

# Strategic Pricing with Rational Inattention to Quality\*

Daniel Martin<sup>†</sup>

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### Abstract

This paper studies, both theoretically and experimentally, the pricing strategy of a firm that faces a consumer who is “rationally inattentive” to product quality (Sims [2003]). In a standard sequential pricing game, rational inattention to quality produces two types of mixed strategy perfect Bayesian equilibria: one where sellers pool at a low price and buyers are fully inattentive to quality and one where there is semi-pooling at a high price and buyers are selectively attentive to quality. I characterize these equilibria for all possible attentional costs and show that the welfare effects of policies designed to lower attentional costs can differ substantially between equilibria. To determine if either type of equilibria can explain actual behavior, I run an experiment in which sellers of hypothetical products face buyers who have real attentional costs in becoming informed about product quality. I find strong evidence of the equilibrium with semi-pooling at a high price. Buyers attend enough to allow high quality sellers to price high, but only enough to make low quality sellers indifferent between pricing high or low. This attentional effort makes the observed semi-pooling a best response for sellers and is shown to be consistent with rational inattention.

## 1 Introduction

Consumers often face a large amount of information about product quality through direct observation, pictures, product specifications, customer reviews, and advertisements. Even if there are no monetary costs

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<sup>†</sup>Center for Experimental Social Science and Department of Economics, New York University. Email address: daniel.martin@nyu.edu.

to gathering this information, a substantial amount of attentional effort is required to attend to all of it. As a result, consumers may limit their attention, even when doing so leaves them uncertain about quality when they make purchase decisions.

In this paper, I study, both theoretically and experimentally, how the limited attention of buyers to information about product quality impacts the prices that sellers offer. On the one hand, sellers of low quality products may try to trick uninformed buyers into paying higher prices by charging the same price as higher quality sellers. On the other hand, sellers of high quality products may be forced to sell at lower prices because buyers do not recognize their products as high quality.

The market setting I examine is a standard sequential pricing game with one seller, one buyer, and one product of uncertain quality. This game can be interpreted as a representative sales encounter in a market with a monopolist and many buyers who interact with the monopolist independently. In the first move of this game, nature determines the quality of the seller’s product according to a commonly known probability distribution. Next, the seller learns the quality of the product and offers a take-it-or-leave-it price to the buyer. Finally, the buyer learns the offered price, attends to product quality, and then decides whether or not to accept the seller’s offer. The seller wants to sell the product at the highest possible price, and the buyer only wants to accept the offer if the price is sufficiently low given quality.<sup>1</sup>

To produce theoretical predictions for this game, I model the attention of buyers abstractly using the “rational inattention” approach introduced by Sims [2003]. This approach is well suited to studying how buyers evaluate product quality because it is a “useful way to describe more subjective evaluations, such as the probability of a crisis, an optimal price, or future productivity” (Hellwig, Kohls, and Veldkamp [2012]). To implement this approach, I assume that buyers choose a joint distribution over signals and quality levels and then receive a signal drawn from this distribution, conditional on the actual quality level. Buyers do not choose perfectly informative joint distributions because distributions generate costs based on how much they reduce uncertainty about quality, as measured by Shannon entropy.<sup>2</sup>

To determine the resulting theoretical predictions, I look for all possible mixed strategy perfect Bayesian equilibria of the game. I find just two types of equilibria, which exist for all possible model parameter values. In one, buyers acquire market power through strategic ignorance, i.e., they never attend to information about quality and hold pessimistic beliefs about sellers that charge a high price, so all sellers charge a low price in equilibrium. In this “pooling low” equilibrium, buyers obtain a larger expected surplus than under full information.

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<sup>1</sup>My assumption that the buyer observes price perfectly is appropriate for retail settings in which there are only a small number of products under consideration and prices are simple (listed in dollars and cents) and prominently posted. Aside from the information about quality conveyed by price, the buyer is ex ante uninformed about product quality, which is appropriate for durable goods that are rarely purchased and change features over time, such as computers, cell phones, and air conditioners.

<sup>2</sup>See Wiederholt [2010] for an introduction to this approach.

In the other equilibrium, sellers have market power. Sellers of high quality products charge a high price and sellers of low quality products sometimes mimic them. In this semi-pooling “mimic high” equilibrium, buyers must undertake attentional effort to distinguish high and low quality products when the price is high. Because limited attention leads to mistakes, buyers have a lower expected surplus in this equilibrium than under full information, and there is some deadweight loss.

I characterize prices in the mimic high equilibrium by identifying the unique rate at which low quality sellers mimic high quality sellers as a function of the attentional cost parameter. While mimicking is generally increasing in this attentional cost parameter, there are situations for which the opposite is true. This occurs when buyers have strong incentives to not attend at all to quality and to not purchase the product. As the attentional cost parameter decreases, these incentives get weaker, and low quality sellers capitalize on this by mimicking more often.

Policy interventions designed to lower attentional costs impact buyer welfare differently in the two equilibria. In the pooling low equilibrium, policy interventions that lower the attentional cost parameter have no effect. In the mimic high equilibrium, policy interventions that lower attentional cost parameter have positive effect on the welfare of the buyer in most cases because buyers make less costly mistakes. However, the size of this benefit varies, and in the extreme case where low quality sellers mimic more often as attentional costs fall, the buyer’s welfare can decrease.

I run a laboratory experiment to determine if either equilibrium predicted by rational inattention can explain actual behavior, and if so, which one. Subjects are randomly and anonymously rematched into pairs in each round, and in each pairing, they are randomly assigned to be the seller or buyer. The seller is assigned a hypothetical product that has an exogenously determined value to the buyer. The seller is shown the product’s value and then offers a price to the buyer for that product. The buyer is shown the price, but must add up 20 numbers to determine the value. In principle, the buyer can become fully informed about value, but in the presence of real attentional costs, buyers may not do so.

I find that high quality sellers charge a high price around 99% of the time, and low quality sellers mimic them by charging a high price around 20% of the time. When the price is high, buyers almost always accept offers from high quality firms, but only reject offers from low quality sellers around half of the time. This implies that buyers are paying some attention to quality at high prices, but not full attention.

To constitute a mimic high equilibrium, these prices and demands must be a best response on both sides of the market. Given the frequency with which offers are accepted at both prices, pricing high is a best response for high quality sellers, and more surprising, mixing is a best response for low quality sellers. On the other side of the market, buyer demands are consistent with the predictions of rational inattention. Thus, buyer demands can be viewed as a best response given seller strategies and attentional costs.

This paper makes two main contributions. The first contribution is establishing that rational inattention to quality produces a unique probability of low quality mimicking in the mimic high equilibrium and showing how mimicking varies with attentional costs and other model parameters in closed form. Central to this contribution is showing how a recently established property of rational inattention, the invariance of threshold beliefs to prior beliefs, greatly simplifies the characterization of the buyer’s best response to different rates of high quality mimicking.<sup>3</sup> The second contribution is finding strong evidence of this mimic high equilibrium in an experiment in which subjects face real attentional costs.

This paper is related to work in four different literatures, which are discussed in more detail in the next section. First, this paper relates to a large recent literature in economics on limited attention to available information about choice alternatives. Second, this paper relates to a small, but growing, literature in economics that tests models of attention using experiments with real attentional costs. It introduces one of the first experiments with real attentional costs in a market setting. Third, this paper relates to an expanding literature based around rational inattention theory. It presents one of the first games with rationally inattentive agents and one of the first experiments to find evidence of rational inattention. Last, this paper relates to a large literature on asymmetric information in market settings. While Akerlof [1970] assumed that buyers could not acquire any information about quality, subsequent models have included many different assumptions for how buyers can gather information. I show that rational inattention produces broadly similar equilibria to those found with commonly used assumptions for information acquisition, but without the discontinuities in existence that are found when model parameters vary. Also, this paper provides the first experiment to select among these equilibria.

In section 2, I provide a review of the related literature. In section 3, I describe the model and the equilibrium concept. In section 4, I solve for buyer demands and information gathering as a function of model parameters and seller strategies. In section 5, I characterize the equilibria of the model in closed form. In section 6, I present the experimental design and results. Section 7 concludes.

## 2 Related Literature

This section describes the four literatures that this paper is most related to: attention in economic decision making, economic experiments with real attentional costs, rational inattention theory, and price signaling.

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<sup>3</sup>For this and other behavioral properties of rational inattention, see Caplin and Dean [2012].

## 2.1 Growing Literature on Attention in Economics

There is a large recent literature in economics on limited attention to available information about choice alternatives. Some of these papers take a “bounded rationality” approach, in which decision makers only consider a subset of the available options. For example, see Manzini and Mariotti [2007], Salant and Rubinstein [2008], Eliaz and Spiegler [2011a,b], and Masatlioglu, Nakajima, and Ozbay [2012]. Other papers assume that limited attention is generated by behavioral biases, such as a tendency to focus on certain information because it is salient or boosts self image. For example, see Rabin and Weizsacker [2009], Eil and Rao [2011], Gottlieb [2011], Bordalo, Gennaioli, and Shleifer [2012], and Koszegi and Szeidl [2012].

Instead, I assume that limited attention is a rational response to the costs associated with encoding or processing information. This assumption is closer in spirit to papers where limited attention is a consequence of thinking costs or exogenous limitations on encoding information, such as Ergin and Sarver [2010], Gennaioli and Shleifer [2010], Caplin and Dean [2011], Compte and Postlewaite [2012], Ortoleva [2012], and Schwartzstein [2012].

Across these approaches, there has been increasing interest in the market implications of this behavior. For example, Eliaz and Spiegler [2011a,b] examine how firms try to influence the consideration sets of consumers with marketing or attention grabbing products. Another example is by Bordalo, Gennaioli, and Shleifer [2012], who look at the impact of salient product dimensions on consumer choices.

## 2.2 Economic Experiments with Real Attentional Costs

Gabaix, Laibson, Moloche, and Weinberg [2006], Caplin, Dean, and Martin [2011], Caplin and Martin [2011], and Caplin and Martin [2012] all use a similar addition task to the one in this paper in order to induce real attentional costs, but they only consider the impact of these costs in individual decision problems. The experiment that Kalaycı and Potters [2011] implement is similar to mine in that it considers a market setting and requires the buyer to solve a math problem to learn quality, but differs because of their model setup: sellers determine the complexity level of the math problem, buyers know little of the seller’s characteristics or objectives, and buyers face extreme time pressure. As a result, there is little to no room for subjects to change attentional effort based on strategic considerations.

## 2.3 Rational Inattention Theory

Sims [2003] introduces rational inattention theory in order to model the constraints that agents face in processing available information.<sup>4</sup> It is based on classic works in the information theory literature which describe a physical constraint on the flow of information. This constraint, called the Shannon capacity or Shannon channel, determines the amount of uncertainty (entropy) that can be reduced by a message, and it has been interpreted as a cognitive limitation for economic agents (Wiederholt [2010], Tutino [2011]). In models of choice, this constraint produces a noisy perception of the underlying state. Woodford [2012] generates a related informational constraint with stronger neurobiological foundations, and Gabaix [2011] provides a related approach to limited attention in which agents simplify the data available to them.

Rational inattention can be modeled as agents choosing the distribution from which they draw an informative signal, with more informative signals being more costly. This cost takes a specific log-linear form (see Veldkamp [2011]). For tractability, many models have assumed a Gaussian relationship in the signal structure and a linear-quadratic utility function, but recent work, including this paper, allows for more general signal structures and classic utility functions (see Sims [2006] for a discussion and Yang [2012b] for an implication). Recently, Matějka and McKay [2011] and Caplin and Dean [2012] have shown that in models with a finite number of states, as in this paper, rational inattention can yield clean solutions.

In other models with rational inattentive buyers, it is assumed that prices are not easily observable (Matějka [2010], Matějka and McKay [2012]). Instead, I assume that prices are observed perfectly, which is appropriate for retail environments where prices are prominent and simple (i.e., dollars and cents). Because prices are observed perfectly by buyers, the interaction between sellers and buyers can be represented with a game. As a result, this paper joins Yang [2012a,b] as one of the first applications of rational inattention to games. Like Yang [2012a], I use a binary action setup with sequential moves and include only one rationally inattentive agent (the second mover). However, a substantial difference in this paper is that the first mover is informed, so that their action choice can reveal information.

One related experimental paper is by Cheremukhin, Popova, and Tutino [2012], who test for rational inattention in cognitive limits of processing information about choices over lotteries. Another related experimental paper is by Treviño and Szkup [2011], who allow subjects to improve the precision of their signal at a cost, which induces an information choice that can be interpreted as rational inattention. My experiment differs from these existing experiments by looking for evidence of rational inattention to available information in a market setting.

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<sup>4</sup>For a brief overview see Wiederholt [2010], for the connection to information theory see Sims [2010], and for a more detailed treatment see Veldkamp [2011].

## 2.4 Price Signaling and Information Acquisition

The fourth related literature is a long literature on strategic pricing games where buyers acquire information about quality. There have been a variety of assumptions for information acquisition employed in this literature. Cooper and Ross [1984] and Bagwell and Riordan [1991] assume that consumers can only be fully informed or fully uninformed. Chan and Leland [1982] and Bester and Ritzberger [2001] endogenize this form of information gathering. Voorneveld and Weibull [2011] consider buyers who get a normally distributed signal of quality. In Wolinsky [1983], consumers get a noisy signal of quality when they sample a price. Kalaycı and Potters [2011] assume consumers observe price perfectly, but quality differences with uniform noise. Bar-Isaac, Caruana, and Cuñat [2012] assume that buyers can pay a cost to learn about a dimension of quality.

Rational inattention can be interpreted as a form of costly information acquisition and produces broadly similar equilibria to many of these existing assumptions. However, commonly used assumptions produce equilibria in this strategic pricing game that have discontinuities as model parameter values vary. For example, if rational inattention is replaced in the model with being fully informed at a cost, then the mimic high equilibrium is not stable at higher costs, as showed by Bester and Ritzberger [2001]. If rational inattention is replaced with a free normally distributed signal of quality, then the model no longer predicts fully inattentive behavior, so there is only pooling low equilibrium when the signal is very uninformative. In addition, this paper provides the first experiment to select among these equilibria.

The experiment in this paper sheds light on how much prices can signal about quality in the presence asymmetric information about quality. In this way, it is related to experiments reported by Miller and Plott [1985], who examine how much prices can signal uncertain quality in an experiment where sellers can also signal quality by adding observable quality. My experiment differs from theirs in that buyers have access to an exogenous source of information about quality and sellers can only signal through price. There have also been many experiments on the effects of exogenous price variation on the choices of real goods; however, these studies are rarely incentivized, and few look at the key role of information acquisition. One exception is by Lynch and Ariely [2000], who conduct an experiment that contains a treatment where information on prices for wines is easy to obtain, but information on quality levels is not, and find that price elasticity decreases with the difficulty of search for information about quality. Another is by Heffetz and Shayo [2009], who control the information that subjects have about an uncertain food product and find that exogenous variation in price does not have a large effect on elasticities.

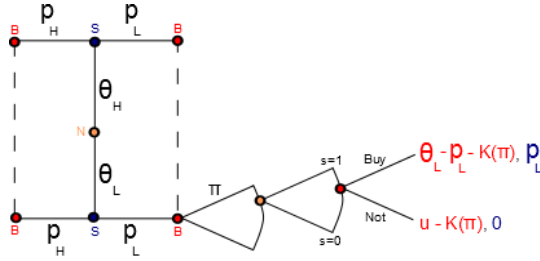


Figure 1: The game tree.

### 3 A Model of Strategic Pricing

To study how sellers react to buyers that are rationally inattentive to quality, I modify a simple and standard sequential pricing game. This modified game and the corresponding equilibrium are defined in the following section.

#### 3.1 Describing the Sales Encounter

One seller and one buyer are engaged in a one-off sales encounter, which is represented by the game tree presented in figure 1. Nature moves first by determining the product’s quality level  $\theta \in \Theta = \{\theta_L, \theta_H\}$ , where  $\theta_L, \theta_H \in \mathbb{R}_+$  and  $\theta_L < \theta_H$ . The probability of the product being of high quality is  $\lambda \in (0, 1)$ , which is commonly known. The seller learns the realized quality level, but the buyer does not.

Next, the seller chooses a price  $p \in P = \{p_L, p_H\}$ , where  $p_L, p_H \in \mathbb{R}_+$  and  $p_L < p_H$ , which is a take it or leave it offer for the product. The buyer learns the price and chooses an attention technology  $\pi$  (shown for just one node in the figure), and then nature generates a posterior belief  $s \in [0, 1]$  that the product is of high quality based on  $\pi$ . Finally, the buyer chooses to buy the product or not.

Notice that there is two sided information asymmetry in that the buyer does not know the realized quality level and the seller does not know the posterior belief of the buyer.

I start with a simple model in order to help isolate the direct impact of rational inattention. In section 5, I briefly consider how the set of equilibria changes with an increase in the number of prices, number of quality levels, number of sellers, or number of buyers.

#### 3.2 Payoffs to the Buyer and Seller

The buyer has the following “purchase” utility function, which is separable from any attentional costs:

$$U(\theta, p, x) = x(\theta - p) + (1 - x)u,$$



where  $u \in \mathbb{R}_+$  is the utility of not purchasing the product, which can be interpreted as the outside option, and  $x = 1$  if the buyer chooses to purchase the product and  $x = 0$  if not. This quasilinear utility function reflects a buyer who is balancing the quality and price of a good and is suitable when prices do not have a big effect on wealth (see Vives [2001]). The buyer is a risk neutral expected utility maximizer.

To focus the analysis on situations where the buyer would want to exert attention effort, I make the following assumptions:

1. The buyer wants to accept high price, high quality offers and low price, low quality offers:  $\theta_H - p_H > u$  and  $\theta_L - p_L > u$ .
2. The buyer wants to reject high price, low quality offers:  $\theta_L - p_H < u$ .

The seller has the following profit function:

$$V(p, x) = xp,$$

and is also a risk neutral expected profit maximizer. This profit function reflects a seller who does not have reputational concerns or marginal costs.

### 3.3 Defining the Market

A market  $\omega$  is defined as

$$\omega := (\lambda, \Theta, P, u).$$

All of the following could change a market: new technologies could alter the probability a product is high quality level or improve the absolute quality levels; subsidies or taxes could change the utility or profit functions or the value of the outside option; and price floors or ceilings could alter the set of prices to choose from.

### 3.4 Product Quality and Information Asymmetries

As in a long literature in microeconomics, industrial organization, and marketing, I assume that product quality can be summarized with a scalar value.<sup>5</sup> This value has been treated both as an objective measure of quality (for example, Cooper and Ross [1984]) and a subjective measure of quality (for example, Judd and Riordan [1994]). I assume that the seller knows this value perfectly, which can be thought of as resulting from long experience with selling the product.

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<sup>5</sup>As in much of the theoretical literature, I assume that both quality and price are measurable in terms of expected utility.

I also assume that the buyer does not know, *ex ante*, this value. As proposed by Wolinsky [1983] and others, this is a suitable assumption when thinking about infrequently purchased products, such as durable goods that have features that change over time. At the same time, I assume the buyer knows the probability that the product is of high quality, which can be interpreted as past experience with similar products or the product's brand or reputation for quality.

In addition, the buyer has access to many different sources of information about the quality of the product: physical inspection, information provided on the packaging, customer reviews, advertisements, etc. For now, I will ignore the possible endogeneity that results from information being supplied by the seller, though this is a potentially interesting extension.

### 3.5 Adding Rational Inattention

The way that a buyer attends to available information is treated abstractly. Unlike other models of rational inattention, I assume that prices are easily observable. Sims [2006] argues that if prices are real numbers, then it would take an infinite capacity to internalize the exact price. Instead, I interpret prices as easy to observe, which reflects retail prices, the environment I am considering. None of the results that follow depend on prices being real numbers.

The buyer chooses an attention technology  $\pi$  after observing the price and forming an interim belief  $\beta_p$  that the product is of high quality. This attention technology produces a range of possible posterior beliefs based on the true state of product quality. A posterior belief  $s$  is a point in the interval  $S = [0, 1]$ .

Technically,  $\pi$  is a function that maps the realized quality level into  $\Delta(S)$ , the set of probability distributions over  $S$  that have finite support, so that

$$\pi : \Theta \rightarrow \Delta(S).$$

Let  $\Pi$  denote the set of all such functions,  $\pi^\theta(s)$  be the probability of posterior  $s \in S$  given quality  $\theta \in \Theta$  for a given  $\pi$ , and  $S(\pi) \subset S$  denote the support of a given  $\pi$ .

With this structure, it is necessary to limit the set of feasible choices to all information technologies  $\Pi(\beta_p) \subset \Pi$  that generate correct posteriors for a given interim belief  $\beta_p$  that the product is of high quality, so that

$$\Pi(\beta_p) = \left\{ \pi \in \Pi \mid \forall s \in S(\pi), s = \frac{\beta_p \pi^{\theta_H}(s)}{\beta_p \pi^{\theta_H}(s) + (1 - \beta_p) \pi^{\theta_L}(s)} \right\}.$$

As discussed in Caplin and Martin [2011], this modeling approach is equivalent to the agent having a subjective signaling technology. However, this prior-posterior approach will be useful for representing the cost function described below.

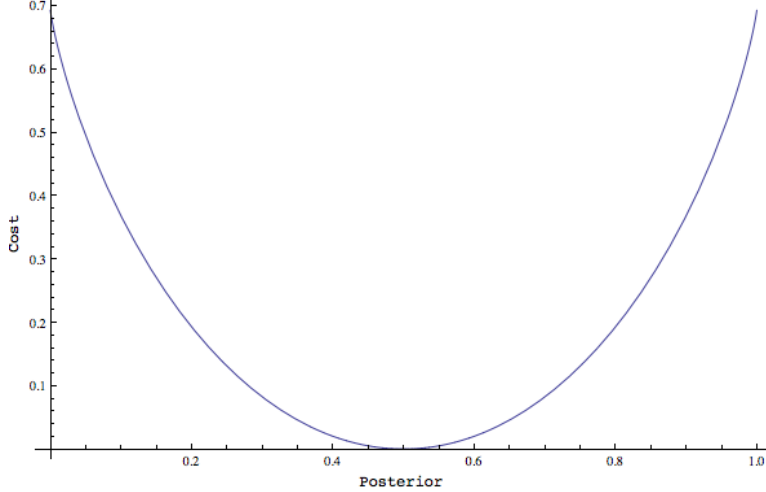


Figure 2: Shannon costs for a posterior  $s$  when the interim belief  $\beta_p$  is 0.5 and the cost parameter  $\kappa$  is 1.

In this model, attention is costly, and these costs are determined using Shannon capacity (see section 2 for a review). According to this approach, a posterior is more costly when it reduces uncertainty more. Here uncertainty is measured by how far a posterior is from revealing that a product is of high or low quality. As  $s$  approaches 0 or 1, it reduces uncertainty more and more.

Formally, each attention technology  $\pi \in \Pi(\beta_p)$  has a cost in expected utility units that is assigned by the function

$$K(\pi, \kappa, \beta_p) = \kappa \left( \left[ \sum_{s \in S} \pi(s) (s \ln(s) + (1-s) \ln(1-s)) \right] - \beta_p \ln(\beta_p) + (1-\beta_p) \ln(1-\beta_p) \right)$$

where  $\kappa \in \mathbb{R}_+$  is a linear cost parameter. The lowest cost attention technology, which has a cost of zero, is one that just produces one posterior and thus returns the interim belief as the posterior.

As shown in figure 2, this form produces u-shaped costs for each posterior, which bottom out at a posterior of 0.5 and increase symmetrically in either direction (towards being certain of high quality or being certain of low quality). Also, this cost has an infinite derivative as it approaches 0 and 1.

### 3.6 The Game

Adding costly attention to a market  $\omega$  produces the game  $G := (\omega, \kappa)$ . Everything about the game, except for the realization of quality, is common knowledge. Thus, the seller is knowledgeable about the buyer's attentional cost parameter. This assumption is more plausible in some settings than others.

### 3.7 Equilibrium Concept

The equilibrium concept employed is mixed strategy perfect Bayesian equilibrium (PBE). The seller has pricing strategy  $\sigma(\theta)$ , which is the probability of pricing high for quality level  $\theta$ . The buyer has information strategy  $\pi_p \in \Pi(\beta_p)$ , which depends on the observed price, and purchasing strategy  $\alpha(p, s)$ , which is the probability of purchasing for each price and posterior. Finally, the buyer has beliefs  $\mu(p)$  of the probability of high quality for each price.<sup>6</sup>

For a game  $G$ , a mixed strategy PBE is a 4-tuple  $(\hat{\sigma}, \hat{\pi}, \hat{\alpha}, \hat{\mu})$  that satisfies seller optimality, buyer optimality, and Bayesian beliefs:

- Seller optimality

–  $\forall \theta \in \Theta, \hat{\sigma}(\theta) > 0$  implies

$$p_H \in \arg \max_{p \in \{p_L, p_H\}} E[V(p, x) | \theta, \hat{\alpha}, \hat{\mu}],$$

and  $\hat{\sigma}(\theta) < 1$  implies

$$p_L \in \arg \max_{p \in \{p_L, p_H\}} E[V(p, x) | \theta, \hat{\alpha}, \hat{\mu}].$$

- Buyer optimality

–  $\forall p \in P, s \in S, \hat{\alpha}(p, s) > 0$  implies

$$1 \in \arg \max_{x \in \{0,1\}} E[U(\theta, p, x) | p, s],$$

and  $\hat{\alpha}(p, s) < 1$  implies

$$0 \in \arg \max_{x \in \{0,1\}} E[U(\theta, p, x) | p, s].$$

–  $\forall p \in P,$

$$\hat{\pi}_p \in \arg \max_{\pi \in \Pi(\hat{\mu}(p))} E[U(\theta, p, x) | p, \pi] - K(\pi, \kappa, \hat{\mu}(p)).$$

- Bayesian beliefs in equilibrium

– If  $\hat{\sigma}(\theta) > 0$  for any  $\theta \in \Theta$ , then

$$\hat{\mu}(p_H) = \frac{\lambda \hat{\sigma}(\theta_H)}{\lambda \hat{\sigma}(\theta_H) + (1 - \lambda) \hat{\sigma}(\theta_L)},$$

and if  $\hat{\sigma}(\theta) < 1$  for any  $\theta \in \Theta$ , then

$$\hat{\mu}(p_L) = \frac{\lambda(1 - \hat{\sigma}(\theta_H))}{\lambda(1 - \hat{\sigma}(\theta_H)) + (1 - \lambda)(1 - \hat{\sigma}(\theta_L))}.$$

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<sup>6</sup>Note that the agent does not mix over information acquisition technologies. This is without loss of generality as Caplin and Dean [2012] show it is not optimal to mix over information acquisition technologies for a Shannon cost function, which follows from the strict concavity of the log function.

## 4 Buyer Best Responses

To find the equilibria of this sequential game, I first determine the best responses of the buyer to every possible interim belief  $\beta_p$  that the product is of high quality for every price  $p$ . The buyer's best response is composed of two parts: the optimal information strategy and optimal purchasing strategy.

By assumption, it is always optimal for the buyer to purchase the product when the price is low, regardless of the quality level. As a result, the buyer has no incentive to expend costly attentional effort if a low price is observed. Also, when the price is high, the buyer only wants to purchase the product if it is of high quality, so there are incentives to undertake costly attentional effort if the product could be of low quality. In what follows, I solve for the optimal information strategy and optimal purchasing strategy at price  $p_H$  given all interim beliefs  $\beta_{p_H}$ . The price will be kept general, however, to indicate that the solution can be applied when there are more than two prices available to the seller.

### 4.1 Choosing the Optimal Attention Technology

Caplin and Dean [2012] show how to determine the optimal attention technology  $\pi$  for rational inattention theory using a posterior-based approach. This problem is greatly simplified by their observations that the optimal technology generates a single posterior for each action, that the corresponding action is strictly optimal for each posterior, and that the solution is unique.

To find the solution in this binary action problem, I first assume that both actions (purchase and not purchase) are taken with positive probability. For this model, let  $s_p^0$  be the posterior for which the buyer does not purchase at price  $p$  and  $s_p^1$  be the posterior for which the buyer purchases at price  $p$ . Because both actions are taken with positive probability, the resulting optimization problem is:

$$\max_{s_p^0, s_p^1, \pi(s_p^1)} \pi(s_p^1) [s_p^1(\theta_H - p) + (1 - s_p^1)(\theta_L - p)] + (1 - \pi(s_p^1))u - K(\pi, \kappa, \beta_p)$$

where again

$$K(\pi, \kappa, \beta) = \kappa \left( \left[ \sum_{s \in \mathcal{S}} \pi(s) (s \ln(s) + (1 - s) \ln(1 - s)) \right] - \beta \ln(\beta) + (1 - \beta) \ln(1 - \beta) \right).$$

Finally, because  $\pi \in \Pi(\beta_p)$ , it must satisfy Bayes rule, so that

$$\pi(s_p^1) s_p^1 + (1 - \pi(s_p^1)) s_p^0 = \beta_p.$$

The first order conditions reduce to the following ratios, which can also be expressed as log likelihood

ratios:

$$\begin{aligned}\frac{s_p^1}{s_p^0} &= \exp\left(\frac{\theta_H - p - u}{\kappa}\right), \\ \frac{1 - s_p^1}{1 - s_p^0} &= \exp\left(\frac{\theta_L - p - u}{\kappa}\right).\end{aligned}$$

Thus, when both actions are taken with positive probability, the optimal posteriors are

$$\begin{aligned}s_p^0 &= \frac{1 - \exp\left(\frac{\theta_L - p - u}{\kappa}\right)}{\exp\left(\frac{\theta_H - p - u}{\kappa}\right) - \exp\left(\frac{\theta_L - p - u}{\kappa}\right)}, \\ s_p^1 &= \exp\left(\frac{\theta_H - p - u}{\kappa}\right) s_p^0.\end{aligned}$$

Note that these posteriors are not impacted by interim beliefs, just price  $p$ , the cost of attention  $\kappa$ , and market parameters  $\theta_L$ ,  $\theta_H$ , and  $u$ . However, the unconditional likelihood  $\pi(s_p^1)$  of posterior  $s_p^1$  is determined by interim beliefs because

$$\pi(s_p^1) = \min\left(\max\left(\frac{\beta_p - s_p^0}{s_p^1 - s_p^0}, 0\right), 1\right).$$

If  $\beta_p \leq s_p^0$  or  $\beta_p \geq s_p^1$ , then the probability  $\pi(s_p^1)$  will equal 0 or 1, so only one posterior will be produced. This means that no attentional effort has been exerted, the interim belief becomes the posterior, and only one action is taken. Because the buyer does not attend to quality above or below these posteriors, they can be interpreted as reservation or threshold beliefs.

As  $p$  converges down to  $\theta_L - u$ , both thresholds converge to 0. This means that for very low prices, the buyer will drop out from attending and will purchase the product even with little information on quality. On the other hand, as  $p$  converges up to  $\theta_H - u$ , both thresholds converge to 1. At these very high prices, the buyer will also drop out from attending, but will instead refrain from purchasing.

#### 4.1.1 Example: Minimize Type I Errors

In the following two examples, I show how threshold posteriors adjust to model parameters to minimize different types of errors. Specify a market as

$$(\lambda, \Theta, P, u) = (0.5, \{100, 200\}, \{50, 100\}, 25),$$

and let  $\kappa = 15$ . In other words, the probability that the seller's product is of high quality is 50%, the possible quality levels are 100 and 200, the feasible prices are 50 and 100, and the buyer's outside option is worth 25. We can use the steps above to determine the optimal attention technology for this market.

First, the two posteriors  $s_{100}^0$  and  $s_{100}^1$  are approximately 0.01 and 0.81 respectively. Because the high price of 100 is not very high, the buyer chooses to be very certain of quality when not buying, but less certain

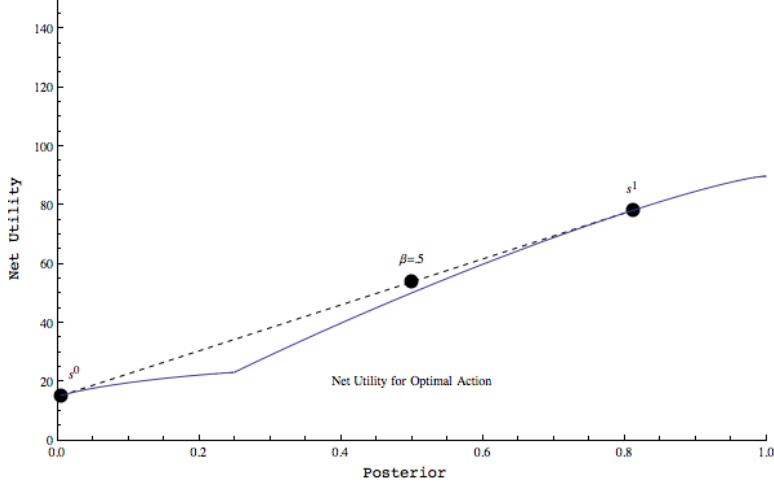


Figure 3: The net utility (purchase utility minus attentional costs) produced by the optimal information processing technology and optimal purchasing strategy at price 100 for market  $(\lambda, \Theta, P, u) = (0.5, \{100, 200\}, \{50, 100\}, 25)$ , cost  $\kappa = 15$ , and prior  $\beta_{100} = .5$ .

of quality when buying. The reason for this asymmetry is that, at this price, the buyer wants to buy if there is a decent chance that the quality level is high. In other words, the buyer will adjust their attention to reduce Type I errors: mistakenly not buying a high quality product.

Assume that all sellers, regardless of their type, pool at a price of 100, so that  $\beta_{100} = 0.5$ . In this case, the interim belief is between the thresholds  $s_{100}^1$  and  $s_{100}^0$ , so the optimal attention technology puts some weight on both posteriors:  $\pi(s_{100}^1)$  is approximately 0.61.

This solution is summarized by figure 3, which is based on the approach of Caplin and Dean [2012]. The figure shows how net utility for the optimal action changes with posterior beliefs. The two optimal posteriors are shown as the left and right dots, and the interim belief is shown between them. The net utility produced by the optimal technology at this interim belief is found by taking the convex combination of the net utilities of the two different posteriors.

#### 4.1.2 Example: Minimize Type II Errors

Now assume that the quality levels are 50 and 150, not 100 and 200, so that the market  $(\lambda, \Theta, P, u)$  is  $(0.5, \{50, 150\}, \{50, 100\}, 25)$ . How does the optimal attention technology change?

In this market, the two posteriors  $s_{100}^0$  and  $s_{100}^1$  are approximately 0.19 and 0.99 respectively. Unlike the previous case, the buyer chooses to be very certain of quality when buying and less certain of quality when not buying. This is because the buyer wants to refrain from buying if there is a decent chance that the

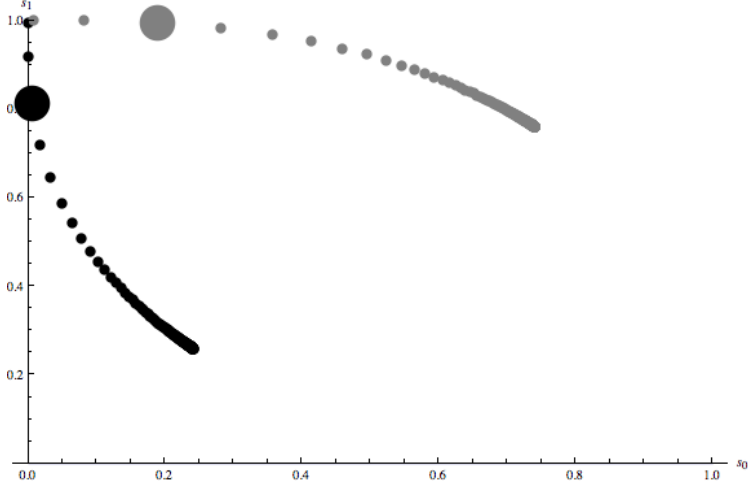


Figure 4: Threshold posteriors for costs  $\kappa \in \{5, 10, \dots, 1000\}$  at price 100 for market  $(\lambda, \Theta, P, u) = (0.5, \{100, 200\}, \{50, 100\}, 25)$  and prior  $\beta_{100} = .5$  (dots below) and at price 100 for market  $(\lambda, \Theta, P, u) = (0.5, \{50, 150\}, \{50, 100\}, 25)$  and prior  $\beta_{100} = .5$  (dots above).

quality is low. In other words, the buyer will adjust their attention to reduce Type II errors: mistakenly buying a low quality product.

These examples illustrate how attention and drop out thresholds can change substantially with model parameters. Also, it shows that they change in response to the costliness of different mistakes.

Figure 4 shows how threshold beliefs change with the attentional cost parameter for these two example markets. The dots representing threshold beliefs in the first market (in black) are below and to the left of the dots for the second market (in grey). As information costs go to zero, thresholds converge smoothly to  $s_{100}^1 = 1$  and  $s_{100}^0 = 0$  for both sets of quality levels. However, as costs increase, the thresholds move away from this point in different directions. Each dot represents an incremental increase of 5 in cost, from 5 to 1000. The large dots represent the solutions for  $\kappa = 15$ , as above. For low information costs, both optimal information technologies give very precise posteriors for one of the actions, but as costs increase, the posteriors for both actions become less precise for both optimal information technologies. As costs increase, the thresholds are converging from above and below to the belief for which the buyer is indifferent between purchasing or not:

$$s = \frac{u - (\theta_L - p_H)}{(\theta_H - \theta_L)}.$$

## 4.2 Conditional Demands

Because both actions are uniquely optimal for their corresponding posteriors, the optimal purchasing strategy is deterministic for a given price and posterior, even though the solution concept allows for stochasticity.



However, the stochasticity in attention means that overall product demand, given by  $\pi(s_{p_H}^1)$ , is not necessarily deterministic.

While  $\pi(s_{p_H}^1)$  gives the unconditional probability of a buyer purchasing at price  $p_H$ , a seller of type  $\theta$  will choose a pricing strategy based on the probability that the buyer will purchase a product of quality  $\theta$  at price  $p_H$ , which is denoted by  $d_{p_H}^\theta$ . Using Bayes rule, it can be determined that conditional demands are

$$d_{p_H}^{\theta_H} = \Pr(\text{buy}|\theta_H) = \frac{s_{p_H}^1 \pi(s_{p_H}^1)}{\beta_{p_H}},$$

and

$$d_{p_H}^{\theta_L} = \Pr(\text{buy}|\theta_L) = \frac{(1 - s_{p_H}^1) \pi(s_{p_H}^1)}{(1 - \beta_{p_H})}.$$

There are two key features of these conditional demands. First, because posteriors are not impacted by interim beliefs with Shannon attentional costs, the conditional demand for the sellers of low quality products at the high price are strictly increasing in  $\beta_{p_H}$ . Second, if more than one action is taken, then the conditional demand for high price, high quality offers is strictly higher than the demand for high price, low quality offers ( $d_{p_H}^{\theta_H} > d_{p_H}^{\theta_L}$ ) because  $\beta_{p_H} < s_{p_H}^1$  implies

$$(1 - \beta_{p_H}) s_{p_H}^1 > \beta_{p_H} (1 - s_{p_H}^1).$$

## 5 Equilibrium

In this section, I will describe the two types of equilibria that are possible in this game and fully characterize them. To start, I will solidify the connection between conditional demands and pricing strategies.

### 5.1 Conditional Demands and Pricing Strategies

An optimal pricing strategy will not put positive weight on price  $p$  if any other price makes a higher expected return, and the expected return from charging price  $p$  for a product of quality  $\theta$  is determined by the probability a product of quality  $\theta$  is purchased at price  $p$ , which is the conditional demand  $d_p^\theta$ . Thus,  $\hat{\sigma}(\theta) > 0$  only if

$$d_{p_H}^\theta \cdot p_H \geq d_{p_L}^\theta \cdot p_L$$

and  $\hat{\sigma}(\theta) < 1$  only if

$$d_{p_H}^\theta \cdot p_H \leq d_{p_L}^\theta \cdot p_L.$$

There are two useful implications that come immediately from this. First, because buyers always purchase when the price is low (i.e.,  $d_{p_L}^\theta = 1$  for all  $\theta$ ), a seller of type  $\theta$  will only mix between prices if the expected

return from pricing high is equal to the low price, which is true when

$$d_{p_H}^\theta = \frac{p_L}{p_H}.$$

Second, because  $d_{p_H}^{\theta_H} > d_{p_H}^{\theta_L}$  when  $d_{p_H}^{\theta_H} > 0$  and  $d_{p_H}^{\theta_L} > 0$ , if it is optimal for sellers of type  $\theta_H$  to mix, then sellers of type  $\theta_L$  will price low with probability 1, and if it is optimal for sellers of type  $\theta_L$  to mix, then sellers of type  $\theta_H$  will price high with probability 1

Finally, pricing strategies impact conditional demands through  $\beta_p$ , the probability of  $\theta_H$  given price  $p$ , where

$$\begin{aligned}\beta_{p_H} &= \frac{\lambda \hat{\sigma}(\theta_H)}{\lambda \hat{\sigma}(\theta_H) + (1 - \lambda) \hat{\sigma}(\theta_L)}, \\ \beta_{p_L} &= \frac{\lambda (1 - \hat{\sigma}(\theta_H))}{\lambda (1 - \hat{\sigma}(\theta_H)) + (1 - \lambda) (1 - \hat{\sigma}(\theta_L))}.\end{aligned}$$

As a result,  $\beta_p = \hat{\mu}(p)$  on the equilibrium path.

## 5.2 Two Types of Equilibria

### 5.2.1 Pooling at a Low Price

For any game  $G$ , there always exists a pooling equilibrium where both types of sellers charge a low price with probability 1, which I will call a “pooling low” equilibrium. A pooling low equilibrium requires pessimistic off-equilibrium path beliefs: if the buyer sees a high price, then they believe the seller is a low quality type with a high enough probability that they choose an uninformative information strategy and rarely purchase the product, regardless of its quality level. Note that this type of equilibrium may not be unique.

Even when there are no costs to attention effort, there exists a pooling low equilibrium in which the buyer is certain that the deviating seller is of low quality and chooses a completely uninformative attention technology if a deviation occurs. This equilibrium cannot be eliminated with the Intuitive Criterion (Cho and Kreps [1987]) or D1 (Banks and Sobel [1987]) because both types of sellers have the same incentive to deviate to the higher price. This would still be true if different types of sellers had different marginal costs.

### 5.2.2 Mimic High Equilibrium

There is just one other type of equilibrium, which I will call the “mimic high” equilibrium. It is the more informative equilibrium because prices are weakly more informative about quality than in the pooling low equilibrium for all values of  $\kappa$  and strictly more informative for some values of  $\kappa$ . In this equilibrium, the high quality seller always puts probability 1 on setting a high price, and the low quality seller mimics the high quality seller by charging a high price with a certain probability  $\eta$ , which is determined by the parameters of the game. Theorem 1 indicates that there is a single mimicking probability for each game  $G$ .

**Theorem 1** *For any game  $G$ , there exists an equilibrium (“mimic high”) in which high quality sellers price high with probability 1 and low quality sellers price high with a unique probability  $\eta \in [0, 1]$ .*

The proof of this theorem is in the appendix. The uniqueness of  $\eta$  comes from the fact that conditional demand has a single crossing property with the indifference condition for low quality sellers to price low or high as the interim belief decreases.

The unique rate of mimicking can be determined in closed form with the Shannon cost function and is given in Corollary 1.

**Corollary 1** *In the mimic high equilibrium, the probability of mimicking is*

$$\eta = \min \left\{ \max \left\{ \frac{\lambda}{1 - \lambda} \frac{(1 - s_{p_H}^1)(1 - s_{p_H}^0)}{(1 - s_{p_H}^1)(s_{p_H}^0) + \frac{p_L}{p_H}(s_{p_H}^1 - s_{p_H}^0)}, 0 \right\}, 1 \right\}.$$

As the attentional cost parameter  $\kappa$  goes to 0, the probability of mimicking  $\eta$  converges to 0. However, as  $\kappa$  goes to  $\infty$ ,  $\eta$  does not always converge to 1. The proof of Theorem 1 exposes a regularity in the relationship between the probability of mimicking  $\eta$  and the attentional cost parameter  $\kappa$ .

In the mimic high equilibrium, strategies are as in the full information equilibrium if the cost parameter is zero and converge to those in the full information equilibrium as the cost parameter goes to zero. Surprisingly, for some model parameters, when the cost parameter gets high enough, an increase in the cost parameter actually decreases the probability that low quality sellers mimic. The reason is that low quality sellers must mimic less to overcome the pull of buyers towards not purchasing.

**Corollary 2** *As  $\kappa$  varies, there are four possible regions of the mimic high equilibrium:*

1. *For  $\kappa = 0$ , separating prices ( $\eta = 0$ )*
2. *For  $\kappa \in (0, \kappa^*)$ , increasing mimicking with  $\kappa$  ( $\eta \in [0, 1]$ ).*
3. *For  $\kappa \in [\kappa^*, \kappa^{**})$ , pooling high ( $\eta = 1$ )*
4. *For  $\kappa \in [\kappa^{**}, \infty)$ , decreasing mimicking with  $\kappa$  ( $\eta \in [0, 1]$ ).*

As Corollary 2 indicates, the mimic high equilibrium can be broken into four regions of  $\kappa$ , which are ordered. If uninformed buyers purchase when sellers pool at the high price, then regions 1, 2, and 3 are nonempty. On the other hand, if buyers do not purchase when sellers pool at the high price, then region 4 can be nonempty too. The surprise is found in region 4: that mimicking can decrease as the cost parameter increases.

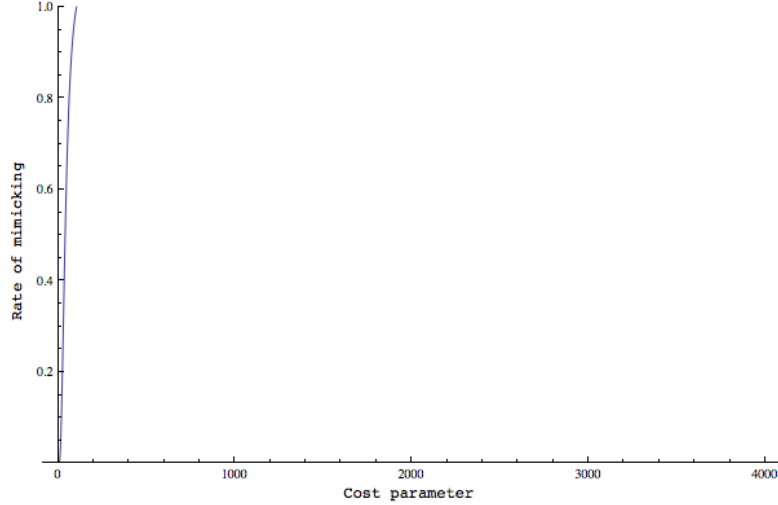


Figure 5: The rate of low quality mimicking in the mimic high equilibrium as the cost parameter varies for the market  $(\lambda, \Theta, P, u) = (0.5, \{100, 250\}, \{50, 150\}, 25)$ .

It remains to be shown that these are the only types of equilibria of this game. As mentioned above, if high quality sellers mix, then low quality sellers must put full weight on pricing low. However, if the low quality sellers put no weight on pricing high, then high quality sellers must put full weight on pricing high. Thus, high quality sellers will not mix in any equilibrium. Also, if high quality sellers put no weight on pricing high, then low quality sellers must also put no weight on pricing high. Thus, pooling low is the only equilibrium if high quality sellers price low with probability 1. Finally, if high quality sellers price high with probability 1, then all of the possible equilibria at different cost parameters are captured by the mimic high equilibrium.

### 5.2.3 Examples

I will now show how changing a model parameter value can change the regions that are nonempty. First, consider the market  $(\lambda, \Theta, P, u)$  to be  $(0.5, \{100, 250\}, \{50, 150\}, 25)$ . In this case, we see the first three regions of attentional costs in the mimic high equilibrium in that order, as shown in figure 5. The second region, which has an interior rate of mimicking, only occurs at small cost parameter values. The third region, where the rate of mimicking is 1, begins where the line disappears.

Figure 6 shows what happens when high quality is 249 instead of 250. For higher cost parameter values, the fourth region, where mimicking decreases with the cost parameter, appears/

Finally, in figure 7 we see what happens when high quality is 200. For this market, region 3 is empty, so there is never pooling at the high price.

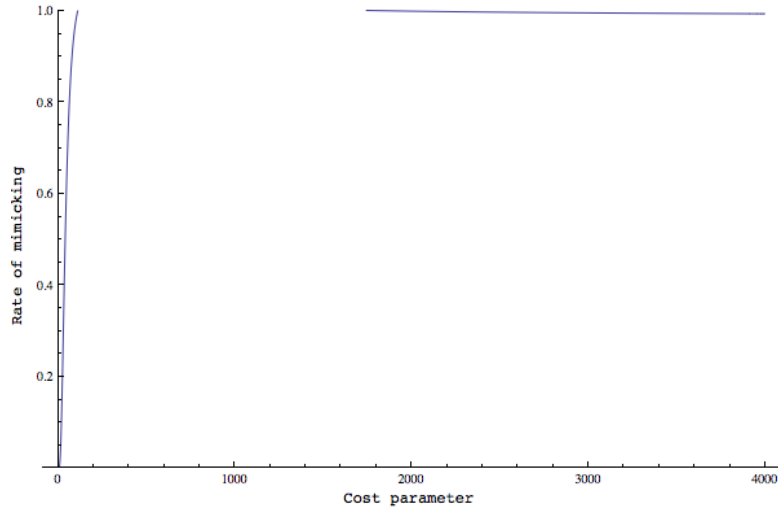


Figure 6: The rate of low quality mimicking in the mimic high equilibrium as the cost parameter varies for the market  $(\lambda, \Theta, P, u) = (0.5, \{100, 249\}, \{50, 150\}, 25)$ .

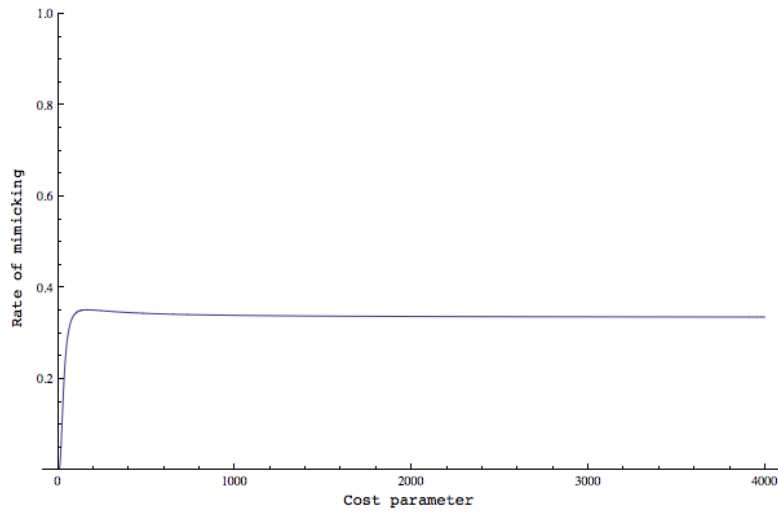


Figure 7: The rate of low quality mimicking in the mimic high equilibrium as the cost parameter varies for the market  $(\lambda, \Theta, P, u) = (0.5, \{100, 200\}, \{50, 150\}, 25)$ .

### 5.3 Increasing the Complexity of the Market

Here I briefly consider how the set of equilibria changes with an increase in the number of prices, number of quality levels, number of sellers, or number of buyers.

The analysis in this paper can be extended to cover a large number of prices, even a continuum of prices. If there are multiple “low” prices (prices where  $\theta_L - p \leq u$ ), then the only active low price will be the full information monopolist price for a low quality seller, which is the highest low price. This is because a monopolist who is known to have a low quality product always has a profitable deviation to that price. There is always a pooling low equilibrium at the highest low price, with off-equilibrium beliefs that a deviation to any other price is made by a low quality seller.

When there are multiple high prices (prices where  $\theta_H - p \geq u$ ), there can be many equilibria, but there always exists a mimic high equilibrium where high quality sellers offer a price equal to *any* of one the high prices and low quality sellers sometimes offer that high price and otherwise offer the highest low price. This is supported with off-equilibrium beliefs that a deviation to any other price is made by a low quality seller.

As the number of quality levels increases, it makes sense to consider a commensurate increase in the number of prices, so that there is a price that corresponds to each quality level, as in this model. The pooling low equilibrium is preserved, with off-equilibrium path beliefs that a deviation to any price above the lowest price must be made by a low quality type.

As the number of sellers increases, there are two possibilities, both of which occur in the literature: sellers know each other’s realized quality levels before setting prices, or they do not. If we treat the seller as an expert in the type of product that is being sold, then it seems reasonable to assume that they know the realized quality level of all sellers. In this case, there exist equilibria with features of both equilibrium described in this paper. When both sellers are low quality, they both set low prices; when both sellers are high quality, they both set high prices; and when one seller is low quality and one high quality, the high quality seller prices high and the low quality seller sometime mimics high.

As the number of buyers increases, there is no change in the set of equilibria, as long as each buyer can be treated separately in their interaction with the seller and the seller is a risk neutral expected utility maximizer.

### 5.4 Welfare and Efficiency

The full information equilibrium coincides with the mimic high equilibrium when the attentional cost parameter is zero: low quality sellers price low, high quality sellers price high, and buyers have no attentional costs. In this case, there is full efficiency, as buyers accept every offer. This is a natural benchmark for

welfare and efficiency comparisons, as it might be considered an attractive target for policy interventions.

Relative to the full information equilibrium, buyer welfare is higher in expectation in a pooling low equilibrium because high quality sellers price low instead of high and attentional costs remain at zero. Clearly, in a pooling low equilibrium, there is lower surplus for high quality sellers and the same surplus for low quality sellers. Because all offers are accepted, the pooling low equilibrium also has full efficiency.

On the other hand, relative to the full information equilibrium, buyer welfare is lower in expectation in the mimic high equilibrium when the attentional cost parameter is positive. This is because buyers have both lower surplus and higher attentional costs. Their surplus is lower because they sometimes accept high price, low quality offers and sometimes reject high price, high quality offers. At the same time, the surplus is *also* lower for high quality sellers relative to the full information equilibrium because their offers are sometimes rejected. In expectation, the surplus for low quality sellers remains the same in regions 1, 2, and 4 of the cost parameter, but they can have higher surplus in region 3, where there is pooling high. Because offers are rejected, there is less than full efficiency in the mimic high equilibrium when the attentional cost parameter is positive.

## 5.5 Policy Implications

The welfare implications of policies designed to lower attentional costs differ between these two types of equilibria. In a pooling low equilibrium, lowering the attentional cost parameter has no impact on buyer or seller welfare because buyers will choose to be uninformed, regardless of the costs of attention. However, in the mimic high equilibrium, lowering the attentional cost parameter can have an impact on buyer and seller welfare, but this impact depends crucially on model parameter values.

For attention costs in region 2 of the mimic high equilibrium, lowering the attentional cost parameter improves the welfare of both buyers and sellers. As noted before, for costs in region 2, lowering the attentional cost parameter decreases the amount that low quality sellers mimic. In equilibrium, buyers still make the same number of mistakes when the quality is low and the price is high, but these mistakes occur less often because there is less mimicking. Also, buyers will mistakenly reject high quality offers less often. As a result, there is higher surplus for both buyers and high quality sellers and no change in surplus for low quality sellers, which means more efficiency and a smaller deadweight loss. In addition, buyers have lower overall attentional costs.

In region 3, as the attentional cost parameter gets smaller buyer welfare increases even though buyers can have higher overall attentional costs. Also, buyers may choose to increase or decrease the frequency of rejecting high and low quality offers as the cost parameter falls, so the impact on seller welfare is ambiguous.

Finally, in region 4, the impact of lowering the attentional cost parameter on buyer welfare and surplus is

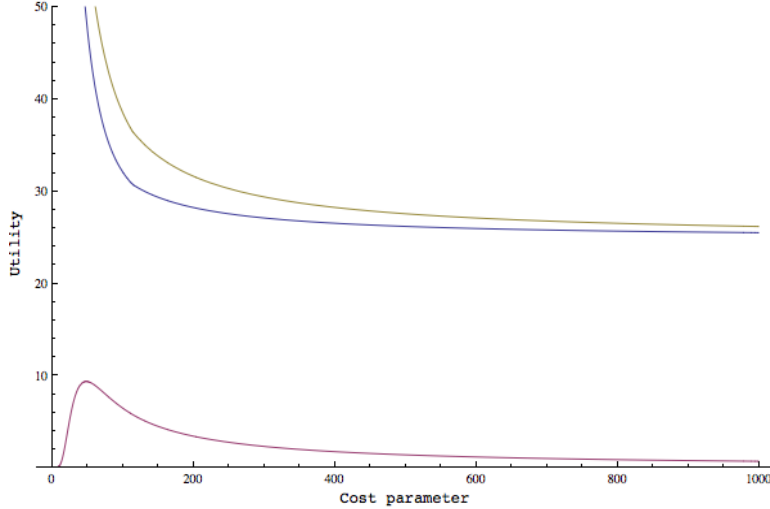


Figure 8: Purchase utility (top), net utility (middle), and attentional costs (bottom) in the mimic high equilibrium as the cost parameter varies for the market  $(\lambda, \Theta, P, u) = (0.5, \{100, 249\}, \{50, 150\}, 25)$ .

ambiguous. As noted before, for costs in region 4, lowering the attentional cost parameter can increase the amount that low quality sellers mimic. In equilibrium, buyers still make the same number of mistakes when the quality is low and the price is high, but these mistakes occur *more* often because there is *less* mimicking. In addition, buyers can have higher attentional costs. On the other hand, buyers will mistakenly reject high quality offers less often. On the other side of the market, there is higher surplus for high quality sellers and no change in surplus for low quality sellers.

In figure 8, purchase utility, net utility, and attentional costs in the mimic high equilibrium are presented as a function of the attentional cost parameter in the market  $(\lambda, \Theta, P, u) = (0.5, \{100, 250\}, \{50, 150\}, 25)$ , which was shown previously to have all four regions. Attentional costs (bottom line) have a hump shape, but purchase utility (top line) and net utility (middle line) are decreasing with the attentional cost parameter.

## 6 Experiment

To see if either equilibrium is a plausible description of actual behavior, I conducted a laboratory experiment in the Center for Experimental Social Science laboratory at New York University with undergraduate students. In the experiment, subjects are assigned to be either the buyer or the seller of a hypothetical product. In order to learn the value of this product, the buyer must add up 20 numbers.<sup>7</sup> In principle, product quality

<sup>7</sup>Similar additional tasks were used to generate real attentional costs in Gabaix, Laibson, Moloche, and Weinberg [2006], Caplin, Dean, and Martin [2011], Caplin and Martin [2011], and Caplin and Martin [2012]. It loosely imitates aggregating disparate information about quality or aggregating multiple dimensions of quality.



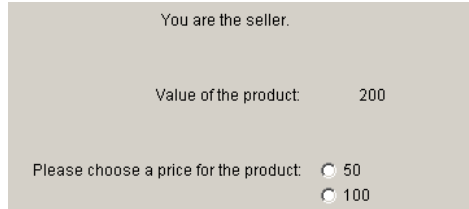


Figure 9: An example of the choice screen for sellers in the experiment.

can be determined perfectly, but in the presence of real attentional costs, buyers may choose not to become fully informed. The goal of the experiment is to see how both sides of the market respond to this possibility.

## 6.1 Implementing the Market Setting

Subjects assigned to be sellers are given the payoffs and actions of the seller in the model, and subjects assigned to be buyers are given those of the buyer in the model. Payoffs are given in Experimental Currency Units (ECU), and subjects are told that 40 ECU are equal to \$1.<sup>8</sup>

The experiment is based on the following market:

$$\omega = (\lambda, \Theta, P, u) = (0.5, \{100, 200\}, \{50, 100\}, 25).$$

In other words, the probability that the seller’s product is of high quality is 50%, the possible quality levels are 100 and 200 ECU, the possible prices are 50 and 100 ECU, and the buyer’s outside option is worth 25 ECU. This market is the same as in the first example of section 4.

In each of the 30 rounds, subjects are randomly and anonymously rematched into pairs, and one player in the pair is randomly assigned to be the seller and the other to be the buyer. Because roles are not fixed, subjects experience role reversal: buyers gain experience as sellers, and sellers gain experience as buyers.

In each round, the seller is randomly assigned a hypothetical “product” that has a value to the buyer of either 100 or 200. After being shown the value of the product, the seller offers a price for that product of 50 or 100. Next, the buyer is shown the price and chooses whether to accept or reject the offer. Before making their choice, the buyer can “check” the value of the product if they click through to a second screen. An example of the seller’s display is shown in figure 9 and of the buyer’s display in figure 10.

<sup>8</sup>Full instructions are available at <https://files.nyu.edu/djm431/public/>.

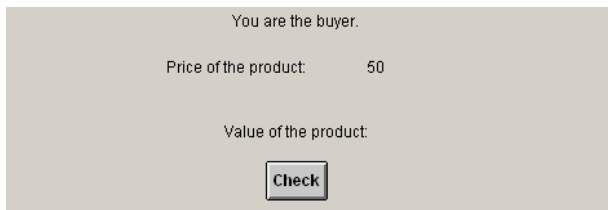


Figure 10: An example of the choice screen for buyers in the experiment.

77	96	-60	-43	-89	-26	7	-1	-51	17
92	7	-81	3	20	81	40	-57	10	58

Figure 11: An example of the 20 number strings used in the experiment.

## 6.2 Available Information

If the buyer clicks the check button, they are presented with a string of 20 numbers, and the sum of these numbers is equal to the value of the product. This addition task was selected because it allows the buyer flexibility how they allocate their attention. Subjects can carefully add up some numbers, or they can just scan the numbers for certain features.

Each of the terms in each calculation is a whole number between -100 and 100. Subjects are told the process by which these numbers are generated, which is that “20 numbers are determined by randomly drawing twenty numbers between -100 and 100 such that they add up to the product’s value.” An example expression is presented in figure 11. The seller does not know the exact string that the buyer faces, and buyers are not allowed to use calculators or scratch paper during the experiment.

## 6.3 Payoffs

Buyers have up to 90 seconds to make a decision, but can submit their decision earlier. If no decision has been made after 90 seconds, the offer will be automatically be rejected.<sup>9</sup> The buyer’s payoff is the value of the option minus the price (in ECU) if they accept the offer or 25 ECU if they reject the offer. The seller’s payoff is the price if their offer is accepted and 0 otherwise. After each round, both players are shown the value of the product, the seller’s and buyer’s choices in their pairing, and their own payoff in that round.

Finally, subjects are paid for 6 random rounds, plus a \$10 show-up fee. The average payoff was approximately \$20 for an experiment that lasted approximately 1.25 hours. The experiment was programmed in

<sup>9</sup>The time limit was reached in less than 1% of rounds.

z-Tree (Fischbacher [2007]).

## 6.4 Observed Prices

**Result #1:** High quality sellers price high, low quality sellers sometimes mimic them.

Over two sessions, 34 subjects completed the experiment, for a total of 510 pairings. Table 1 shows the fraction of rounds in which each price was offered by round number and quality level. Even in the first 15 rounds, the vast majority of sellers with high quality products charged a high price. This only increased in the second half of rounds. On the other hand, in most rounds where the seller had a low quality product, the seller offered a low price. However, in approximately 19% of rounds, sellers with a low quality product offered a high price. This frequency rose slightly to around 20% in the second half of rounds.

Table 1. Fraction of rounds each price is offered by quality level.

Price \ Quality	Rounds 1-15		Rounds 15-30		Overall	
	100	200	100	200	100	200
50	83%	5%	80%	1%	81%	3%
100	17%	95%	20%	99%	19%	97%

While there are not enough observations per subject to produce a reliable estimate of individual level pricing, there is some evidence of mixing at the individual level. Around 47% of subjects priced high at least once when the quality was low, and of the subjects who were a low quality seller at least 8 times (the median), around 62% priced high at least once when the quality was low. In addition, every subject who priced high at least once when the quality was low also priced low at least once when quality was low.

## 6.5 Observed Demands

**Result #2:** Buyers pay partial attention to quality.

Table 2 shows the percentage of offers accepted by round number and quality level. Even from the beginning of the experiment, buyers almost never made the mistake of rejecting an offer with a low price, as can be seen in the first row. The percentage of times that the buyer mistakenly accepted a high price, low quality offer stays around 50% over the course of the experiment. However, the frequency with which buyers mistakenly rejected a high price, high quality offer decreased over the course of the experiment. In

the second half of rounds, 8% of such offers were rejected.

Table 2. Fraction of offers accepted at each price and quality level.

	Rounds 1-15		Rounds 15-30		Overall	
Price \ Quality	100	200	100	200	100	200
50	97%	100%	100%	100%	99%	100%
100	47%	79%	54%	92%	51%	85%

These mistakes indicate that buyers are paying some attention to quality, but not full attention. If buyers paid full attention, we would expect an acceptance rate of 0% for low quality and 100% for high quality. On the other hand, if subjects paid no attention to quality and just randomized over accept and reject at a price of 100, we would expect the acceptance rates to be similar between the two quality levels. Instead, they are significantly different at the 1% level using a t-test.

## 6.6 Checking for Best Responses

Observed prices are in line with the mimic high equilibrium, in which high quality sellers price high with probability 1 and low quality sellers sometimes mimic high quality sellers. To constitute an equilibrium, behavior on both sides of the market should be a best response.

**Result #3:** Seller prices are a best response to buyer demands.

Looking first at high quality sellers, for pricing high to be a best response, it must be that

$$\frac{d_{p_H}^{\theta_H}}{d_{p_L}^{\theta_H}} > \frac{p_L}{p_H} = 0.50.$$

Indeed,  $\frac{d_{p_H}^{\theta_H}}{d_{p_L}^{\theta_H}}$  is far above this threshold, as  $d_{p_H}^{\theta_H} = 85\%$  and  $d_{p_L}^{\theta_H} = 100\%$ .

Looking at low quality sellers, for mixing over prices to be a best response, it should be that

$$\frac{d_{p_H}^{\theta_H}}{d_{p_L}^{\theta_H}} = \frac{p_L}{p_H} = 0.50.$$

Overall,  $\frac{d_{p_H}^{\theta_H}}{d_{p_L}^{\theta_H}} = 52\%$ , which appears close to a best response. In fact, for low quality sellers, pricing low is less than 2 ECU (\$0.05) from a best response.

**Result #4:** Buyer demands are a best response given rational inattention.

To determine if buyer behavior is a best response, we can look to see whether there is a cost parameter value that explains the observed choices. Looking just at the second half of rounds, the value of  $\kappa$  that

minimizes the distance between actual demands and predicted demands is 11.5. Given that the actual probability of high quality given a price of 100 is 0.8, rationally inattentive buyers with  $\kappa = 11.5$  should accept high price, low quality offers 52% of the time and high price, high quality offers 99% of the time. The actual demands, as shown in table 2, are 54% and 92% respectively.

These estimates are based on the assumption that all subjects have the same attentional cost parameter. As noted in Sims [2003], if individuals have different attentional costs, they cannot be modeled with a representative agent who has a single attentional cost parameter.

## 6.7 Revealing Changes in Attention

As shown in Caplin and Martin [2012], one way to infer attention is through consideration times. Because subjects must click a button to check quality and then click back to make their choice, the time spent looking at the information about quality can be somewhat separated from the time spent contemplating a choice. Across prices and whether or not the offer was accepted, subjects spent on average 6 seconds on the choice screen. However, as table 3 shows, the average time spent checking quality varied both by price and whether or not the offer was accepted. For a price of 100, the average times are significantly different at the 5% level using a t-test.

Table 3. Average time (in seconds) spent checking quality.

Price	Accept offer	Reject offer
50	3.8	
100	41.3	53.4

Although the theory is silent on how attention relates to consideration time, the comparative statics of consideration time have two intuitive features. First, consideration times are very short at a price of 50, which echoes the theoretical prediction of full inattention at a price of 50. Second, consideration times are 29% higher before rejecting than before accepting. This makes sense because at a price of 100 the buyer wants to minimize Type I errors: mistakenly rejecting a high quality product.

## 7 Conclusion

In this paper, I present a simple and standard sequential pricing game in which buyers are rationally inattentive to product quality, which is one of the first games where agents have attentional costs based on Shannon entropy. I show that with this assumption there are unique closed form solutions for attentional effort and seller prices for two possible types of equilibria. Finally, I test these predictions in a lab experiment in which subjects faced real attentional costs and find strong evidence of one of these equilibria. In other words, buyer

demands are consistent with rational inattention, and seller prices are consistent with best responding to rational inattentive buyers.

As an extension of this game, it would be interesting to consider what happens if the seller can influence the buyer's attentional cost parameter (as in Carlin and Manso [2011], Kalaycı and Potters [2011], and Ellison and Wolitzky [2012]) or can bias the information that is available to the buyer. Both possibilities seem realistic given the control that sellers often have over the retail environment. A complication of adding these features to the model is that the seller can communicate information through these actions, giving another channel over which the buyer must have beliefs. One solution employed in the literature is to have buyers be nonstrategic over these actions. Another possibility is to have sellers take these actions before they become aware of the quality of their product.

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## 8 Appendix

**Theorem:** For all  $\kappa$ , there exists an equilibrium (“mimic high”) where high quality sellers price high with probability 1 and low quality sellers price high with a unique probability  $\eta \in [0, 1]$ .

**Proof.** When  $\kappa = 0$ , the unique value of  $\eta$  for the mimic high equilibrium is  $\eta = 0$ . In other words, sellers separate by offering different prices. Buyers exert full attentional effort at no cost, so they can distinguish between sellers perfectly. Thus, low quality sellers have no profitable deviation to any  $\eta > 0$ , and high quality sellers make the maximal return. This is the unique value of  $\eta$  because if low quality sellers set  $\eta > 0$ , then buyers have a strict incentive to exert full attentional effort, in which case low quality sellers have a profitable deviation to  $\eta = 0$ .

Note that for  $\kappa > 0$ , there will never be separating prices because buyers would not exert any attentional effort and never buy the low quality product, giving low quality sellers an incentive to deviate up to charging the high price.

For  $\kappa > 0$ , I will examine three cases separately: (1) when uninformed buyers purchase given “pooling high”, which is when both sellers charge a high price with probability 1, (2) when uninformed buyers do not purchase given pooling high, and (3) when uninformed buyers are indifferent between purchasing or not given “pooling high”, which is when both sellers charge a high price with probability 1.

Also it will be useful to divide the space of attentional costs into four possible regions: ■

1. For  $\kappa = 0$ , separating prices ( $\eta = 0$ )
2. For  $\kappa \in (0, \kappa^*)$ , increasing mimicking with  $\kappa$  ( $\eta \in [0, 1]$ ).
3. For  $\kappa \in [\kappa^*, \kappa^{**})$ , pooling high ( $\eta = 1$ )
4. For  $\kappa \in [\kappa^{**}, \infty)$ , decreasing mimicking with  $\kappa$  ( $\eta \in [0, 1]$ ).

**Proof.** Starting with case 1, where uninformed buyers purchase given pooling high, the first three regions appear in order as  $\kappa$  increases. The threshold  $\kappa^*$  between regions 2 and 3 is the value of  $\kappa$  at which low quality sellers are indifferent between pricing low and pricing high given conditional demands for pooling high. Because region 4 is empty, the upper limit on region 3 is  $\kappa^{**} = \infty$ .

In region 2, where  $\kappa \in (0, \kappa^*)$ , the unique value of  $\eta$  in the mimic high equilibrium is

$$\eta = \frac{\lambda}{1 - \lambda} \frac{(1 - s_{pH}^1)(1 - s_{pH}^0)}{(1 - s_{pH}^1)(s_{pH}^0) + \frac{p_L}{p_H}(s_{pH}^1 - s_{pH}^0)},$$

which converges to 0 as  $\kappa$  goes to zero and to 1 as  $\kappa$  goes to  $\kappa^*$ . For this value of  $\eta$ , low quality sellers have an incentive to mix, which is only true when  $d_{p_H}^{\theta_L} = \frac{p_L}{p_H}$ . For this to hold, it must be that

$$\frac{(1 - s_{p_H}^1) \frac{\beta_{p_H} - s_{p_H}^0}{s_{p_H}^1 - s_{p_H}^0}}{(1 - \beta_{p_H})} = \frac{p_L}{p_H},$$

which implies that

$$\beta_{p_H} = \frac{s_{p_H}^0 + s_{p_H}^1 - s_{p_H}^0 s_{p_H}^1 - 1}{s_{p_H}^0 s_{p_H}^1 + s_{p_H}^0 \frac{p_L}{p_H} - s_{p_H}^1 \frac{p_L}{p_H} - s_{p_H}^0}.$$

To find  $\eta$ , it just remains to note that the interim probability that a seller is of high quality at the high price is

$$\beta_{p_H} = \frac{\lambda}{\lambda + \eta(1 - \lambda)}.$$

To show that no other value of  $\eta$  supports the mimic high equilibrium in this region, it is enough to show the existence of a single crossing property for  $\frac{p_L}{p_H}$  and  $d_{p_H}^{\theta_L}$  as  $\beta_{p_H}$  increases. When  $\kappa \in (0, \kappa^*)$ , if  $\beta_{p_H} = \lambda$ , then  $d_{p_H}^{\theta_L} < \frac{p_L}{p_H}$ . Also, for some  $\beta_{p_H} > \lambda$ ,  $d_{p_H}^{\theta_L} > \frac{p_L}{p_H}$ . Thus, because  $d_{p_H}^{\theta_L}$  is strictly increasing in  $\beta_{p_H}$ , there exists a single  $\beta_{p_H}$  where  $d_{p_H}^{\theta_L} = \frac{p_L}{p_H}$ .

In this region,  $d_{p_H}^{\theta_L}$  is also strictly increasing with the cost parameter, so the distance to the crossing point decreases with cost. This shows why there is a decrease in mimicking as cost rises in this region.

For  $\kappa \in [\kappa^*, \infty)$ , both types charge a high price, so that  $\eta = 1$ . In this region,  $\beta_{p_H} = \lambda$ , so  $d_{p_H}^{\theta_L} > \frac{p_L}{p_H}$ . As a result, neither low nor high quality sellers will deviate to charging a lower price.

Case 2 is much like case 1, except that there is also a region 4 of the equilibrium, and regions 3 and 4 can be empty. If region 3 is nonempty, then the threshold  $\kappa^*$  between regions 2 and 3 is the lower value of  $\kappa$  at which low quality sellers are indifferent between pricing low and pricing high given conditional demands for pooling high and  $\kappa^{**}$  is the upper value (if just one value then  $\kappa^* = \kappa^{**}$ ). If low quality sellers always prefer pricing low given conditional demands for pooling high, then region 3 is empty.

In case 3, just regions 1 and 2 are nonempty, so that  $\kappa^* = \infty$ . This is because  $d_{p_H}^{\theta_L}$  is increasing in cost and reaches  $\frac{p_L}{p_H}$  in the limit as  $\kappa$  goes to  $\infty$ . ■