\partial \hat{N}/\partial \hat{A}_Y}{\partial \hat{w}/\partial \hat{A}_Y} \approx \frac{2.879}{-0.117} \approx -24.61.$$

We are not aware of any estimates of this elasticity, although works by Saks (2008) and others has highlighted the importance of housing supply in accommodating worker inflows.

4.2 Local Housing Supply and Demand

As labor and housing markets both clear in the neoclassical model, the population is closely tied to the amount of housing, which we interpret as home goods. The difference between population and housing is due to substitution and income effects in consumption, calibrated as

$$\hat{Y}^j = \hat{N}^j - 0.43\hat{p}^j - \hat{Q}^j \tag{34}$$

$$= 6.17\hat{Q}^j + 2.38 s_x \hat{A}_X^j + 8.21 s_y \hat{A}_Y^j. \tag{35}$$

Relative to population, housing responds less strongly to quality of life and trade-productivity differences and more strongly to home-productivity differences. Most empirical studies ignore changes in per-capita housing consumption and instead equate housing with population.

It is worth repeating that the source of the shift in housing supply is important. Housing supply responds over twice as much to an increase of quality of life as to an increase in trade-productivity.[22]

$$\frac{\partial \hat{Y}}{\partial \hat{p}} = \frac{\partial \hat{Y}/\partial \hat{A}_X}{\partial \hat{p}/\partial \hat{A}_X} \approx \frac{1.524}{1.608} \approx 0.95$$

$$\frac{\partial \hat{Y}}{\partial \hat{p}} = \frac{\partial \hat{Y}/\partial \hat{Q}}{\partial \hat{p}/\partial \hat{Q}} \approx \frac{6.175}{2.544} \approx 2.43.$$

These calibrated values are within the range seen in Saiz (2010) of 0.80 to 5.45 for different cities. His empirical strategy uses shifts in industrial composition, immigrant enclaves, and sunshine as sources of exogenous variation to identify these elasticities. Thus, he appears to estimate a hybrid of the two elasticities above; places deemed to have a greater housing supply elasticity may have instead experienced a greater quality of life change than trade-productivity change. Part of the observed variation also stems from cross-metro differences in σ_Y and $\varepsilon_{L,r}$, which we ignore.

Our log-linearization predicts that housing prices will not be affected by increasing housing supply through the production elasticity σ_Y, since there is no first-order dependence.[23] An increase

[22] When land supply is fixed, the total home-good differential represents a housing density differential.

[23] See equation (19).

in home-productivity does lower prices, but by much less than it increases the amount of housing,

$$\frac{\partial \hat{Y}}{\partial \hat{p}} = \frac{\partial \hat{Y}/\partial \hat{A}_Y}{\partial \hat{p}/\partial \hat{A}_Y} \approx \frac{2.953}{-0.172} \approx -17.17.$$

Thus, with homogenous preferences and mobile factors, measures that increase housing productivity, such as reducing regulations, will be seen much more in quantities than prices.

The frictionless neoclassical model generates own-price demand elasticities which are roughly an order of magnitude larger than supply elasticities.

5 Empirical Relationship between Density, Prices, and Amenities

5.1 Data

We define cities at the Metropolitan Statistical Area (MSA) level using 1999 Office of Management and Budget (OMB) definitions of consolidated MSAs (e.g., San Francisco is combined with Oakland and San Jose), of which there are 276. We use the 5-percent sample of 2000 United States Census from Ruggles et al. (2004) to calculate wage and housing price differentials, controlling for relevant covariates.[24] Population density also comes from the 2000 Census. Density is calculated at the census tract level, then averaged according to population to form an MSA density value. All of our empirical results below use MSA population weights.

5.2 Population Density and Calibrated Substitution Elasticities

We first consider how well the model predicts population densities using price information and examine the accuracy of our calibrated substitution elasticities. Following the discussion above, we assume $\hat{A}_Y^j = 0$ and use $\hat{w}^j$ and $\hat{p}^j$ to identify $\hat{Q}^j$ and $\hat{A}_X^j$ from equations (1*) and (25). Given

[24] See Appendix D for more details on the calculation of wage and price differentials.

these amenity estimates and our calibrated elasticities, we predict $\hat{N}^j$ for each city and compare this with actual population density differences.

As shown in Table 4, the variance in log population density differences across MSAs is 0.770. The variance predicted by the model is 0.359, which means that a remarkable 47-percent of density variation is explained by the neoclassical model, based on a calibration pre-set in Albouy (2009a).[25]

To determine whether we could explain more variation by choosing different elasticities of substitution, we consider how well different combinations of $\sigma_D, \sigma_X, \sigma_Y$ predict densities. In Figure 2, we graph the variance of the prediction error as a function of the elasticities of substitution. If, for simplicity's sake, we restrict $\sigma_D = \sigma_X = \sigma_Y = \sigma$, as in equation (33), prediction error is minimized at roughly $\sigma = 0.667$, our initial specification. Other values increase prediction error, including the Cobb-Douglas case $\sigma = 1$. The next curve fixes $\sigma_X = 0.667$, which reduces the prediction error, but only by a small amount. Also fixing $\sigma_D = 0.667$, as in the last curve, reduces prediction error by roughly the same amount. The greatest reduction comes from setting $\sigma_Y = 0.667$, further emphasizing its importance. If σ_Y is set to zero, the model reduces prediction error by only half as much relative to when $\sigma_Y = 0.667$. The takeaway from this exercise is that our the pre-set calibration does quite well relative to other potential calibrations.[26]

5.3 Trade and Home-Productivity Estimates

We now exploit density information to identify trade and home-productivity separately, using the method proposed in Section 2.6. Figure 3 displays estimated measures of inferred cost and excess density for different MSAs from the left-hand sides of equations (25) and (26). The figure in-

[25]It is worth repeating that the values here were taken from the literature, and were not estimated from population density.

[26]An unrestricted regression of log density on wages and housing costs naturally produces a higher R-squared of $0.72 > 0.47$, with $\hat{N}^j = 4.40\hat{w}^j + 0.90\hat{p}^j + e^j = 0.63\hat{Q}^j + 6.26\hat{A}_X^j + e^j$. Relative to the calibration, this produces an estimate of $\varepsilon_{N,Q}$ that is far too low and ε_{N,A_X} that is far too high. The two estimated reduced-form elasticities are insufficient for identifying the three elasticity parameters. Furthermore, since the estimated coefficient on $\hat{Q}^j$ of 0.63 is less than the constant 0.77 in equation (29), then at least one of the substitution elasticities would have to be negative, which is untenable. The calibrated model suggests that e^j, which includes $\hat{A}_Y^j$, is positively correlated with A_X^j or negatively correlated with $\hat{Q}^j$. If we instead constrain the estimates to fit the restriction $\sigma_D = \sigma_X = \sigma_Y = \sigma$, as in equation (33), then we obtain $\hat{N}^j = 8.57\hat{Q}^j + 2.30\hat{A}_X^j + e^j$ implying a $\sigma = 0.680$, quite close to the calibration.

cludes iso-productivity lines for both tradable and non-tradable sectors. To understand how trade-productivity is inferred, consider the downward-sloping iso-trade-productivity line, along which cities have the average trade-productivity. Above and to the right of this line, cities have higher excess density or inferred costs, indicating above-average trade-productivity. Above and to the left of the upward-sloping iso-home-productivity line, cities have high excess density or low inferred costs, indicating high home-productivity. For example, San Francisco has high inferred costs and average excess density, or high trade-productivity and below-average home-productivity.

Figure 4 uses the same data as Figure 3, but graphs trade and home-productivity directly.[27] Overall, New York is the most productive city. Philadelphia and Chicago have high levels of trade and home-productivity as well, while Santa Fe and Myrtle Beach are unproductive in both sectors. San Antonio has low trade-productivity and high home-productivity. Figure 4 also includes isoclines for excess density and inferred costs, which correspond to the axes in Figure 3. Holding quality of life constant, trade-productivity and home-productivity must move in opposite directions to keep population density constant. Holding quality of life constant, home-productivity must rise faster than trade-productivity to keep inferred costs constant.

Two important points should be made about about the home-productivity estimates. First, they strongly reflect the residual measure of population density.[28] Second, the measure is highly indicative of the accumulated housing stock of a city. Older cities, like New York, Chicago, and Philadelphia, have high home-productivity. We can explain part of this by noting that these cities have been built up over the past century, when building and land use regulations were less restrictive.[29]

[27]Figure A.2 displays the distribution of quality of life, trade-productivity, and home-productivity. Home-productivity displays the greatest variance, though this could be due to the estimation procedure, as described below.

[28]Recall that we can estimate $\hat{Q}^j$ perfectly and $\hat{A}_X^j$ quite well with only wage and housing price data.

[29]Some of these findings appear to conflict with recent work by Albouy and Ehrlich (2012), who use data on land values to infer productivity in the housing sector, which comprises most of the non-tradable sector. While the two approaches largely agree on which large areas have high home-productivity, the land values approach suggests that larger, denser cities generally have lower, rather than higher housing productivity. This apparent contradiction actually highlights what the two methodologies infer differently. Productivity measures based on land values provide a better insight into the marginal cost of increasing the housing supply, by essentially inferring the replacement cost. Productivity measures based on density are more strongly related to the average cost of the housing supply, thereby reflecting the whole history of building in a city. The distinction matters particularly for cities with older housing built on the easiest terrain in the decades prior to the diffusion of residential land-use regulations when factor prices were

The estimation procedure outlined above also refines estimates of trade-productivity over those provided in Albouy (2009a), which assumes that home-productivity is constant. Table 6 contains estimates of population density, wages, housing costs, land values, and amenity differentials for a selected sample of metropolitan areas. Table A.4 contains a full list of metropolitan and non-metropolitan areas; the table also compares the trade-productivity estimates from the two approaches.

5.4 Determinants of Population Density

We now explore the determinants of household location decisions using straightforward variance decomposition, which we present in Tables 4 and 5. The first relies on the simpler estimates of $\hat{Q}^j$ and $\hat{A}_X^j$ based only on price data, while Table 5 uses density information to identify $\hat{A}_Y^j$, providing a fuller decomposition. In Table 4, quality of life explains nearly half of the total variance in predicted population density, even though the variance of trade-productivity is an order of magnitude larger than the variance of quality of life (not reported, but see Figure A.2). Relatively small differences in quality of life explain a large amount of the population distribution. In other words, the constant home-productivity frictionless neoclassical model predicts that "jobs follow people" much more than "people follow jobs."

In Table 5 Panel A, we decompose the variance of observed (which now equals predicted) population density. Quality of life explains more than twice as much population density compared to trade-productivity. The relatively large fraction of variance explained by home-productivity suggests that there remains some portion of household location decisions which our simple model does not explain. Nevertheless, quality of life and trade-productivity explain nearly half of the total variation in population density.

Table 5 Panel B explores how population density would change if federal taxes were made geographically neutral.[30] Trade-productivity now explains a larger fraction of population density than

relatively low.

[30] In particular, we use our amenity estimates and calibrated model to predict prices and quantities (including population density) for each city in the absence of location-distorting federal income taxes. Because we estimate amenities

does quality of life. Federal taxes introduce a wedge between trade-productivity and the benefits that households receive by locating in productive cities. Eliminating the geographic distortion in the tax code would allow households to benefit more from trade-productive cities, which would encourage migration to those places.

When we simultaneously account for agglomeration and congestion forces, we find that these increase the importance of natural advantages in quality of life and lower the importance of those in trade-productivity (Table A.2).

6 Conclusion

Although Rosen and Roback designed the neoclassical model to explain price differences across metropolitan areas, it provides a surprisingly accurate general-equilibrium basis for estimates of elasticities of labor and housing demand and supply. Using only two simple measures of wages and housing costs, the model explains half of the variation in population density using a pre-set calibration. Within the neoclassical framework, the other half may be explained by differences in home-good productivity or heterogeneity in structural parameters, like the elasticity of substitution in home-good production. Additional variation in population density might be explained by extensions such as heterogeneous households, non-neutral productivity differences, or non-homothetic preferences. Beyond the neoclassical framework, issues such as historical path dependence, flexible preference heterogeneity, and mobility frictions might also account for the remaining variation.

Even small differences in quality of life seem to explain large differences in population density, especially when we account for agglomeration economies. The calibrated model suggests that funds spent to attract households may be more effective at boosting metropolitan population than funds spent to attract firms. Federal taxes explain much of the stronger appeal offered by quality of life. The results also imply that available land and flexible housing production are important in accommodating high population and density levels. In other words, differences in population lev-

using observed density, wage, and housing price data, we cannot estimate amenities in the absence of distortionary federal taxes.

els across cities unaccounted for by wages and prices may be due to land and housing availability, rather than preference heterogeneity or dynamic adjustment costs. An important step in our understanding of household location decisions is the development of models capable of consistently explaining both levels and changes in population, while also accounting for housing availability and general-equilibrium adjustments.

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Table 1: Calibrated Parameters

Parameter Name	Notation	Calibrated Value
Cost and Expenditure Shares		
Home-good expenditure share	s_y	0.36
Income share to land	s_R	0.10
Income share to labor	s_w	0.75
Traded-good cost share of land	θ_L	0.025
Traded-good cost share of labor	θ_N	0.825
Home-good cost share of land	ϕ_L	0.233
Home-good cost share of labor	ϕ_N	0.617
Share of land used in traded good	λ_L	0.17
Share of labor used in traded good	λ_N	0.70
Tax Parameters		
Average marginal tax rate	τ	0.361
Average deduction level	δ	0.291
Structural Elasticities		
Elasticity of substitution in consumption	σ_D	0.667
Elasticity of traded-good production	σ_X	0.667
Elasticity of home-good production	σ_Y	0.667
Elasticity of land supply	$\varepsilon_{L,r}$	0.0

Table 2: Sensitivity Analysis

	$\varepsilon_{N,Q}$	ε_{N,A_X}	ε_{N,A_Y}
σ_D	1.258	0.795	-0.085
σ_X	1.954	0.467	0.636
σ_Y	8.015	2.052	2.608
$\varepsilon_{L,r}$	11.853	4.009	3.857
Constant	0.773	0.000	0.773

Table 2 describes the effect on reduced-form elasticities of increasing each structural elasticity by one, e.g., $\varepsilon_{N,Q} = 0.773 + 1.258\sigma_D + 1.954\sigma_X + 8.015\sigma_Y + 11.853\varepsilon_{L,r}$.

Table 3: Base Elasticities

	A: With Taxes			B: Neutral Taxes		
	$\hat{Q}$	$\hat{A}_X$	$\hat{A}_Y$	$\hat{Q}$	$\hat{A}_X$	$\hat{A}_Y$
$\hat{r}$	11.853	4.009	3.857	10.001	6.400	3.600
$\hat{w}$	-0.359	1.091	-0.117	-0.303	1.018	-0.109
$\hat{p}$	2.544	1.608	-0.172	2.146	2.121	-0.227
$\hat{x}$	-0.389	0.386	-0.041	-0.654	0.728	-0.078
$\hat{y}$	-2.086	-0.686	0.074	-1.916	-0.905	0.097
$\hat{N}$	8.261	2.210	2.879	7.091	3.721	2.717
$\hat{L}$	0.000	0.000	0.000	0.000	0.000	0.000
$\hat{K}$	8.008	2.907	2.774	6.864	4.385	2.616
$\hat{X}$	8.085	3.414	2.926	7.008	4.805	2.777
$\hat{Y}$	6.175	1.524	2.953	5.175	2.816	2.815
$\hat{N}_X$	8.324	2.354	3.004	7.210	3.792	2.850
$\hat{N}_Y$	8.112	1.869	2.583	6.809	3.551	2.403
$\hat{L}_X$	0.178	0.407	0.354	0.337	0.203	0.376
$\hat{L}_Y$	-0.034	-0.078	-0.067	-0.064	-0.039	-0.071
$\hat{K}_X$	8.045	3.081	2.926	7.008	4.471	2.777
$\hat{K}_Y$	7.872	2.596	2.505	6.607	4.230	2.330

Each value in Table 3 represents the partial effect that a one-percent increase in each amenity has on each price or quantity, e.g., $\hat{N}^j = 8.261\hat{Q}^j + 2.210\hat{A}_X^j + 2.879\hat{A}_Y^j$. The values in panel A are derived using the parameters in Table 1. The values in panel B are derived using geographically neutral taxes.

Table 4: Variance Decomposition, Two Amenity

	Fraction of $Var(\hat{N})$ explained by		
$Var(\hat{N})$	$Var(\varepsilon_{N,Q}\hat{Q})$	$Var(\varepsilon_{N,A_X}\hat{A}_X)$	$Cov(\varepsilon_{N,Q}\hat{Q}, \varepsilon_{N,A_X}\hat{A}_X)$
0.359	0.498	0.184	0.318

Table 4 presents the variance decomposition of predicted population density using data on wages and house prices only. See text for more details.

Table 5: Variance Decomposition, Three Amenity

A: Observed Population Density and Prices. $Var(\hat{N}) = 0.770$ Fraction of $Var(\hat{N})$ explained by			
	$Cov(\varepsilon_{N,Q}\hat{Q}, \cdot)$	$Cov(\varepsilon_{N,A_X}\hat{A}_X, \cdot)$	$Cov(\varepsilon_{N,A_Y}\hat{A}_Y, \cdot)$
$Cov(\cdot, \varepsilon_{N,Q}\hat{Q})$	0.232	·	·
$Cov(\cdot, \varepsilon_{N,A_X}\hat{A}_X)$	0.137	0.102	·
$Cov(\cdot, \varepsilon_{N,A_Y}\hat{A}_Y)$	-0.145	0.235	0.439
B: Counterfactual Density and Prices. $Var(\hat{N}) = 1.005$ Fraction of $Var(\hat{N})$ explained by			
	$Cov(\varepsilon_{N,Q}\hat{Q}, \cdot)$	$Cov(\varepsilon_{N,A_X}\hat{A}_X, \cdot)$	$Cov(\varepsilon_{N,A_Y}\hat{A}_Y, \cdot)$
$Cov(\cdot, \varepsilon_{N,Q}\hat{Q})$	0.131	·	·
$Cov(\cdot, \varepsilon_{N,A_X}\hat{A}_X)$	0.151	0.221	·
$Cov(\cdot, \varepsilon_{N,A_Y}\hat{A}_Y)$	-0.090	0.287	0.300

Panel A presents the variance decomposition using data on population density, wages, and house prices. Panel B presents the variance decomposition under geographically neutral taxes and associated price and quantity predictions.

Table 6: List of Selected Metropolitan Areas Ranked by Inferred Land Value

Name of Metropolitan Area	$\hat{N}^j$	$\hat{w}^j$	$\hat{p}^j$	$\hat{r}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	$\hat{A}_Y^j$
New York, Northern New Jersey, Long Island, NY-NJ-CT-PA	2.285	0.209	0.411	3.366	0.029	0.264	0.509
San Francisco-Oakland-San Jose, CA	1.209	0.256	0.813	2.000	0.138	0.269	-0.182
Los Angeles-Riverside-Orange County, CA	1.250	0.129	0.450	1.913	0.081	0.159	0.079
Chicago-Gary-Kenosha, IL-IN-WI	1.191	0.136	0.224	1.767	0.005	0.161	0.276
Salinas (Monterey-Carmel), CA	0.863	0.103	0.590	1.435	0.137	0.124	-0.189
San Diego, CA	0.872	0.058	0.479	1.403	0.123	0.085	-0.114
Philadelphia-Wilmington-Atlantic City, PA-NJ-DE-MD	0.958	0.114	0.052	1.393	-0.040	0.133	0.346
Miami-Fort Lauderdale, FL	0.964	0.001	0.126	1.352	0.041	0.035	0.191
Santa Barbara-Santa Maria-Lompoc, CA	0.713	0.068	0.662	1.255	0.176	0.090	-0.326
...							
Goldsboro, NC	-1.509	-0.183	-0.297	-2.226	-0.007	-0.213	-0.340
Johnson City-Kingsport-Bristol, TN-VA	-1.485	-0.179	-0.363	-2.236	-0.028	-0.209	-0.273
Dothan, AL	-1.533	-0.181	-0.404	-2.314	-0.040	-0.214	-0.253
Hickory-Morganton-Lenoir, NC	-1.624	-0.127	-0.220	-2.356	-0.008	-0.168	-0.412
Ocala, FL	-1.582	-0.170	-0.298	-2.363	-0.010	-0.205	-0.362
Florence, SC	-1.606	-0.120	-0.341	-2.381	-0.049	-0.162	-0.292
Rocky Mount, NC	-1.640	-0.111	-0.246	-2.384	-0.024	-0.155	-0.381
Anniston, AL	-1.579	-0.183	-0.424	-2.385	-0.046	-0.216	-0.250
Texarkana, TX-Texarkana, AR	-1.556	-0.185	-0.498	-2.388	-0.068	-0.219	-0.178
Jonesboro, AR	-1.651	-0.240	-0.452	-2.533	-0.026	-0.269	-0.293

Table 6 includes the top and bottom ten metropolitan areas ranked by inferred land value ($\hat{r}^j$). The first three columns ($\hat{N}^j, \hat{w}^j, \hat{p}^j$) are estimated from Census data, while the last four columns come from the calibrated model. See text for estimation procedure.

Figure 1: Distribution, 2000

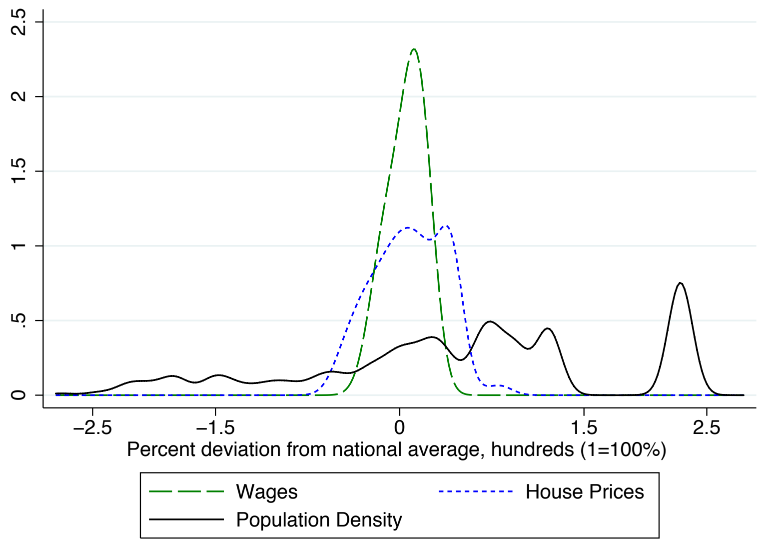


Figure 1 is smoothed with a Gaussian kernel, bandwidth=0.1.

Figure 2: Error in Fitting Pop. Density using $\hat{Q}$ and $\hat{A}_X$ Only

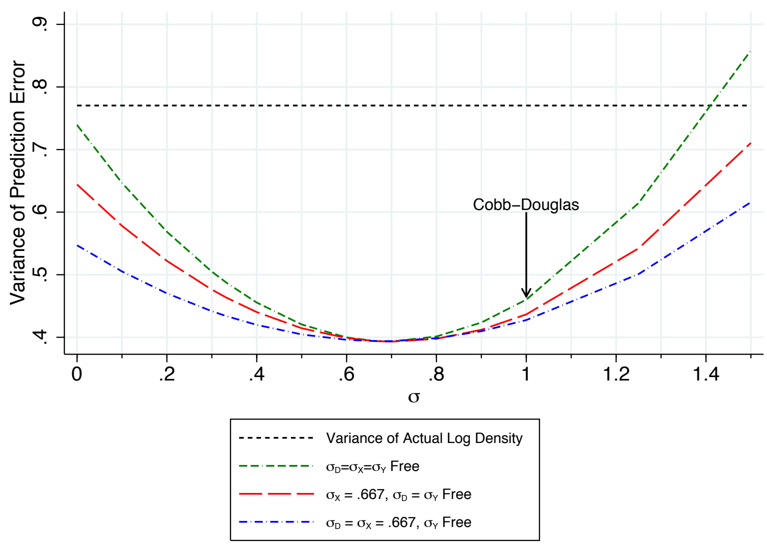
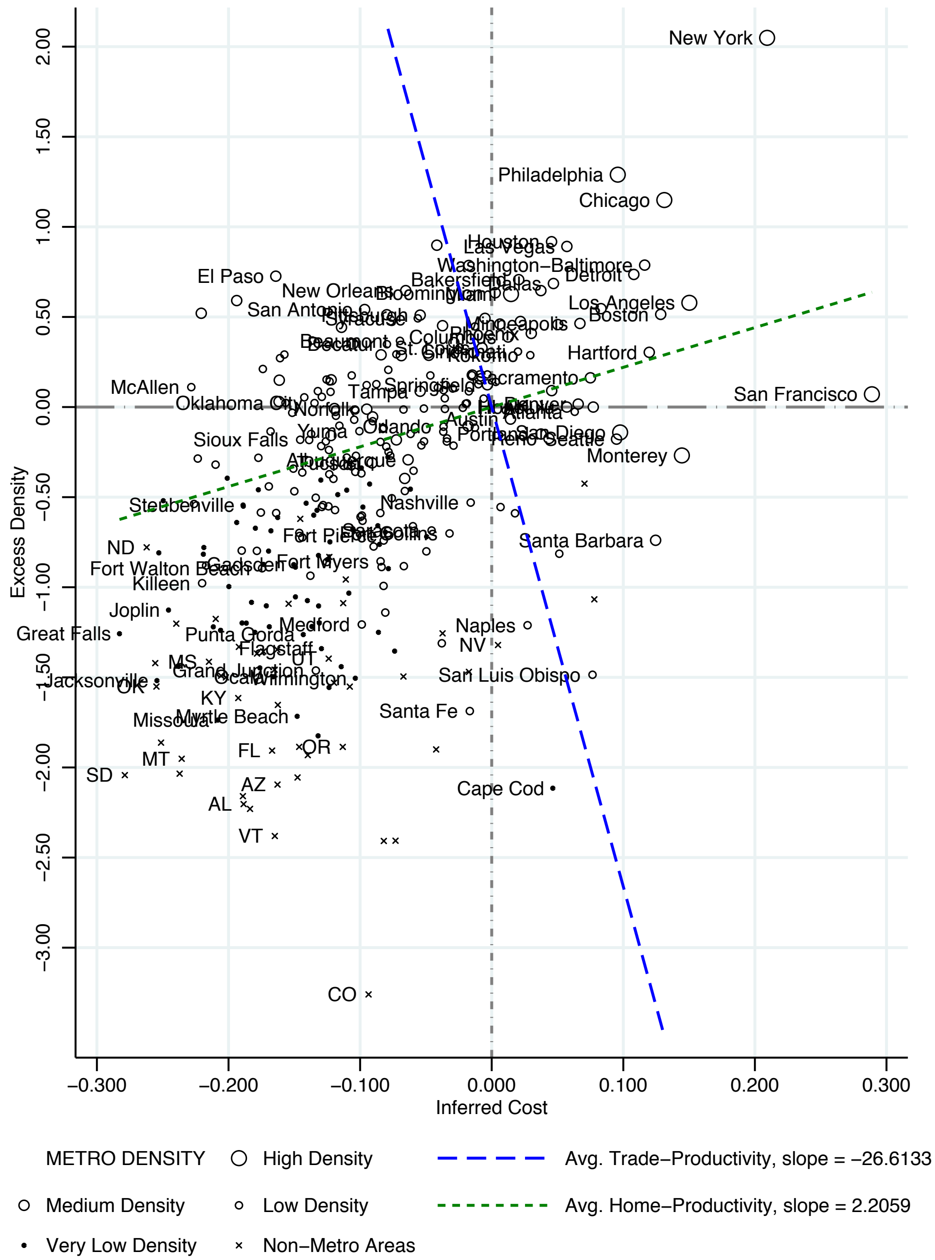
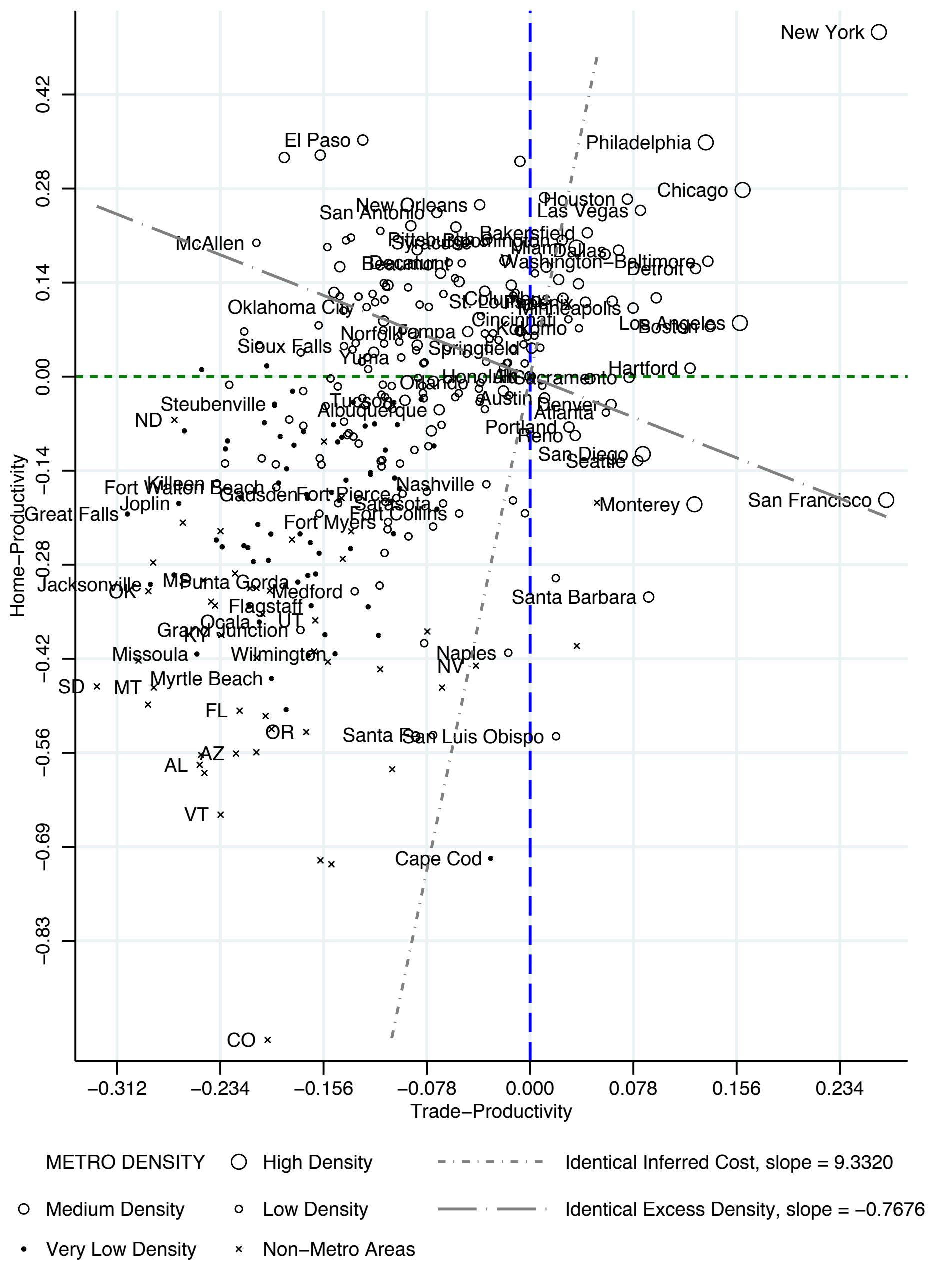


Figure 3: Excess Density and Inferred Cost Estimates, 2000



See text for estimation details. High density metros have population density which exceeds the national average by 80-percent, medium density metros are between the national average and 80-percent. Low density and very low density metros are defined symmetrically.

Figure 4: Trade- and Home-Productivity Estimates, 2000



See note to Figure 3.

Online Appendix - Not for Publication

A Comparison of Nonlinear and Linear Models

We employ a two-step simulation method to solve a nonlinear version of the model.[31] We assume that utility and production functions display constant elasticity of substitution,

$$U(x, y; Q) = Q(\eta_x x^{\alpha} + (1 - \eta_x)y^{\alpha})^{1/\alpha} \tag{A.1}$$

$$F_X(L_X, N_X, K_X; A_X) = A_X(\gamma_L L^{\beta} + \gamma_N N^{\beta} + (1 - \gamma_L - \gamma_N)K^{\beta})^{1/\beta} \tag{A.2}$$

$$F_Y(L_Y, N_Y, K_Y; A_Y) = A_Y(\rho_L L^{\chi} + \rho_N N^{\chi} + (1 - \rho_L - \rho_N)K^{\chi})^{1/\chi} \tag{A.3}$$

where

$$\alpha \equiv \frac{\sigma_D - 1}{\sigma_D}$$
$$\beta \equiv \frac{\sigma_X - 1}{\sigma_X}$$
$$\chi \equiv \frac{\sigma_Y - 1}{\sigma_Y}$$

Throughout, we assume that $\sigma_D = \sigma_X = \sigma_Y = 0.667$. We first consider a "large" city with amenity values normalized so that $Q = A_X = A_Y = 1$. We fix land supply, population, and the rental price of capital $\bar{\iota}$. We then solve a nonlinear system of fifteen equations, corresponding to equations (1)-(14) and (16), for fifteen unknown variables: $\bar{u}, w, r, p, x, y, X, Y, N_X, N_Y, L_X, L_Y, K_X, K_Y, K$. We simultaneously choose values of $\eta_x, \gamma_L, \gamma_N, \rho_L$, and ρ_N so that the model matches values of $s_y, \theta_L, \theta_N, \phi_L$, and ϕ_N in Table 1. The large city calibration also yields values for R, I, and T.[32]

We then consider a "small" city, which we endow with land equal to one one-millionth of the large city's land.[33] The population for the small city is endogenous, and the reference utility level $\bar{u}$ is exogenous. The average amenity values are $Q = A_X = A_Y = 1$. While holding two amenities fixed at the average, we solve the model after setting the third amenity to be somewhere between 0.8 and 1.2. We solve the same system as for the large city, but now solve for $w, r, p, x, y, X, Y, N_X, N_Y, L_X, L_Y, K_X, K_Y, N, K$.

We compare the nonlinear model to a one-city linear model. We use parameter values from Table 1, but set the marginal tax rate $\tau = 0$ and deduction level $\delta = 0$. The average amenity differentials are $\hat{Q} = \hat{A}_X = \hat{A}_Y = 0$. As with the nonlinear model, we vary a single amenity while holding the other amenities at their average value. Given a choice of amenity differentials $\hat{Q}, \hat{A}_X$, and $\hat{A}_Y$, equations (17), (18), and (19) determine prices $\hat{r}, \hat{w}$, and $\hat{p}$. Equations (20) and (21) next determine per-capita consumption quantities $\hat{x}$ and $\hat{y}$. We express equations (6*)-(16*) in matrix form as

[31] Rappaport (2008a, 2008b) follows a similar procedure.

[32] To simulate the model, we solve a mathematical program with equilibrium constraints, as described in Su and Judd (2012). MATLAB code for the simulation is available at **WEBSITE**.

[33] We do this to avoid any feedback effects from the small city to the large one. In particular, this permits use of values of $\bar{u}, \bar{\iota}, R, I$, and T from the large city calibration, which simplifies the procedure considerably.

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ \lambda_N & 0 & 0 & 0 & 1-\lambda_N & 0 & 0 & 0 & -1 & 0 \\ 0 & \lambda_L & 0 & 0 & 0 & 1-\lambda_L & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_K & 0 & 0 & 0 & 1-\lambda_K & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \hat{N}_X \\ \hat{L}_X \\ \hat{K}_X \\ \hat{X} \\ \hat{N}_Y \\ \hat{L}_Y \\ \hat{K}_Y \\ \hat{Y} \\ \hat{N} \\ \hat{K} \end{bmatrix} = \begin{bmatrix} -\hat{A}_X + \theta_L \sigma_X (\hat{r} - \hat{w}) - \theta_K \sigma_X \hat{w} \\ -\hat{A}_X + \theta_N \sigma_X (\hat{w} - \hat{r}) - \theta_K \sigma_X \hat{r} \\ -\hat{A}_X + \theta_L \sigma_X \hat{r} + \theta_N \sigma_X \hat{w} \\ -\hat{A}_Y + \phi_L \sigma_Y (\hat{r} - \hat{w}) - \phi_K \sigma_Y \hat{w} \\ -\hat{A}_Y + \phi_N \sigma_Y (\hat{w} - \hat{r}) - \phi_K \sigma_Y \hat{r} \\ -\hat{A}_Y + \phi_L \sigma_Y \hat{r} + \phi_N \sigma_Y \hat{w} \\ 0 \\ \varepsilon_{L,r} \hat{r} \\ 0 \\ \hat{y} \end{bmatrix}$$

We can simply invert the above matrix system to solve for aggregate production quantity differentials, including population.

Figure A.1 presents results of both models in terms of reduced-form population elasticities with respect to each amenity.[34] The linear model does quite well in approximating population responses to trade- and home-productivity. The linear model approximates the quality of life reduced-form elasticity less precisely. As quality of life increases, population increases at an increasing rate, while population decreases at a decreasing rate as quality of life falls. Similar, but weaker, patterns exist for trade- and home-productivity.

B Additional Theoretical Details

B.1 Reduced-Form Elasticities

The analytic solutions for reduced-form elasticities of population with respect to amenities are given below.

$$\begin{aligned} \varepsilon_{N,Q} = &\left[\frac{\lambda_N - \lambda_L}{\lambda_N}\right] + \sigma_D \left[\frac{s_x(\lambda_N - \lambda_L)^2}{s_y \lambda_N (\lambda_N - \lambda_L \tau)}\right] + \sigma_X \left[\frac{\lambda_L^2}{s_w(\lambda_N - \lambda_L \tau)} + \frac{\lambda_L \lambda_N}{s_R(\lambda_N - \lambda_L \tau)}\right] \\ &+ \sigma_Y \left[\frac{\lambda_L^2 (1-\lambda_N)}{s_w \lambda_N (\lambda_N - \lambda_L \tau)} + \frac{\lambda_N (1-\lambda_L)}{s_R(\lambda_N - \lambda_L \tau)} - \frac{(\lambda_N - \lambda_L)^2}{s_y \lambda_N (\lambda_N - \lambda_L \tau)}\right] \\ &+ \varepsilon_{L,r} \left[\frac{\lambda_N}{s_R(\lambda_N - \lambda_L \tau)}\right] \end{aligned}$$

$$\begin{aligned} \varepsilon_{N,A_X} = &\ \sigma_D \left[\frac{s_x^2(\lambda_N - \lambda_L)(1-\lambda_L)(1-\tau)}{s_y \lambda_N (\lambda_N - \lambda_L \tau)}\right] + \sigma_X \left[\frac{s_x \lambda_L (\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L \tau)} - \frac{s_x \lambda_L (1-\lambda_L)}{s_w(\lambda_N - \lambda_L \tau)}\right] + \\ &\sigma_Y \left[\frac{s_x (1-\lambda_L)(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L \tau)} - \frac{s_x \lambda_L (1-\lambda_L)(1-\lambda_N)}{s_w \lambda_N (\lambda_N - \lambda_L \tau)} - \frac{s_x (1-\lambda_L)(\lambda_N - \lambda_L)(1-\tau)}{s_y \lambda_N (\lambda_N - \lambda_L \tau)}\right] \\ &+ \varepsilon_{L,r} \left[\frac{s_x(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L \tau)}\right] \end{aligned}$$

[34]We normalize the elasticities in Figure A.1 for trade- and home-productivity by s_x and s_y.

$$\varepsilon_{N,A_Y} = \left[\frac{\lambda_N - \lambda_L}{\lambda_N}\right] + \sigma_D \left[\frac{-s_x\lambda_L(\lambda_N - \lambda_L)(1-\tau)}{\lambda_N(\lambda_N - \lambda_L\tau)}\right] + \sigma_X \left[\frac{s_y\lambda_N\lambda_L}{s_R(\lambda_N - \lambda_L\tau)} + \frac{s_y\lambda_L^2}{s_w(\lambda_N - \lambda_L\tau)}\right]$$
$$+ \sigma_Y \left[-\left(\frac{\lambda_N - \lambda_L}{\lambda_N}\right) + \frac{s_y\lambda_L^2(1-\lambda_N)}{s_w\lambda_N(\lambda_N - \lambda_L\tau)} + \frac{s_y\lambda_N(1-\lambda_L)}{s_R(\lambda_N - \lambda_L\tau)} + \frac{\lambda_L(\lambda_N - \lambda_L)(1-\tau)}{\lambda_N(\lambda_N - \lambda_L\tau)}\right]$$
$$+ \varepsilon_{L,r}\left[\frac{s_y\lambda_N}{s_R(\lambda_N - \lambda_L\tau)}\right]$$

B.2 Special Case: Fixed Per-Capita Housing Consumption

Consider the case in which per-capita housing consumption is fixed, $\hat{y}^j = 0$. The model then yields $\hat{N}^j = \tilde{\varepsilon}_{N,Q}\hat{Q}^j + \tilde{\varepsilon}_{N,A_X}\hat{A}_X^j + \tilde{\varepsilon}_{N,A_Y}\hat{A}_Y^j$, where the coefficients are defined as:

$$\tilde{\varepsilon}_{N,Q} = \sigma_X \left[\frac{\lambda_L^2}{s_w(\lambda_N - \lambda_L\tau)} + \frac{\lambda_L\lambda_N}{s_R(\lambda_N - \lambda_L\tau)}\right] + \varepsilon_{L,r}\left[\frac{\lambda_N}{s_R(\lambda_N - \lambda_L\tau)}\right]$$
$$+ \sigma_Y \left[\frac{\lambda_L^2(1-\lambda_N)}{s_w\lambda_N(\lambda_N - \lambda_L\tau)} + \frac{\lambda_N(1-\lambda_L)}{s_R(\lambda_N - \lambda_L\tau)} - \frac{(\lambda_N - \lambda_L)^2}{s_y\lambda_N(\lambda_N - \lambda_L\tau)}\right]$$
$$\tilde{\varepsilon}_{N,A_X} = \sigma_X \left[\frac{s_x\lambda_L(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L\tau)} - \frac{s_x\lambda_L(1-\lambda_L)}{s_w(\lambda_N - \lambda_L\tau)}\right] + \varepsilon_{L,r}\left[\frac{s_x(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L\tau)}\right]$$
$$+ \sigma_Y \left[\frac{s_x(1-\lambda_L)(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L\tau)} - \frac{s_x\lambda_L(1-\lambda_L)(1-\lambda_N)}{s_w\lambda_N(\lambda_N - \lambda_L\tau)} - \frac{s_x(1-\lambda_L)(\lambda_N - \lambda_L)(1-\tau)}{s_y\lambda_N(\lambda_N - \lambda_L\tau)}\right]$$
$$\tilde{\varepsilon}_{N,A_Y} = \sigma_X \left[\frac{s_y\lambda_N\lambda_L}{s_R(\lambda_N - \lambda_L\tau)} + \frac{s_y\lambda_L^2}{s_w(\lambda_N - \lambda_L\tau)}\right] + \varepsilon_{L,r}\left[\frac{s_y\lambda_N}{s_R(\lambda_N - \lambda_L\tau)}\right]$$
$$+ \sigma_Y \left[\frac{s_y\lambda_L^2(1-\lambda_N)}{s_w\lambda_N(\lambda_N - \lambda_L\tau)} + \frac{s_y\lambda_N(1-\lambda_L)}{s_R(\lambda_N - \lambda_L\tau)} + \frac{\lambda_L(\lambda_n - \lambda_L)(1-\tau)}{\lambda_N(\lambda_N - \lambda_L\tau)}\right]$$

These reduced-form elasticities no longer depend on the elasticity of substitution in consumption σ_D. In addition, above-average quality of life and/or home-productivity no longer lead to higher population independently of the substitution elasticities, i.e., the term $(\lambda_N - \lambda_L)/\lambda_N$ drops out of the elasticities.

B.3 Identification of Elasticity of Substitution in Non-Tradable Production

Consider setting home-productivity constant across cites, $\hat{A}_Y^j = 0$, and using population density to estimate the elasticity of non-tradable good production σ_Y^j for each city. In particular, we have

$$\hat{N}^j = \varepsilon_{N,Q}\hat{Q}^j + \varepsilon_{N,A_X}\hat{A}_X^j, \tag{A.4}$$

where $\varepsilon_{N,Q}$ and ε_{N,A_X} are defined in Section B.1, but now depend on a city-specific σ_Y^j. When home-productivity is constant, we can identify trade-productivity using information on wages and housing prices,

$$\hat{A}_X^j = \frac{\theta_L}{\phi_L}\hat{p}^j + \left(\theta_N - \phi_N \frac{\theta_L}{\phi_L}\right)\hat{w}^j$$

and so equation (A.4) is a single equation in one unknown variable, σ_Y^j. We do not pursue this approach further, but note that it yields city-specific estimates of the elasticity of substitution in non-tradable production, under the restrictive assumption that home-productivity does not vary across cities. Without accurate data on land values, which is necessary to jointly identify trade and home-productivity, the model does not permit simultaneous identification of $\hat{A}_Y^j$ and σ_Y^j using population data alone. Data on capital differentials $\hat{K}^j$ would permit identification of a city-specific elasticity of non-tradable production and both types of productivity, but we are not aware of reliable data on capital stock differentials across cities.

B.4 Deduction

Tax deductions are applied to the consumption of home goods at the rate $\delta \in [0, 1]$, so that the tax payment is given by $\tau(m - \delta p y)$. With the deduction, the mobility condition becomes

$$\begin{aligned}\hat{Q}^j &= (1 - \delta\tau')s_y\hat{p}^j - (1 - \tau')s_w\hat{w}^j \\ &= s_y\hat{p}^j - s_w\hat{w}^j + \frac{d\tau^j}{m}\end{aligned}$$

where the tax differential is given by $d\tau^j/m = \tau'(s_w\hat{w}^j - \delta s_y p^j)$. This differential can be solved by noting

$$\begin{aligned}s_w\hat{w}^j &= s_w\hat{w}_0^j + \frac{\lambda_L}{\lambda_N}\frac{d\tau^j}{m} \\ s_y\hat{p}^j &= s_y\hat{p}_0^j - \left(1 - \frac{\lambda_L}{\lambda_N}\right)\frac{d\tau^j}{m}\end{aligned}$$

and substituting them into the tax differential formula, and solving recursively,

$$\begin{aligned}\frac{d\tau^j}{m} &= \tau' s_w\hat{w}_0^j - \delta\tau' s_y\hat{p}_0^j + \tau'\left[\delta + (1-\delta)\frac{\lambda_L}{\lambda_N}\right] \\ &= \tau'\frac{s_w\hat{w}_0^j - \delta s_y\hat{p}_0^j}{1 - \tau'\left[\delta + (1-\delta)\lambda_L/\lambda_N\right]}\end{aligned}$$

We can then solve for the tax differential in terms of amenities:

$$\frac{d\tau^j}{m} = \tau'\frac{1}{1 - \tau'\left[\delta + (1-\delta)\lambda_L/\lambda_N\right]}\left[(1-\delta)\left(\frac{1-\lambda_L}{\lambda_N}s_x\hat{A}_X^j - \frac{\lambda_L}{\lambda_N}s_y A_Y^j\right) - \frac{(1-\delta)\lambda_L + \delta\lambda_N}{\lambda_N}\hat{Q}^j\right]$$

This equation demonstrates that the deduction reduces the dependence of taxes on productivity and increases the implicit subsidy for quality-of-life.

B.5 State Taxes

The tax differential with state taxes is computed by including an additional component based on wages and prices relative to the state average, as if state tax revenues are redistributed lump-sum to households within the state. This produces the augmented formula

$$\frac{d\tau^j}{m} = \tau' \left(s_w \hat{w}^j - \delta \tau' s_y \hat{p}^j \right) + \tau'_S [s_w (\hat{w}^j - \hat{w}^S) - \delta_S s_y (\hat{p}^j - \hat{p}^S)] \quad \text{(A.5)}$$

where τ'_S and δ_S are are marginal tax and deduction rates at the state-level, net of federal deductions, and $\hat{w}^S$ and $\hat{p}^S$ are the differentials for state S as a whole relative to the entire country.

C Additional Calibration Details

C.1 Cost and Expenditure Shares

We calibrate the model using the data described below and national-level parameters. Starting with income shares, Krueger (1999) argues that s_w is close to 75 percent. Poterba (1998) estimates that the share of income from corporate capital is 12 percent, so s_I should be higher and is taken as 15 percent. This leaves 10 percent for s_R, which is roughly consistent with estimates in Keiper et al. (1961) and Case (2007).[35]

Turning to expenditure shares, Albouy (2008), Moretti (2008), and Shapiro (2006) find that housing costs approximate non-housing cost differences across cities. The cost-of-living differential is $s_y \hat{p}^j$, where $\hat{p}^j$ equals the housing-cost differential and s_y equals the expenditure share on housing plus an additional term which captures how a one percent increase in housing costs predicts a $b = 0.26$ percent increase in non-housing costs.[36] In the Consumer Expenditure Survey (CEX), the share of income spent on shelter and utilities, s_{hous}, is 0.22, while the share of income spent on other goods, s_{oth}, is 0.56, leaving 0.22 spent on taxes or saved (Bureau of Labor Statistics 2002).[37] Thus, our coefficient on the housing cost differential is $s_y = s_{hous} + s_{oth} b = 0.22 + 0.56 \times 0.26 = 36$ percent. This leaves s_x at 64 percent.

We choose the cost shares to be consistent with the expenditure and income shares above. θ_L appears small: Beeson and Eberts (1986) use a value of 0.027, while Rappaport (2008a, 2008b) uses a value of 0.016. Valentinyi and Herrendorff (2008) estimate the land share of tradables at 4 percent, although their definition of tradables differs from the one here. We use a value of 2.5 percent for θ_L here. Following Carliner (2003) and Case (2007), the cost-share of land in home-goods, ϕ_L, is taken at 23.3 percent; this is slightly above values from McDonald (1981), Roback (1982), and Thorsnes (1997) to account for the increase in land cost shares over time described by Davis and Palumbo (2007). Together the cost and expenditure shares imply λ_L is 17 percent, which appears reasonable since the remaining 83 percent of land for home goods includes all residential land and much commercial land; the cost and expenditure shares also agree with s_R at

[35]The values Keiper reports were at a historical low. Keiper et al. (1961) find that total land value was found to be about 1.1 times GDP. A rate of return of 9 percent would justify using $s_R = 0.10$. Case (2007), ignoring agriculture, estimates the value of land to be \$5.6 trillion in 2000 when personal income was \$8.35 trillion.

[36]See Albouy (2008) for details.

[37]Utility costs account for one fifth of s_{hous}, which means that without them this parameter would be roughly 0.18.

10 percent.[38] Finally, we choose the cost shares of labor and capital in both production sectors. As separate information on ϕ_K and θ_K does not exist, we set both cost shares of capital at 15 percent to be consistent with s_I. Accounting identities then determine that θ_N is 82.5 percent, ϕ_N is 62 percent, and λ_N is 70.4 percent.

The federal tax rate, when combined with relevant variation in wages with state tax rates, produces an approximate marginal tax rate, τ, of 36.1 percent. Details on this tax rate, as well as housing deductions, are discussed in Appendix C.2.

C.2 Calibration of Tax Parameters

The federal marginal tax rate on wage income is determined by adding together federal marginal income tax rate and the effective marginal payroll tax rate. TAXSIM gives an average marginal federal income tax rate of 25.1 percent in 2000. In 2000, Social Security (OASDI) and Medicare (HI) tax rates were 12.4 and 2.9 percent on employer and employee combined. Estimates from Boskin et al. (1987, Table 4) show that the marginal benefit from future returns from OASDI taxes is fairly low, generally no more than 50 percent, although only 85 percent of wage earnings are subject to the OASDI cap. HI taxes emulate a pure tax (Congressional Budget Office 2005). These facts suggest adding 37.5 percent of the Social Security tax and all of the Medicare tax to the federal income tax rate, adding 8.2 percent. The employer half of the payroll tax (4.1 percent) has to be added to observed wage levels to produce gross wage levels. Overall, this puts an overall federal tax rate, τ' , of 33.3 percent tax rate on gross wages, although only a 29.2 percent rate on observed wages.

Determining the federal deduction level requires taking into account the fact that many households do not itemize deductions. According to the Statistics on Income, although only 33 percent of tax returns itemize, they account for 67 percent of reported Adjusted Gross Income (AGI). Since the income-weighted share is what matters, 67 percent is multiplied by the effective tax reduction given in TAXSIM, in 2000 of 21.6 percent. Thus, on average these deductions reduce the effective price of eligible goods by 14.5 percent. Since eligible goods only include housing, this deduction applies to only 59 percent of home goods. Multiplying 14.5 percent times 59 percent gives an effective price reduction of 8.6 percent for home goods. Divided by a federal tax rate of 33.3 percent, this produces a federal deduction level of 25.7 percent.

State income tax rates from 2000 are taken from TAXSIM, which, per dollar, fall at an average marginal rate of 4.5 percent. State sales tax data in 2000 are taken from the Tax Policy Center, originally supplied by the Federation of Tax Administrators. The average state sales tax rate is 5.2 percent. Sales tax rates are reduced by 10 percent to accommodate untaxed goods and services other than food or housing (Feenberg et al. 1997), and by another 8 percent in states that exempt food. Overall state taxes raise the marginal tax rate on wage differences within state by an average of 5.9 percentage points, from zero points in Alaska to 8.8 points in Minnesota.

State-level deductions for housing expenditures, explicit in income taxes, and implicit in sales

[38] These proportions are roughly consistent with other studies. In the base calibration of the model, 51 percent of land is devoted to actual housing, 32 percent is for non-housing home goods, and 17 percent is for traded goods, including those purchased by the federal government. Keiper et al. (1961) find that about 52.5 of land value is in residential uses, a 22.9 percent in industry, 20.9 percent in agriculture. Case (2007), ignoring agriculture, finds that in 2000 residential real estate accounted for 76.6 percent of land value, while commercial real estate accounted for the remaining 23.4 percent.

taxes, should also be included. At the state level, deductions for income taxes are calculated in an equivalent way using TAXSIM data. Furthermore, all housing expenditures are deducted from the sales tax. Overall this produces an average effective deduction level of $\delta = 0.291$.

D Data and Estimation

We use United States Census data from the 2000 Integrated Public-Use Microdata Series (IPUMS), from Ruggles et al. (2004), to calculate wage and housing price differentials. The wage differentials are calculated for workers ages 25 to 55, who report working at least 30 hours a week, 26 weeks a year. The MSA assigned to a worker is determined by their place of residence, rather than their place of work. The wage differential of an MSA is found by regressing log hourly wages on individual covariates and indicators for which MSA a worker lives in, using the coefficients on these MSA indicators. The covariates consist of

- 12 indicators of educational attainment;
- a quartic in potential experience, and potential experience interacted with years of education;
- 9 indicators of industry at the one-digit level (1950 classification);
- 9 indicators of employment at the one-digit level (1950 classification);
- 4 indicators of marital status (married, divorced, widowed, separated);
- an indicator for veteran status, and veteran status interacted with age;
- 5 indicators of minority status (Black, Hispanic, Asian, Native American, and other);
- an indicator of immigrant status, years since immigration, and immigrant status interacted with black, Hispanic, Asian, and other;
- 2 indicators for English proficiency (none or poor).

All covariates are interacted with gender.

This regression is first run using census-person weights. From the regressions a predicted wage is calculated using individual characteristics alone, controlling for MSA, to form a new weight equal to the predicted wage times the census-person weight. These new income-adjusted weights are needed since workers need to be weighted by their income share. The new weights are then used in a second regression, which is used to calculate the city-wage differentials from the MSA indicator variables. In practice, this weighting procedure has only a small effect on the estimated wage differentials.

Housing price differentials are calculated using the logarithm reported gross rents and housing values. Only housing units moved into within the last 10 years are included in the sample to ensure that the price data are fairly accurate. The differential housing price of an MSA is calculated in a manner similar to wages, except using a regression of the actual or imputed rent on a set of covariates at the unit level. The covariates for the adjusted differential are

- 9 indicators of building size;
- 9 indicators for the number of rooms, 5 indicators for the number of bedrooms, number of rooms interacted with number of bedrooms, and the number of household members per room;
- 2 indicators for lot size;
- 7 indicators for when the building was built;
- 2 indicators for complete plumbing and kitchen facilities;
- an indicator for commercial use;
- an indicator for condominium status (owned units only).

A regression of housing values on housing characteristics and MSA indicator variables is first run using only owner-occupied units, weighting by census-housing weights. A new value-adjusted weight is calculated by multiplying the census-housing weights by the predicted value from this first regression using housing characteristics alone, controlling for MSA. A second regression is run using these new weights for all units, rented and owner-occupied, on the housing characteristics fully interacted with tenure, along with the MSA indicators, which are not interacted. The house-price differentials are taken from the MSA indicator variables in this second regression. As with the wage differentials, this adjusted weighting method has only a small impact on the measured price differentials.

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Table A.1: Agglomeration and Congestion Elasticities

	A: Agglomeration Elasticities						B: Congestion Elasticities					
	I: With Taxes			II: Neutral Taxes			I: With Taxes			II: Neutral Taxes		
	$\hat{Q}$	$\hat{A}_{X0}$	$\hat{A}_Y$	$\hat{Q}$	$\hat{A}_{X0}$	$\hat{A}_Y$	$\hat{Q}_0$	$\hat{A}_X$	$\hat{A}_Y$	$\hat{Q}_0$	$\hat{A}_X$	$\hat{A}_Y$
$\hat{r}$	14.144	4.622	4.655	13.507	8.240	4.944	10.546	3.659	3.401	9.040	5.896	3.232
$\hat{w}$	0.264	1.257	0.100	0.255	1.311	0.105	-0.319	1.101	-0.103	-0.274	1.033	-0.098
$\hat{p}$	3.463	1.854	0.148	3.308	2.731	0.218	2.263	1.533	-0.270	1.940	2.013	-0.306
$\hat{x}$	-0.168	0.445	0.035	-0.255	0.938	0.075	-0.346	0.397	-0.026	-0.591	0.761	-0.054
$\hat{y}$	-2.478	-0.791	-0.063	-2.412	-1.166	-0.093	-1.856	-0.625	0.154	-1.732	-0.809	0.168
$\hat{N}$	9.524	2.548	3.320	9.130	4.791	3.499	7.350	1.967	2.562	6.409	3.363	2.456
$\hat{L}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\hat{K}$	9.669	3.351	3.353	9.265	5.645	3.537	7.125	2.670	2.467	6.204	4.038	2.363
$\hat{X}$	10.036	3.936	3.606	9.640	6.186	3.786	7.193	3.176	2.615	6.334	4.451	2.519
$\hat{Y}$	7.046	1.757	3.256	6.717	3.625	3.406	5.494	1.342	2.716	4.677	2.554	2.624
$\hat{N}_X$	9.669	2.714	3.473	9.288	4.883	3.646	7.406	2.108	2.684	6.517	3.429	2.584
$\hat{N}_Y$	9.180	2.155	2.955	8.754	4.572	3.148	7.217	1.630	2.271	6.154	3.208	2.152
$\hat{L}_X$	0.411	0.469	0.435	0.448	0.261	0.418	0.159	0.402	0.347	0.305	0.185	0.363
$\hat{L}_Y$	-0.078	-0.089	-0.083	-0.085	-0.050	-0.080	-0.030	-0.076	-0.066	-0.058	-0.035	-0.069
$\hat{K}_X$	9.845	3.552	3.540	9.457	5.757	3.716	7.193	2.843	2.615	6.334	4.118	2.519
$\hat{K}_Y$	9.356	2.993	3.022	8.924	5.446	3.218	7.004	2.364	2.202	5.971	3.897	2.086

Endogenous productivity: $A_X^j = A_{X0}^j (N^j)^\alpha$, $\alpha = 0.06$. See text for details.
Congestion costs: $Q^j = Q_0^j (N^j)^{-\gamma}$, $\gamma = 0.015$. See text for details.

Table A.2: Variance Decomposition, Three Amenity, With Feedback Effects

A: Observed Population Density and Prices. $Var(\hat{N}) = 0.770$			
Fraction of $Var(\hat{N})$ explained by			
	$Cov(\varepsilon_{N,Q}\hat{Q}, \cdot)$	$Cov(\varepsilon_{N,A_X}\hat{A}_X, \cdot)$	$Cov(\varepsilon_{N,A_Y}\hat{A}_Y, \cdot)$
$Cov(\cdot, \varepsilon_{N,Q}\hat{Q})$	0.309	·	·
$Cov(\cdot, \varepsilon_{N,A_X}\hat{A}_X)$	0.116	0.044	·
$Cov(\cdot, \varepsilon_{N,A_Y}\hat{A}_Y)$	-0.025	0.109	0.447
B: Counterfactual Density and Prices. $Var(\hat{N}) = 0.825$			
Fraction of $Var(\hat{N})$ explained by			
	$Cov(\varepsilon_{N,Q}\hat{Q}, \cdot)$	$Cov(\varepsilon_{N,A_X}\hat{A}_X, \cdot)$	$Cov(\varepsilon_{N,A_Y}\hat{A}_Y, \cdot)$
$Cov(\cdot, \varepsilon_{N,Q}\hat{Q})$	0.213	·	·
$Cov(\cdot, \varepsilon_{N,A_X}\hat{A}_X)$	0.157	0.116	·
$Cov(\cdot, \varepsilon_{N,A_Y}\hat{A}_Y)$	-0.019	0.161	0.372

Panel A presents the variance decomposition using data on population density, wages, and house prices. Panel B presents the variance decomposition under geographically neutral taxes.

Table A.3: Relationship between Observed and Predicted Values

	$\hat{Q}$	$\hat{A}_X$	$\hat{A}_Y$	$\hat{r}$	$\hat{x}$	$\hat{y}$	$\hat{L}$	$\hat{K}$	$\hat{X}$	$\hat{Y}$	$\hat{N}_X$	$\hat{N}_Y$	$\hat{K}_X$	$\hat{K}_Y$	$\hat{L}_X$	$\hat{L}_Y$
$\hat{w}$	-0.478	0.837	0.730	0.485	0.478	0.478	0.000	0.618	1.117	0.468	0.171	-0.442	0.838	0.225	0.515	-0.098
	(0.002)	(0.001)	(0.006)	(0.026)	(0.002)	(0.002)	(0.000)	(0.017)	(0.017)	(0.020)	(0.017)	(0.018)	(0.017)	(0.018)	(0.000)	(0.000)
$\hat{p}$	0.324	0.007	-0.935	0.278	-0.084	-0.751	-0.000	0.030	-0.084	-0.741	-0.086	0.237	-0.086	0.237	-0.271	0.052
	(0.001)	(0.000)	(0.002)	(0.010)	(0.001)	(0.001)	(0.000)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.000)	(0.000)
$\hat{N}$	0.000	0.034	0.320	1.373	-0.000	-0.000	-0.000	0.989	1.055	0.998	1.044	0.892	1.044	0.892	0.128	-0.024
	(0.000)	(0.000)	(0.001)	(0.002)	(0.000)	(0.000)	(0.000)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.000)	(0.000)

Each column presents coefficients (standard errors) from an OLS regression of (unobserved) estimated amenity, price, or quantity on observed prices and population density ($\hat{w}, \hat{p}, \hat{N}$).

Table A.4: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}^j_X$	Restricted $\hat{A}^j_X$	$\hat{A}^j_Y$	$\hat{r}^j$
New York, Northern New Jersey, Long Island, NY-NJ-CT-PA	2.285	0.029	0.209	0.264	0.509	3.366
San Francisco-Oakland-San Jose, CA	1.209	0.138	0.289	0.269	-0.182	2.000
Los Angeles-Riverside-Orange County, CA	1.250	0.081	0.150	0.159	0.079	1.913
Chicago-Gary-Kenosha, IL-IN-WI	1.191	0.005	0.131	0.161	0.276	1.767
Salinas (Monterey-Carmel), CA	0.863	0.137	0.144	0.124	-0.189	1.435
San Diego, CA	0.872	0.123	0.098	0.085	-0.114	1.403
Philadelphia-Wilmington-Atlantic City, PA-NJ-DE-MD	0.958	-0.040	0.096	0.133	0.346	1.393
Miami-Fort Lauderdale, FL	0.964	0.041	0.015	0.035	0.191	1.352
Santa Barbara-Santa Maria-Lompoc, CA	0.713	0.176	0.125	0.090	-0.326	1.255
Boston-Worcester-Lawrence, MA-NH-ME-CT	0.797	0.034	0.128	0.136	0.074	1.241
Washington-Baltimore, DC-MD-VA-WV	0.683	-0.013	0.116	0.135	0.170	1.044
Las Vegas, NV-AZ	0.684	-0.025	0.057	0.083	0.246	0.987
New Orleans, LA	0.686	0.005	-0.065	-0.038	0.254	0.864
Providence-Fall River-Warwick, RI-MA	0.591	0.014	0.022	0.037	0.137	0.850
Stockton-Lodi, CA	0.529	-0.002	0.083	0.095	0.116	0.794
Milwaukee-Racine, WI	0.573	-0.009	0.037	0.057	0.181	0.792
Phoenix-Mesa, AZ	0.507	0.012	0.030	0.042	0.110	0.714
Denver-Boulder-Greeley, CO	0.467	0.054	0.066	0.061	-0.041	0.711
Sacramento-Yolo, CA	0.434	0.033	0.075	0.075	-0.001	0.694
Buffalo-Niagara Falls, NY	0.448	-0.054	-0.042	-0.008	0.318	0.620
Modesto, CA	0.389	-0.008	0.050	0.062	0.111	0.575
Seattle-Tacoma-Bremerton, WA	0.324	0.061	0.095	0.082	-0.124	0.570
Provo-Orem, UT	0.447	0.019	-0.048	-0.034	0.126	0.565
Champaign-Urbana, IL	0.435	-0.009	-0.080	-0.056	0.221	0.562
Detroit-Ann Arbor-Flint, MI	0.346	-0.047	0.108	0.125	0.160	0.554
Laredo, TX	0.524	-0.008	-0.194	-0.159	0.327	0.532
Salt Lake City-Ogden, UT	0.394	0.026	-0.015	-0.008	0.067	0.515
Reading, PA	0.403	-0.046	-0.017	0.011	0.265	0.515
Madison, WI	0.333	0.053	-0.018	-0.020	-0.021	0.483
Dallas-Fort Worth, TX	0.318	-0.044	0.047	0.067	0.187	0.463
Reno, NV	0.263	0.053	0.043	0.034	-0.087	0.454
Houston-Galveston-Brazoria, TX	0.323	-0.072	0.045	0.074	0.262	0.449
Cleveland-Akron, OH	0.329	-0.016	0.006	0.022	0.143	0.443
Allentown-Bethlehem-Easton, PA	0.308	-0.022	-0.005	0.012	0.162	0.413
West Palm Beach-Boca Raton, FL	0.235	0.017	0.046	0.045	-0.003	0.365

Table A.4: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Portland-Salem, OR-WA	0.241	0.047	0.037	0.029	-0.075	0.361
El Paso, TX	0.386	-0.041	-0.164	-0.127	0.349	0.349
State College, PA	0.292	0.036	-0.120	-0.111	0.082	0.341
Fresno, CA	0.232	-0.008	-0.014	-0.002	0.106	0.321
Lincoln, NE	0.330	0.022	-0.122	-0.108	0.135	0.307
Minneapolis-St. Paul, MN-WI	0.203	-0.032	0.067	0.078	0.101	0.292
Lafayette, IN	0.236	-0.006	-0.069	-0.054	0.140	0.266
Hartford, CT	0.087	-0.026	0.120	0.121	0.013	0.255
Springfield, MA	0.143	0.002	-0.003	0.001	0.042	0.235
Norfolk-Virginia Beach-Newport News, VA-	0.210	0.027	-0.095	-0.088	0.064	0.228
Bakersfield, CA	0.189	-0.063	0.020	0.043	0.212	0.226
Columbus, OH	0.156	-0.028	0.013	0.025	0.116	0.200
San Antonio, TX	0.222	-0.039	-0.097	-0.071	0.242	0.193
Bloomington-Normal, IL	0.126	-0.061	0.003	0.024	0.201	0.159
Austin-San Marcos, TX	0.069	0.016	0.014	0.011	-0.032	0.128
Tucson, AZ	0.122	0.052	-0.091	-0.095	-0.035	0.119
Erie, PA	0.153	-0.035	-0.114	-0.090	0.223	0.101
Toledo, OH	0.113	-0.041	-0.037	-0.019	0.171	0.099
Pittsburgh, PA	0.119	-0.047	-0.054	-0.033	0.202	0.094
Tampa-St. Petersburg-Clearwater, FL	0.109	0.003	-0.054	-0.047	0.067	0.081
Iowa City, IA	0.103	0.034	-0.072	-0.073	-0.007	0.075
Albuquerque, NM	0.113	0.049	-0.064	-0.069	-0.049	0.072
Rochester, NY	0.019	-0.041	-0.029	-0.014	0.135	0.047
Colorado Springs, CO	0.060	0.055	-0.066	-0.075	-0.080	0.041
Omaha, NE-IA	0.136	-0.019	-0.084	-0.068	0.153	0.037
St. Louis, MO-IL	0.052	-0.034	-0.007	0.005	0.111	0.016
Non-metro, HI	0.000	0.126	0.013	0.000	0.000	0.000
Anchorage, AK	0.000	0.023	0.077	0.000	0.000	0.000
Non-metro, AK	0.000	0.012	0.037	0.000	0.000	0.000
Honolulu, HI	0.000	0.204	0.057	0.000	0.000	0.000
Bryan-College Station, TX	0.063	0.027	-0.122	-0.118	0.036	-0.007
Albany-Schenectady-Troy, NY	-0.026	-0.041	-0.026	-0.013	0.120	-0.013
Lancaster, PA	-0.002	-0.011	-0.017	-0.013	0.040	-0.016
Corpus Christi, TX	0.074	-0.034	-0.106	-0.085	0.187	-0.017
Cincinnati-Hamilton, OH-KY-IN	-0.007	-0.038	0.020	0.029	0.085	-0.031

Table A.4: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}^j_X$	Restricted $\hat{A}^j_X$	$\hat{A}^j_Y$	$\hat{r}^j$
Non-metro, RI	-0.094	0.040	0.071	0.051	-0.186	-0.042
Lubbock, TX	0.075	-0.009	-0.161	-0.144	0.162	-0.054
Fort Collins-Loveland, CO	-0.045	0.079	-0.032	-0.054	-0.202	-0.058
Spokane, WA	0.011	0.008	-0.090	-0.085	0.047	-0.081
Bloomington, IN	-0.015	0.032	-0.110	-0.111	-0.012	-0.087
Louisville, KY-IN	-0.016	-0.023	-0.047	-0.037	0.089	-0.097
Syracuse, NY	-0.081	-0.069	-0.056	-0.035	0.198	-0.126
Orlando, FL	-0.057	0.006	-0.037	-0.038	-0.009	-0.126
Memphis, TN-AR-MS	-0.044	-0.060	-0.013	0.004	0.153	-0.130
Visalia-Tulare-Porterville, CA	-0.087	-0.016	-0.036	-0.031	0.041	-0.140
Pueblo, CO	0.006	-0.003	-0.162	-0.148	0.124	-0.156
Green Bay, WI	-0.093	-0.011	-0.022	-0.020	0.014	-0.163
Scranton–Wilkes-Barre–Hazleton, PA	-0.048	-0.027	-0.106	-0.092	0.132	-0.163
Amarillo, TX	-0.028	-0.010	-0.142	-0.130	0.118	-0.177
Brownsville-Harlingen-San Benito, TX	0.048	-0.057	-0.221	-0.186	0.324	-0.183
Des Moines, IA	-0.085	-0.022	-0.037	-0.031	0.056	-0.203
Sarasota-Bradenton, FL	-0.138	0.066	-0.046	-0.066	-0.187	-0.208
South Bend, IN	-0.094	-0.047	-0.072	-0.057	0.145	-0.219
Dayton-Springfield, OH	-0.146	-0.030	-0.030	-0.024	0.054	-0.244
Eugene-Springfield, OR	-0.159	0.088	-0.084	-0.108	-0.225	-0.250
Kansas City, MO-KS	-0.125	-0.037	-0.015	-0.008	0.067	-0.259
Altoona, PA	-0.082	-0.045	-0.158	-0.136	0.205	-0.263
Yuma, AZ	-0.121	0.002	-0.100	-0.097	0.028	-0.266
Indianapolis, IN	-0.183	-0.039	0.003	0.008	0.043	-0.277
Merced, CA	-0.215	-0.012	-0.013	-0.016	-0.028	-0.298
Lansing-East Lansing, MI	-0.218	-0.046	-0.008	-0.002	0.059	-0.304
Appleton-Oshkosh-Neenah, WI	-0.185	-0.021	-0.052	-0.048	0.034	-0.318
Grand Rapids-Muskegon-Holland, MI	-0.236	-0.044	-0.010	-0.005	0.048	-0.328
Lexington, KY	-0.150	-0.033	-0.095	-0.084	0.107	-0.333
Harrisburg-Lebanon-Carlisle, PA	-0.222	-0.029	-0.020	-0.018	0.020	-0.334
Waterloo-Cedar Falls, IA	-0.132	-0.023	-0.129	-0.117	0.110	-0.343
Richmond-Petersburg, VA	-0.228	-0.033	-0.006	-0.004	0.020	-0.354
Fargo-Moorhead, ND-MN	-0.115	-0.039	-0.174	-0.153	0.191	-0.355
Rockford, IL	-0.237	-0.069	-0.024	-0.011	0.124	-0.367
Boise City, ID	-0.185	0.010	-0.077	-0.080	-0.032	-0.376

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Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}^j_X$	Restricted $\hat{A}^j_X$	$\hat{A}^j_Y$	$\hat{r}^j$
Odessa-Midland, TX	-0.154	-0.063	-0.136	-0.113	0.215	-0.382
Atlanta, GA	-0.291	-0.032	0.063	0.057	-0.053	-0.382
Oklahoma City, OK	-0.146	-0.020	-0.135	-0.124	0.103	-0.386
Davenport-Moline-Rock Island, IA-IL	-0.215	-0.041	-0.088	-0.077	0.103	-0.397
Wichita, KS	-0.187	-0.048	-0.079	-0.066	0.122	-0.402
York, PA	-0.277	-0.032	-0.036	-0.033	0.022	-0.425
Portland, ME	-0.239	0.051	-0.060	-0.078	-0.170	-0.429
San Luis Obispo-Atascadero-Paso Robles, CA	-0.458	0.124	0.077	0.020	-0.531	-0.436
Jacksonville, FL	-0.265	-0.009	-0.051	-0.054	-0.025	-0.438
Lawrence, KS	-0.240	0.038	-0.129	-0.138	-0.086	-0.440
Yakima, WA	-0.287	-0.009	-0.029	-0.034	-0.048	-0.442
Binghamton, NY	-0.302	-0.054	-0.123	-0.109	0.133	-0.458
Cedar Rapids, IA	-0.266	-0.002	-0.078	-0.081	-0.024	-0.469
Sheboygan, WI	-0.302	-0.019	-0.062	-0.062	-0.004	-0.483
Savannah, GA	-0.307	-0.011	-0.080	-0.082	-0.013	-0.484
Charlottesville, VA	-0.329	0.054	-0.090	-0.109	-0.185	-0.491
Rochester, MN	-0.325	-0.061	-0.003	0.003	0.060	-0.494
Sioux Falls, SD	-0.230	-0.006	-0.146	-0.141	0.045	-0.501
Muncie, IN	-0.272	-0.043	-0.122	-0.110	0.114	-0.501
Naples, FL	-0.424	0.095	0.027	-0.016	-0.408	-0.509
Canton-Massillon, OH	-0.322	-0.024	-0.083	-0.081	0.020	-0.517
Sioux City, IA-NE	-0.220	-0.060	-0.161	-0.139	0.201	-0.523
Gainesville, FL	-0.301	0.024	-0.134	-0.141	-0.067	-0.547
Yuba City, CA	-0.394	0.009	-0.066	-0.077	-0.103	-0.555
Tulsa, OK	-0.280	-0.032	-0.104	-0.096	0.068	-0.559
Abilene, TX	-0.256	0.004	-0.223	-0.216	0.067	-0.562
Utica-Rome, NY	-0.375	-0.064	-0.125	-0.111	0.138	-0.568
Chico-Paradise, CA	-0.444	0.053	-0.067	-0.092	-0.236	-0.579
La Crosse, WI-MN	-0.355	-0.020	-0.126	-0.123	0.028	-0.592
Peoria-Pekin, IL	-0.399	-0.061	-0.041	-0.034	0.064	-0.597
Janesville-Beloit, WI	-0.395	-0.050	-0.019	-0.017	0.020	-0.612
Melbourne-Titusville-Palm Bay, FL	-0.360	-0.000	-0.104	-0.108	-0.042	-0.614
Medford-Ashland, OR	-0.425	0.095	-0.099	-0.133	-0.317	-0.623
Elmira, NY	-0.415	-0.061	-0.132	-0.119	0.122	-0.624
Decatur, IL	-0.385	-0.089	-0.080	-0.062	0.168	-0.627

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Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}^j_X$	Restricted $\hat{A}^j_X$	$\hat{A}^j_Y$	$\hat{r}^j$
Richland-Kennewick-Pasco, WA	-0.440	-0.051	0.011	0.009	-0.014	-0.642
Topeka, KS	-0.349	-0.024	-0.137	-0.132	0.050	-0.642
St. Joseph, MO	-0.347	-0.026	-0.168	-0.160	0.076	-0.652
Springfield, IL	-0.435	-0.039	-0.082	-0.080	0.021	-0.659
Billings, MT	-0.329	0.013	-0.169	-0.172	-0.021	-0.674
Fort Walton Beach, FL	-0.383	0.062	-0.174	-0.192	-0.163	-0.675
Corvalis, OR	-0.474	0.081	-0.081	-0.114	-0.309	-0.688
Columbia, MO	-0.401	0.023	-0.164	-0.172	-0.073	-0.691
Fort Myers-Cape Coral, FL	-0.454	0.049	-0.084	-0.107	-0.215	-0.692
Saginaw-Bay City-Midland, MI	-0.489	-0.074	-0.035	-0.028	0.064	-0.716
Tallahassee, FL	-0.451	0.022	-0.098	-0.112	-0.133	-0.719
Raleigh-Durham-Chapel Hill, NC	-0.503	0.011	0.018	-0.004	-0.202	-0.723
Burlington, VT	-0.453	0.065	-0.082	-0.110	-0.260	-0.730
Baton Rouge, LA	-0.465	-0.031	-0.053	-0.057	-0.030	-0.730
Waco, TX	-0.439	-0.047	-0.118	-0.111	0.069	-0.744
Roanoke, VA	-0.470	-0.017	-0.107	-0.110	-0.030	-0.745
Evansville-Henderson, IN-KY	-0.458	-0.047	-0.104	-0.099	0.051	-0.749
Williamsport, PA	-0.471	-0.031	-0.130	-0.127	0.022	-0.766
Nashville, TN	-0.539	-0.001	-0.016	-0.033	-0.159	-0.767
Grand Junction, CO	-0.518	0.114	-0.134	-0.174	-0.374	-0.771
Lewiston-Auburn, ME	-0.435	-0.008	-0.123	-0.127	-0.032	-0.776
Cheyenne, WY	-0.414	0.056	-0.217	-0.231	-0.128	-0.789
Youngstown-Warren, OH	-0.512	-0.052	-0.090	-0.086	0.039	-0.804
Charleston-North Charleston, SC	-0.534	0.025	-0.082	-0.101	-0.179	-0.805
Columbus, GA-AL	-0.488	-0.055	-0.152	-0.141	0.097	-0.819
Fort Wayne, IN	-0.533	-0.063	-0.067	-0.062	0.043	-0.829
Birmingham, AL	-0.561	-0.047	-0.034	-0.037	-0.032	-0.852
San Angelo, TX	-0.487	-0.025	-0.177	-0.174	0.036	-0.853
Beaumont-Port Arthur, TX	-0.526	-0.108	-0.070	-0.052	0.167	-0.857
Kalamazoo-Battle Creek, MI	-0.602	-0.056	-0.037	-0.039	-0.018	-0.859
Santa Fe, NM	-0.641	0.127	-0.017	-0.073	-0.529	-0.861
Columbia, SC	-0.566	-0.007	-0.076	-0.088	-0.108	-0.874
Kokomo, IN	-0.618	-0.110	0.029	0.037	0.072	-0.901
Fayetteville, NC	-0.566	0.028	-0.178	-0.192	-0.130	-0.906
Daytona Beach, FL	-0.562	0.019	-0.144	-0.158	-0.130	-0.919

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Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Montgomery, AL	-0.578	-0.003	-0.124	-0.134	-0.089	-0.921
Pittsfield, MA	-0.689	0.014	-0.050	-0.073	-0.222	-0.929
Springfield, MO	-0.559	0.003	-0.175	-0.182	-0.063	-0.930
Fort Pierce-Port St. Lucie, FL	-0.620	0.011	-0.078	-0.096	-0.173	-0.938
New London-Norwich, CT-RI	-0.765	0.006	0.051	0.019	-0.298	-0.944
Jamestown, NY	-0.633	-0.079	-0.157	-0.144	0.119	-0.950
Eau Claire, WI	-0.613	-0.026	-0.120	-0.125	-0.042	-0.954
Shreveport-Bossier City, LA	-0.583	-0.042	-0.124	-0.122	0.011	-0.956
Charlotte-Gastonia-Rock Hill, NC-SC	-0.660	-0.013	0.007	-0.013	-0.183	-0.960
Bellingham, WA	-0.701	0.074	-0.038	-0.080	-0.394	-0.962
Sharon, PA	-0.610	-0.033	-0.151	-0.151	-0.003	-0.975
Victoria, TX	-0.623	-0.074	-0.104	-0.096	0.069	-1.002
Jackson, MS	-0.627	-0.031	-0.099	-0.104	-0.048	-1.007
McAllen-Edinburg-Mission, TX	-0.541	-0.079	-0.228	-0.207	0.198	-1.015
Jackson, MI	-0.722	-0.064	-0.034	-0.038	-0.037	-1.029
Duluth-Superior, MN-WI	-0.676	-0.069	-0.116	-0.110	0.049	-1.045
Wichita Falls, TX	-0.601	-0.008	-0.226	-0.228	-0.012	-1.047
Elkhart-Goshen, IN	-0.707	-0.043	-0.059	-0.067	-0.071	-1.049
Mobile, AL	-0.676	-0.016	-0.128	-0.137	-0.084	-1.068
Killeen-Temple, TX	-0.645	0.040	-0.220	-0.237	-0.158	-1.069
Las Cruces, NM	-0.638	0.019	-0.190	-0.203	-0.121	-1.071
Athens, GA	-0.729	0.016	-0.125	-0.145	-0.187	-1.077
Terre Haute, IN	-0.676	-0.060	-0.139	-0.134	0.040	-1.081
Pensacola, FL	-0.676	0.003	-0.146	-0.159	-0.121	-1.090
Tuscaloosa, AL	-0.714	-0.013	-0.099	-0.112	-0.123	-1.093
Dubuque, IA	-0.667	-0.024	-0.150	-0.155	-0.044	-1.096
Mansfield, OH	-0.722	-0.048	-0.110	-0.112	-0.027	-1.102
Little Rock-North Little Rock, AR	-0.699	-0.011	-0.100	-0.113	-0.125	-1.107
Lake Charles, LA	-0.721	-0.064	-0.085	-0.085	-0.002	-1.126
Panama City, FL	-0.723	0.026	-0.138	-0.159	-0.203	-1.131
Owensboro, KY	-0.702	-0.041	-0.144	-0.145	-0.015	-1.136
Greenville, NC	-0.766	-0.022	-0.085	-0.098	-0.129	-1.164
Grand Forks, ND-MN	-0.700	-0.046	-0.210	-0.205	0.046	-1.188
Lakeland-Winter Haven, FL	-0.759	-0.023	-0.119	-0.130	-0.099	-1.193
Lima, OH	-0.787	-0.062	-0.103	-0.105	-0.014	-1.198

Table A.4: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Greensboro–Winston Salem–High Point, NC	-0.848	-0.016	-0.049	-0.070	-0.196	-1.253
Macon, GA	-0.844	-0.068	-0.079	-0.082	-0.033	-1.260
St. Cloud, MN	-0.859	-0.048	-0.110	-0.118	-0.070	-1.269
Non-metro, CA	-0.982	0.059	-0.017	-0.067	-0.459	-1.282
Biloxi-Gulfport-Pascagoula, MS	-0.818	-0.026	-0.135	-0.146	-0.097	-1.289
Albany, GA	-0.860	-0.063	-0.099	-0.103	-0.038	-1.293
Punta Gorda, FL	-0.859	0.049	-0.143	-0.176	-0.304	-1.307
Wilmington, NC	-0.914	0.071	-0.104	-0.148	-0.409	-1.313
Tyler, TX	-0.884	-0.025	-0.106	-0.121	-0.142	-1.329
Benton Harbor, MI	-0.938	-0.029	-0.081	-0.099	-0.165	-1.332
Monroe, LA	-0.867	-0.036	-0.133	-0.142	-0.090	-1.350
Augusta-Aiken, GA-SC	-0.895	-0.057	-0.093	-0.100	-0.071	-1.351
Huntsville, AL	-0.908	-0.055	-0.062	-0.073	-0.102	-1.360
Lafayette, LA	-0.874	-0.057	-0.130	-0.134	-0.038	-1.369
Barnstable-Yarmouth (Cape Cod), MA	-1.111	0.121	0.046	-0.030	-0.712	-1.371
Redding, CA	-1.011	0.041	-0.074	-0.115	-0.382	-1.375
Casper, WY	-0.833	-0.002	-0.219	-0.231	-0.107	-1.399
Non-metro, CT	-1.122	-0.007	0.078	0.035	-0.398	-1.420
Knoxville, TN	-0.937	-0.011	-0.125	-0.144	-0.183	-1.426
Charleston, WV	-0.917	-0.052	-0.117	-0.125	-0.073	-1.446
Auburn-Opelika, AL	-0.950	-0.015	-0.132	-0.150	-0.171	-1.448
Missoula, MT	-0.905	0.101	-0.208	-0.252	-0.410	-1.450
Hattiesburg, MS	-0.910	-0.029	-0.180	-0.189	-0.088	-1.450
Johnstown, PA	-0.911	-0.062	-0.201	-0.199	0.016	-1.451
Chattanooga, TN-GA	-0.970	-0.035	-0.105	-0.121	-0.145	-1.457
Rapid City, SD	-0.948	0.033	-0.212	-0.237	-0.241	-1.536
Lawton, OK	-0.942	-0.016	-0.253	-0.262	-0.080	-1.547
Bismarck, ND	-0.918	-0.048	-0.250	-0.248	0.010	-1.566
Wausau, WI	-1.066	-0.049	-0.086	-0.103	-0.150	-1.576
Flagstaff, AZ-UT	-1.089	0.030	-0.129	-0.166	-0.338	-1.595
Non-metro, PA	-1.057	-0.053	-0.145	-0.156	-0.096	-1.601
Steubenville-Weirton, OH-WV	-1.023	-0.058	-0.189	-0.193	-0.041	-1.603
Wheeling, WV-OH	-1.026	-0.058	-0.189	-0.193	-0.043	-1.613
Fayetteville-Springdale-Rogers, AR	-1.066	0.005	-0.132	-0.160	-0.261	-1.621
Jackson, TN	-1.073	-0.063	-0.098	-0.110	-0.109	-1.622

Table A.4: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Great Falls, MT	-0.957	0.036	-0.283	-0.305	-0.203	-1.624
Fort Smith, AR-OK	-1.010	-0.045	-0.194	-0.201	-0.068	-1.635
Jacksonville, NC	-1.094	0.051	-0.254	-0.287	-0.307	-1.659
Pocatello, ID	-1.042	-0.061	-0.141	-0.149	-0.071	-1.660
Danville, VA	-1.088	-0.057	-0.163	-0.171	-0.081	-1.661
Glens Falls, NY	-1.201	-0.020	-0.109	-0.136	-0.254	-1.663
Enid, OK	-1.041	-0.032	-0.219	-0.229	-0.095	-1.674
Non-metro, WA	-1.186	0.037	-0.067	-0.113	-0.432	-1.683
Huntington-Ashland, WV-KY-OH	-1.072	-0.074	-0.177	-0.180	-0.022	-1.695
Greenville-Spartanburg-Anderson, SC	-1.149	-0.031	-0.078	-0.103	-0.232	-1.701
Alexandria, LA	-1.127	-0.031	-0.173	-0.190	-0.157	-1.738
Non-metro, NY	-1.246	-0.050	-0.123	-0.143	-0.179	-1.755
Pine Bluff, AR	-1.128	-0.053	-0.168	-0.179	-0.101	-1.788
Houma, LA	-1.194	-0.054	-0.123	-0.139	-0.153	-1.802
Non-metro, MA	-1.376	0.063	-0.042	-0.104	-0.580	-1.833
Non-metro, ND	-1.113	-0.041	-0.262	-0.269	-0.064	-1.839
Joplin, MO	-1.216	-0.011	-0.246	-0.266	-0.187	-1.889
Dover, DE	-1.327	-0.009	-0.086	-0.123	-0.340	-1.896
Non-metro, ID	-1.256	0.012	-0.174	-0.207	-0.312	-1.902
Non-metro, UT	-1.315	0.010	-0.124	-0.162	-0.360	-1.909
Asheville, NC	-1.343	0.058	-0.132	-0.185	-0.492	-1.927
Non-metro, NV	-1.409	-0.011	0.005	-0.041	-0.427	-1.928
Clarksville-Hopkinsville, TN-KY	-1.275	-0.004	-0.206	-0.233	-0.251	-1.955
Lynchburg, VA	-1.331	-0.031	-0.140	-0.166	-0.245	-1.962
Non-metro, OR	-1.377	0.062	-0.113	-0.169	-0.525	-1.966
Non-metro, MD	-1.441	-0.022	-0.037	-0.078	-0.376	-1.973
Decatur, AL	-1.356	-0.072	-0.085	-0.105	-0.184	-2.008
Non-metro, OH	-1.387	-0.052	-0.111	-0.135	-0.228	-2.020
Myrtle Beach, SC	-1.402	0.038	-0.148	-0.196	-0.446	-2.028
Longview-Marshall, TX	-1.360	-0.057	-0.149	-0.168	-0.179	-2.048
Bangor, ME	-1.365	-0.018	-0.169	-0.198	-0.271	-2.084
Florence, AL	-1.398	-0.042	-0.149	-0.174	-0.232	-2.098
Cumberland, MD-WV	-1.434	-0.040	-0.171	-0.196	-0.233	-2.099
Sumter, SC	-1.391	-0.037	-0.182	-0.206	-0.218	-2.106
Parkersburg-Marietta, WV-OH	-1.394	-0.072	-0.170	-0.184	-0.136	-2.121

Table A.4: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Non-metro, WY	-1.402	0.007	-0.165	-0.203	-0.351	-2.121
Sherman-Denison, TX	-1.449	-0.028	-0.137	-0.168	-0.294	-2.138
Gadsden, AL	-1.446	-0.069	-0.150	-0.169	-0.174	-2.185
Non-metro, IN	-1.500	-0.050	-0.113	-0.142	-0.269	-2.187
Non-metro, IL	-1.519	-0.052	-0.154	-0.180	-0.241	-2.214
Goldsboro, NC	-1.509	-0.007	-0.176	-0.213	-0.340	-2.226
Johnson City-Kingsport-Bristol, TN-VA	-1.485	-0.028	-0.180	-0.209	-0.273	-2.236
Non-metro, NM	-1.482	0.002	-0.202	-0.238	-0.338	-2.255
Non-metro, MT	-1.461	0.059	-0.236	-0.285	-0.459	-2.262
Non-metro, KS	-1.488	-0.035	-0.240	-0.263	-0.216	-2.276
Dothan, AL	-1.533	-0.040	-0.186	-0.214	-0.253	-2.314
Non-metro, WV	-1.523	-0.042	-0.210	-0.234	-0.228	-2.324
Non-metro, IA	-1.554	-0.027	-0.192	-0.223	-0.291	-2.351
Hickory-Morganton-Lenoir, NC	-1.624	-0.008	-0.124	-0.168	-0.412	-2.356
Ocala, FL	-1.582	-0.010	-0.166	-0.205	-0.362	-2.363
Florence, SC	-1.606	-0.049	-0.131	-0.162	-0.292	-2.381
Rocky Mount, NC	-1.640	-0.024	-0.114	-0.155	-0.381	-2.384
Anniston, AL	-1.579	-0.046	-0.190	-0.216	-0.250	-2.385
Texarkana, TX-Texarkana, AR	-1.556	-0.068	-0.200	-0.219	-0.178	-2.388
Non-metro, NE	-1.592	-0.021	-0.256	-0.285	-0.275	-2.448
Non-metro, MN	-1.735	-0.047	-0.163	-0.197	-0.316	-2.498
Jonesboro, AR	-1.651	-0.026	-0.238	-0.269	-0.293	-2.533
Non-metro, WI	-1.761	-0.028	-0.120	-0.163	-0.406	-2.535
Non-metro, AZ	-1.789	0.037	-0.163	-0.222	-0.557	-2.580
Non-metro, VT	-1.775	0.073	-0.165	-0.234	-0.647	-2.599
Non-metro, MI	-1.864	-0.038	-0.108	-0.153	-0.421	-2.632
Non-metro, FL	-1.823	0.010	-0.167	-0.220	-0.493	-2.683
Non-metro, LA	-1.846	-0.058	-0.178	-0.212	-0.312	-2.745
Non-metro, TX	-1.848	-0.043	-0.206	-0.241	-0.333	-2.767
Non-metro, VA	-1.908	-0.031	-0.163	-0.207	-0.415	-2.771
Non-metro, OK	-1.830	-0.034	-0.255	-0.289	-0.317	-2.782
Non-metro, NH	-2.059	0.042	-0.082	-0.159	-0.715	-2.915
Non-metro, ME	-2.004	0.027	-0.184	-0.246	-0.585	-2.934
Non-metro, MS	-1.956	-0.066	-0.215	-0.247	-0.301	-2.950
Non-metro, MO	-2.048	-0.023	-0.251	-0.296	-0.419	-3.045

Table A.4: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Non-metro, KY	-2.086	-0.057	-0.193	-0.233	-0.382	-3.091
Non-metro, NC	-2.164	-0.013	-0.148	-0.207	-0.555	-3.115
Non-metro, SD	-2.036	0.001	-0.279	-0.328	-0.458	-3.119
Non-metro, GA	-2.219	-0.040	-0.146	-0.200	-0.501	-3.178
Non-metro, SC	-2.203	-0.033	-0.140	-0.196	-0.521	-3.192
Non-metro, CO	-2.333	0.112	-0.094	-0.199	-0.980	-3.236
Non-metro, DE	-2.322	0.010	-0.073	-0.150	-0.720	-3.242
Non-metro, AR	-2.267	-0.028	-0.237	-0.289	-0.485	-3.383
Non-metro, TN	-2.470	-0.038	-0.189	-0.249	-0.559	-3.614
Non-metro, AL	-2.761	-0.067	-0.189	-0.250	-0.573	-4.025

See text for estimation procedure. $\hat{A}_X^j$ corresponds to the trade-productivity estimates obtained using wage, housing price, and density data, while Restricted $\hat{A}_X^j$ corresponds to trade-productivity estimates obtained using wage and housing price data plus the constant home-productivity assumption.

Figure A.1: Comparison of Nonlinear and Linear Model

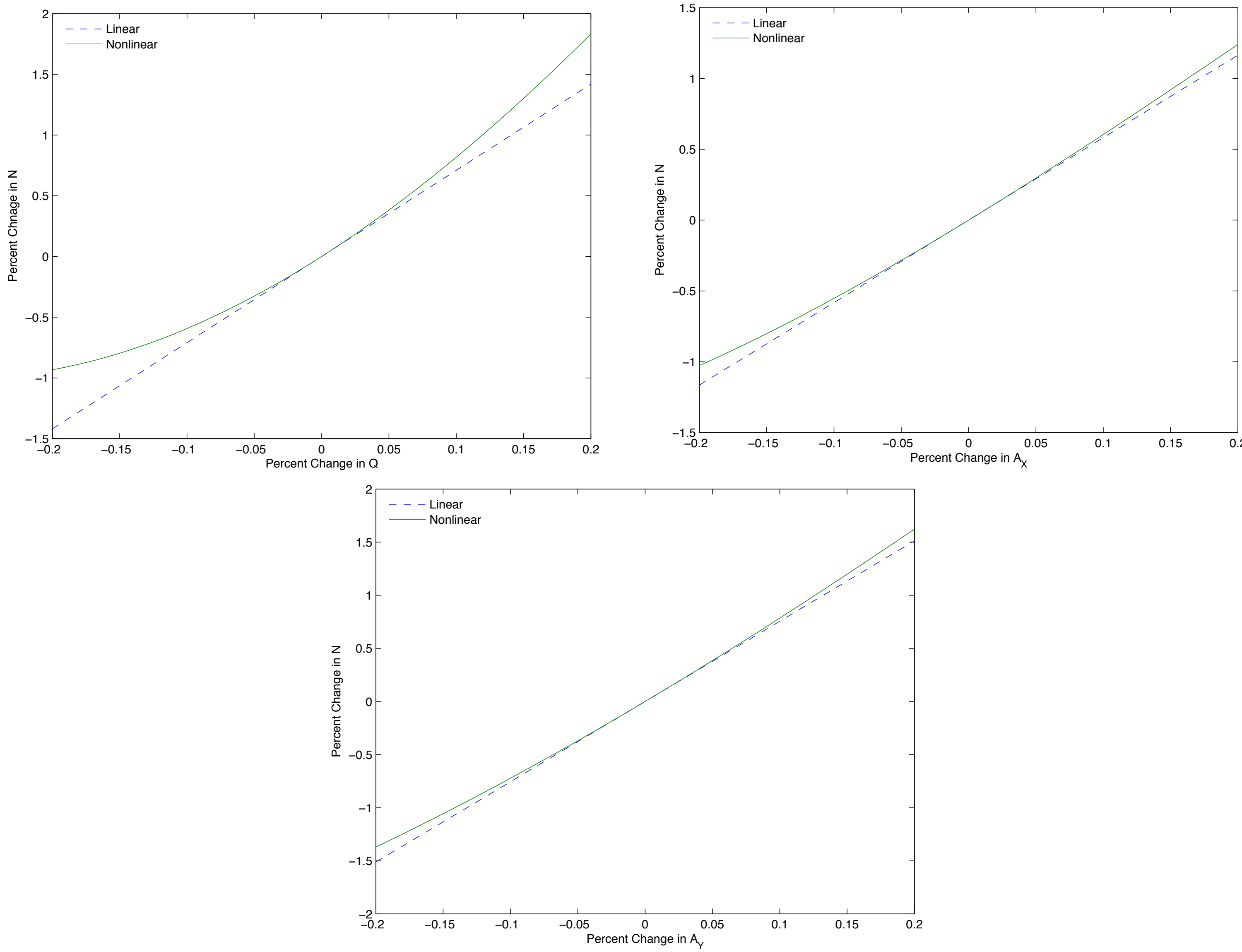

Figure A.2: Estimated Amenity Distribution, 2000

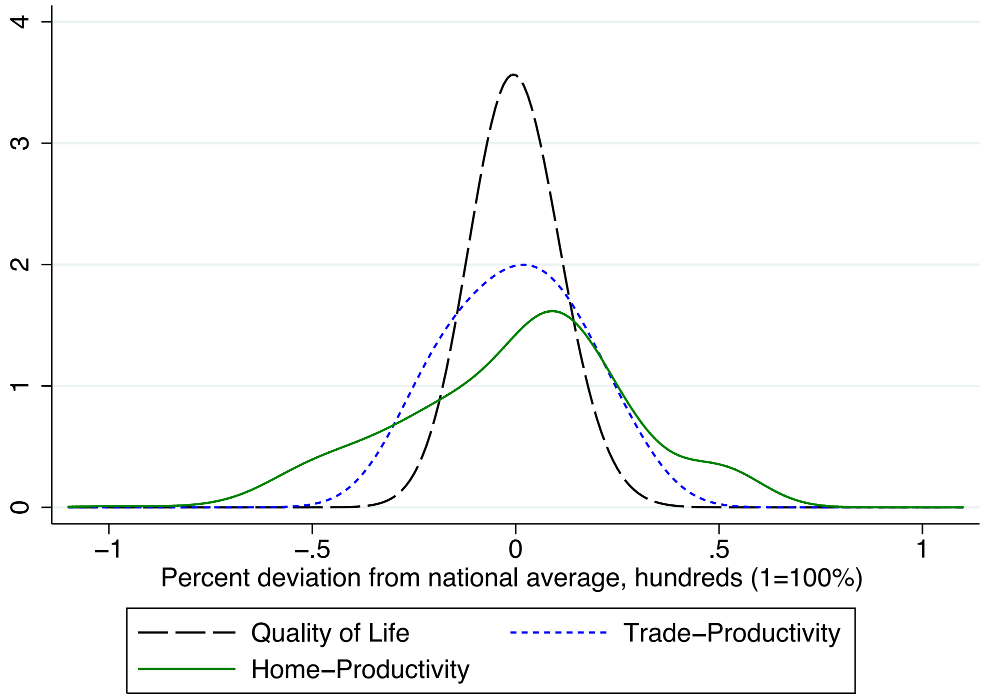

Figure A.2 is smoothed with a Gaussian kernel, bandwidth=0.1.