On Manipulation from an Unacceptable Social Choice to an Acceptable One (Preliminary Version)

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Abstract

Non-manipulability in collective decision making problems has been analyzed mainly through the axiom of strategy-proofness. In this paper, we propose a new concept of non-manipulability. We postulate that each agent misreports his preferences if and only if the misrepresentation leads to a change of the social outcome from an unacceptable one for this agent to an acceptable one. For the formulation of this idea the preference-approval framework is used. Possibility and impossibility results for the existence of a non-manipulable rule are provided.

Keywords: non-manipulability, strategy-proofness, preference-approval

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1 Introduction

In collective decision making problems, non-manipulability of the aggregation rules has been widely studied in the literature.[1] *Strategy-proofness*, one of the central axioms in the theory of social choice, leads to impossibility results in most environments.

For example, by the Gibbard–Satterthwaite theorem (Gibbard [10] and Satterthwaite [12]) we know that on the universal domain of preferences, each *strategy-proof* social choice function whose range contains more than two alternatives is dictatorial. Therefore, the standard notion of *strategy-proofness* does not lead to any normatively appealing rule which is robust to misrepresentation. In other words, *strategy-proofness* does not serve as a practically useful criterion of the robustness to misrepresentation.

We propose a new non-manipulability condition by using approval notion rather than rankings. We postulate that each agent manipulates the social outcome if and only if the social outcome changes from an unacceptable one for himself to an acceptable one. Since the standard model of social choice does not contain this type of binary evaluation, we use a framework which has richer informational content than the standard Arrovian framework.

In the recent literature of Voting Theory, there are some new models of aggregation mechanisms which use different formulations of inputs as individuals' messages in balloting procedures. For example, *Approval voting* ([6], [5]) allows voters to express themselves through two labels as approved or not approved. *Majority judgement* [1] method extends this freedom of expression to seven labels as Finally, *Range Voting* [16] asks voters to provide a numerical score for the candidates within a fixed interval such as 0-100. Non-manipulability of these rules are also investigated in the literature as in [3], [1]. However, the difference between

[1]For an eloquent survey for the related literature, one can check Barbera [2].

the standard and nonstandard settings in the formulation of inputs asks for further analysis of issue.

Brams and Sanver [7] suggest a model by combining the standard ordinal world of rankings with evaluation through approval in a hybrid system called *preference-approval.* Each agent is assumed to have an ordering on a given set of alternatives and a cut-off line to distinguish acceptable and unacceptable alternatives for them. An alternative which is ranked above (resp. below) the line is qualified as acceptable (resp. unacceptable).

We use the preference-approval model as a basis for our study and we investigate the existence of of non-manipulable aggregation rules in this framework.

Our results contribute to two lines of research. One is about non-manipulability of social choice rules and the other is about non-standard formulations of agents' characteristics. In the literature, various paths can be noticed for the non-manipulability analysis. One of them, as in Sato [14, 15], analyses different notions of non-manipulability in Arrovian framework and provides further examination of *strategy-proofness.*. Another stream, investigates the manipulability under some domain conditions, such as *dichotomous preferences* and for specific rules as in In this study, we attempt to investigate a new notion of non-manipulability, with a domain condition in preference-approval framework.

This paper is organized as follows. Section 2 introduces the basic notion and definitions. Section 3 is devoted to the analysis of non-manipulability of preference-approvals. Finally, Section 4 includes concluding remarks.

2 Basic notions and definitions

We consider a set of agents $N = \{1, \ldots, n\}$ with $n \geq 2$ confronting a finite set of alternatives X where $|X| = m \geq 3$. By 2^X we denote the set of all subsets of X.

We write $\mathcal{L}$ for the set of linear orders [2] on X. For each $R \in \mathcal{L}$, we denote the strict part of R by P. Finally, for each $R \in \mathcal{L}$ and each $k \in \{1, \ldots, m\}$, we write $r_k(R)$ for the kth ranked alternative in R.

Now, we introduce the primitive of our model, namely *preference-approval* which incorporates hybrid information of ordinal rankings and the approval notion.

2.1 Preference-approval framework

We consider a framework in which each agent not only ranks the alternatives in X by means of a linear order but also evaluates each alternative as either acceptable or unacceptable.[3]

We provide the formal definition of a preference-approval, as the following.

Definition 2.1

A *preference-approval* is a pair $p = (R, A) \in \mathcal{L} \times 2^X$ satisfying the following condition

$$\forall x, y \in X \ \big((x\ R\ y \text{ and } y \in A) \ \Rightarrow \ x \in A\big).$$

Let $U = X \setminus A$.

We interpret A as the set of *acceptable* alternatives and U as the set of *unacceptable* alternatives. So, the above condition says that if an alternative is approved, all alternatives preferred to this alternative should be approved as well. Similarly we have if $x\,R\,y$ and $x \in U$, then $y \in U$.

One can note that, with this definition, we embed the notion of approval to the primitives of the individual preferences. Therefore, strategic behavior of "approving an alternative in an approval ballot" (or in any other aggregation rule which uses this notion) becomes a different issue from the evaluation of the alternatives during formation of the "preference-approval"of an individual. An analogy would

[2] A linear order is a complete, transitive, and antisymmetric binary relation.

[3] We interchangeably use the terms "approved","acceptable", "eligible", "appropriate", and so on.

be the difference between the strategic behavior of providing a ranking of alternatives in an election method and the primitives as standard preferences (linear or weak orders over alternatives).

We denote a profile of preference-approvals by $\boldsymbol{p} = (p_1, \ldots, p_n)$ where $p_i = (R_i, A_i)$ is a preference-approval of agent i.$\mathcal{A}$ denotes the set of all preference-approvals.

Considering misreprestation, when $p_i \in \mathcal{A}$ in $\boldsymbol{p} \in \mathcal{A}^N$ is replaced by $p'_i \in \mathcal{A}$, we write $(p'_i, \boldsymbol{p}_{-i})$ for the new profile.

$\mathcal{D} \subseteq \mathcal{L}$ denotes the set of *admissible preferences* and we write $\mathcal{P}(\mathcal{D}) = \{(R_i, A_i) \in \mathcal{A} \mid R_i \in \mathcal{D}\}$ for the set of admissible preference-approvals. Interchangeably, we write $\mathcal{P}$ for $\mathcal{P}(\mathcal{D})$ when the meaning is clear.

Providing the basic model, next we discuss aggregation rules defined for "preference-approval" profiles and we propose our notion of non-manipulability in the following part.

2.2 Preference-approval aggregation

We consider single-valued functions defined over preference-approval profiles. For each $\mathcal{D} \subseteq \mathcal{L}$, a *rule* is a mapping f from $\mathcal{P}(\mathcal{D})^N$ into X. Let f be our generic notation for a rule.

We say that a rule f is *approval-invariant* if for each $\boldsymbol{p}, \boldsymbol{p}' \in \mathcal{P}^N$ such that $R_i = R'_i$ for each $i \in N$, we have $f(\boldsymbol{p}) = f(\boldsymbol{p}')$. So, an approval-invariant rule depends only on the linear orderings part of preference-approvals and ignore the positions of approval thresholds.

We call an agent i as *decisive for* $x \in X$ if for each $\boldsymbol{p} \in \mathcal{P}^N$ such that $A_i = \{x\}$, $f(\boldsymbol{p}) = x$. Agent i is *decisive* if he is decisive for each alternative. So an agent who is not approving any alternative cannot be decisive for the outcome of the rule, which will be compatible with the notion of non-manipulability we work in this paper.

Furthermore, we define the following standard axioms for our framework.

- **Efficiency.** For each distinct pair $x, y \in X$ and each $\boldsymbol{p} \in \mathcal{P}^N$ such that $x \; R_i \; y$ for each $i \in N$, we have $f(\boldsymbol{p}) \neq y$.

  So, *efficiency* simply means that when an alternative is dominated by another alternative for every agents, then the dominated one cannot be the social outcome.

- **Unanimity.** For each $x \in X$ and each $\boldsymbol{p} \in \mathcal{P}^N$ such that $r_1(R_i) = x$ for each $i \in N$, we have $f(\boldsymbol{p}) = x$.

  Unanimity, as a weaker condition than *efficiency*, means that when the top ranked alternatives are the same for every agent, the rule respects this agreement.

- **Anonymity.** For each $\boldsymbol{p} \in \mathcal{P}$ and each permutation π of N, we have $f(\boldsymbol{p}) = f(\boldsymbol{p}')$, where $p'_i = p_{\pi^{-1}(i)}$ for each $i \in N$.

  Anonymity simply means that the agents are treated symmetrically and name-tags of them should not matter.

Now, we introduce our notion of manipulability in the preference-approval framework.

Definition 2.2

A rule is **manipulable** if there are $\boldsymbol{p} \in \mathcal{P}^N$, $i \in N$, and $p'_i \in \mathcal{P}$, such that

$$f(\boldsymbol{p}) \notin A_i \;\&\; f(p'_i, \boldsymbol{p}_{-i}) \in A_i.$$

We say thay a rule is **non-manipulable** if it is not manipulable.

According to the above formulation, each agent i manipulates the social outcome if and only if he can change the social outcome from an unacceptable one for himself to an acceptable one.

In this sense, one can note that the above definition of *nonmanipulability* is quite different than the standard notion of *strategy-proofness* which would be defined in this framework as for each $\boldsymbol{p} \in \mathcal{P}^N$, each $i \in N$, and each $p_i' \in \mathcal{P}$, $f(\boldsymbol{p})\ R_i\ f(p_i', \boldsymbol{p}_{-i})$. On the other hand, we will show the relation between these two notions in Section 3, where we provide our results.

2.3 Circular domains

In this section, we introduce a domain condition, which is first proposed by Sato [13].

A set of preferences is called a *circular domain* if the alternatives can be arranged on a circle so that for every alternative on the circle, we have two preferences in the domain in which this alternative is top ranked, and additionally, the second ranked alternative in one of these preference is the bottom ranked in the other one and the bottom ranked alternative in the considered preference is the second ranked in the other one.

Formally, we say the following:

Definition 2.3

$\mathcal{D} \subseteq \mathcal{L}$ is *circular* if the alternatives can be indexed $x_1, x_2, \ldots, x_m$ so that for each $k \in \{1, \ldots, m\}$, there exist two preferences R and R' in $\mathcal{D}$ such that

1. $r_1(R) = x_k$, $r_2(R) = x_{k+1}$, $r_m(R) = x_{k-1}$,
2. $r_1(R') = x_k$, $r_2(R') = x_{k-1}$, and $r_m(R') = x_{k+1}$.

(Let $x_{m+1} = x_1$ and $x_0 = x_m$.) $\mathcal{P}(\mathcal{D})$ is *circular* if $\mathcal{D}$ is *circular*.

It is important to note that this condition is a restriction for only the linear order part of preference-approvals.

Example: For a set of three alternatives, $\{x_1, x_2, x_3\}$, the *minimal circular domain* would be the following set of preferences where the most preferred alternative is written as the leftmost one:

$\{x_1x_2x_3, x_1x_3x_2, x_2x_1x_3, x_2x_3x_1, x_3x_1x_2, x_3x_2x_1\}$

By Sato [13], we know that on any circular domain, any strategy-proof and unanimous social choice function should be dictatorial. So, a natural question is whether the above result extends to the preference-approval framework with the non-manipulability notion that we use in this paper.

Before investigating this question in the next section, we note some properties of circular domains.

- The universal domain is a circular domain.
- The minimal circular domains consist of $2n$ preferences since each alternative should be top ranked in at least two distinct preferences.
- One of the necessary conditions for a domain $\mathcal{D}$ to be circular is that for every $x \in X$, there exists $y \in X$ such that $r_1(R) = x, r_2(R) = y, r_1(R') = x$, and $r_m(R') = y$ for some $R, R' \in \mathcal{D}$. If we cannot find such y for some x, then the domain cannot be circular.

3 Results

We present possibility and impossibility results on constructing *nonmanipulable* rules.

First, we show that for each approval-invariant rule on each domain, our *non-manipulability* definition is logically equivalent to *strategy-proofness*.

Theorem 3.1

Let $\mathcal{D} \subseteq \mathcal{L}$. Let f be an approval-invariant rule on $\mathcal{P}(\mathcal{D})^N$. Then, f is nonmanipulable *if and only if it is* strategy-proof.

Proof. It is trivial to show that *strategy-proofness* implies *non-manipulability*. Thus, we will show the only if part,
non-manipulability implies *strategy-proofness*. We prove the contrapositive. So, assume that f violates *strategy-proofness*. Then, there exist $\boldsymbol{p} \in \mathcal{P}^N$, $i \in N$, and $p_i' \in \mathcal{P}$ such that $f(p_i', \boldsymbol{p}_{-i})\ P_i\ f(\boldsymbol{p})$. Let $p_i^* = (R_i^*, A_i^*) \in \mathcal{P}$ be such that $R_i^* = R_i$ and $f(p_i', \boldsymbol{p}_{-i}) \in A_i^*$, and $f(\boldsymbol{p}) \in U_i^*$. For f is approval-invariant, we have $f(\boldsymbol{p}) = f(p_i^*, \boldsymbol{p}_{-i})$. Then, we get $f(p_i^*, \boldsymbol{p}_{-i}) \in U_i^*$ and $f(p_i', \boldsymbol{p}_{-i}) \in A_i^*$ implying that f is manipulable. ■

By Theorem 3.1, since approval-invariant rules are as the standard rules of Arrovian framework, the Gibbard–Satterthwaite theorem implies that only dictatorship is *nonmanipulable* on $\mathcal{A}^N$. Hence, for a *nonmanipulable* and *nondictatorial* rule on $\mathcal{A}^N$, one has to investigate among rules that essentially depend on position of the cut-off lines between acceptable and unacceptable alternatives.

To state differently, for a positive result of non-manipulable rules, approval information should be taken into account for the rules under consideration.

Our next result shows an example of such a rule.

Theorem 3.2
Let $n \geq 3$ and $R, R', R'' \in \mathcal{D}$ be such that the top ranked alternatives in these preferences are distinct from each other. On $\mathcal{P}(\mathcal{D})$, there exists an efficient *and* nonmanipulable *rule under which no agent is decisive.*

Proof. Let $\boldsymbol{p} \in \mathcal{P}^N$. We consider the following steps for constructing the rule.

STEP 1: If $\bigcap_{i=1}^n A_i \neq \emptyset$, let $f(\boldsymbol{p})$ be any *efficient* alternative[4] in $\bigcap_{i=1}^n A_i \neq \emptyset$. If $\bigcap_{i=1}^n A_i = \emptyset$, proceed to the next step.

[4]Given a profile $\boldsymbol{p}$, an alternative x is *efficient in* $Y \subset X$ if there is no $y \in Y$ such that $y\ R_i\ x$ for each $i \in N$.

STEP k ($1 < k < n-1$): If $\bigcap_{i=k}^{n} A_i \neq \emptyset$, let $f(\boldsymbol{p})$ be any *efficient* alternative in $\bigcap_{i=k}^{n} A_i \neq \emptyset$. If $\bigcap_{i=k}^{n} A_i = \emptyset$, proceed to the next step.

STEP $n-1$: If $\bigcap_{i=n-1}^{n} A_i \neq \emptyset$, let $f(\boldsymbol{p})$ be any *efficient* alternative in $\bigcap_{i=n-1}^{n} A_i \neq \emptyset$.

If $f(\boldsymbol{p})$ is not determined after Step $n-1$, $f(\boldsymbol{p})$ is decided according to the following. If $A_i \neq \emptyset$ for some $i \in N$, let i^* be the agent with the least index such that $A_{i^*} \neq \emptyset$, and let $f(\boldsymbol{p}) = r_1(R_{i^*})$. If $A_i = \emptyset$ for each $i \in N$, let $f(\boldsymbol{p})$ be any *efficient* alternative.

CLAIM 1: f is *efficient*.

Proof of Claim 1. By construction, f always chooses *efficient* alternatives. Hence, f is *efficient*.

CLAIM 2: f is *nonmanipulable*.

Proof of Claim 2. Let $\boldsymbol{p} \in \mathcal{P}^N$. Assume that the social choice is determined at Step $k \in \{1, \ldots, n-1\}$. Then, for each $i \in \{k, \ldots, n\}$, $f(\boldsymbol{p})$ is acceptable for agent i. Thus, agent i does not have an incentive to lie. Let $i \in \{1, \ldots, k-1\}$. Let $p'_i = (R'_i, A'_i) \in \mathcal{P}$. The social choice changes only if A'_i is such that $A'_i \cap \bigcap_{j=i+1}^{n} A_i \neq \emptyset$.

Let $B = A'_i \cap \bigcap_{j=i+1}^{n} A_i$. Since $A_i \cap \bigcap_{j=i+1}^{n} A_i = \emptyset$, $B \subset X \setminus A_i$. Thus, $f(p'_i, \boldsymbol{p}_{-i}) \in X \setminus A_i$.

In the remaining case where $A_{n-1} \cap A_n = \emptyset$, we can see that each agent does not have an incentive to lie.

CLAIM 3: There is no decisive agent under f.

Proof of Claim 3. Let $x, y, z \in X$ denote distinct alternatives which are top ranked at some preference relation in $\mathcal{D}$. (Such x, y, z exist by the assumption.)

Let $\boldsymbol{p} \in \mathcal{P}$ be such that $A_i = \{x\}$ for each $i \in \{1, \ldots, n-2\}$, and $A_{n-1} = A_n = \{y\}$. Then, $f(\boldsymbol{p}) = y$. Thus, each $i \in \{1, \ldots, n-2\}$ is not decisive.

Let $\boldsymbol{p}' \in \mathcal{P}$ be such that $A'_i = \{x\}$ for each $i \in \{1, \ldots, n-2\}$, $A'_{n-1} = \{y\}$, and $A'_n = \{z\}$. Then, $f(\boldsymbol{p}') = x$. Thus, neither agent $n-1$ nor agent n is decisive. ■

Under the rule f constructed in the proof of Theorem 3.2, an agent with a larger index is treated better than those with smaller indices.

For example, let $n = 10$, and $p_1 = p_2 = \cdots = p_8$ be such that $A_1 = A_2 = \cdots = A_8 = \{y\}$, and $p_9 = p_{10}$ be such that $A_9 = A_{10} = \{x\}$. Then, $f(\boldsymbol{p}) = x$. In this sense, f doesn't satisfy an equal treatment of the agents.

The next result shows that the agents cannot be treated equally under each *efficient* and *nonmanipulable* rule when n is even and $\mathcal{P}(\mathcal{D})$ is circular.

Theorem 3.3

Assume that n is even and $\mathcal{P}$ is circular. Then, there is no anonymous, efficient, *and* nonmanipulable *rule on $\mathcal{P}^N$.*

Proof. Let f be an *anonymous*, *efficient*, and *nonmanipulable* rule on $\mathcal{P}^N$. Let $\{N_1, N_2\}$ be a partition of N such that $|N_1| = |N_2|$. Assign a number from 1 to m to each alternative so that it makes $\mathcal{P}$ circular.

CLAIM 1: It is impossible that both N_1 and N_2 are decisive.

Proof of Claim 1. It is easy to derive a contradiction when N_1 and N_2 are both decisive. □

CLAIM 2: Neither N_1 nor N_2 is decisive.

Proof of Claim 2. Suppose that one of N_1 and N_2 is decisive. Without loss of generality, assume that N_1 is decisive. We claim that N_2 is also decisive. Let $x \in X$

Table 1: Profiles of preference-approvals

	$\boldsymbol{p}$		$\boldsymbol{p'}$		$\boldsymbol{p''}$	
	N_1	N_2	N_1	N_2	N_1	N_2
Best	x_k	$[x_{k+1}]$	x_k	$[x_{k+1}]$	x_k	$[x_{k+1}]$
2nd	$[x_{k+1}]$	x_{k+2}	$[x_{k+1}]$	x_k	x_{k-1}	x_k
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Worst	x_{k-1}	x_k	x_{k-1}	x_{k+2}	$[x_{k+1}]$	x_{k+2}

and $\boldsymbol{p} \in \mathcal{P}$ be such that $A_i = \{x\}$ for each $i \in N_2$. Let π be a permutation of N such that for each $i \in N_1$, $\pi(i) \in N_2$. By *anonymity*, $f(\boldsymbol{p}) = f(\pi(\boldsymbol{p}))$. Since N_1 is decisive, $f(\pi(\boldsymbol{p})) = x$. Thus, $f(\boldsymbol{p}) = x$. This implies that N_2 is decisive for x. Since x was arbitrary, N_2 is decisive. Therefore, both N_1 and N_2 are decisive, which is a contradiction to Claim 1. $\square$

CLAIM 3: For each $k \in \{1, \dots, m\}$, either N_1 is decisive for x_k or N_2 is decisive for x_{k+1}.

Proof of Claim 3. The following arguments are modification of those by Sato (2010). Let $x_k \in X$. Assume that agent N_1 is not decisive for x_k. Then, at $\boldsymbol{p}$ in Table 1,[5] $f(\boldsymbol{p}) \neq x_k$. By *efficiency*, $f(\boldsymbol{p}) = x_{k+1}$. Next, consider $\boldsymbol{p'}$ in Table 1. (For each $i \in N_1$, $p_i = p'_i$.) By *nonmanipulability*, $f(\boldsymbol{p'}) = x_{k+1}$. Finally, consider $\boldsymbol{p''}$. (For each $i \in N_2$, $p'_i = p''_i$.) By *efficiency*, $f(\boldsymbol{p''}) \in \{x_k, x_{k+1}\}$. Since $f(\boldsymbol{p''}) = x_k$ is a contradiction to *nonmanipulability*, we have $f(\boldsymbol{p''}) = x_{k+1}$. By *nonmanipulability*, N_2 is decisive for x_{k+1}. $\square$

CLAIM 4: Either N_1 is decisive or N_2 is decisive.

Proof of Claim 4. For each $x_k \in X$, either N_1 is decisive for x_k or N_2 is decisive

[5] In Table 1, the horizontal lines between alternatives represent a boundary between the acceptable and the unacceptable range. The alternative between the brackets is a social outcome at each profile.

for x_k. (If not, then by Claim 3, N_1 is decisive for x_{k-1} and N_2 is decisive for x_{k+1}. However, this cannot be the case.) Let $x \in X$. Then, either N_1 is decisive for x or N_2 is decisive for x. Consider the former case. Let $y \in X \setminus \{x\}$. Then, either N_1 or N_2 is decisive for y. Since N_2 cannot be decisive for y, N_1 is decisive for y. This implies that N_1 is decisive. By similar arguments, when N_2 is decisive for x, N_2 is decisive. □

Clearly, Claim 4 is a contradiction to Claim 2. ■

When $n = 2$, the impossibility in Theorem 3.3 disappears if *efficiency* is replaced by *unanimity*.

Proposition 3.1

Assume $N = \{1, 2\}$. *There is an* anonymous, unanimous, *and* nonmanipulable *rule.*

Proof. Let $x^* \in X$ be fixed in the following. Let $\boldsymbol{p} \in \mathcal{P}^N$.

CASE 1: There is $x \in X$ such that $r_1(R_i) = x$ for each $i \in N$. Let $f(\boldsymbol{p}) = x$.

CASE 2: $A_1 \cap A_2 \neq \emptyset$. Let $f(\boldsymbol{p})$ be any efficient alternative in $A_1 \cap A_2$.

CASE 3: One of A_1 and A_2 is empty and the other is nonempty. Let $A_i \neq \emptyset$. Then, let $f(\boldsymbol{p}) = r_1(R_i)$.

CASE 4: Cases 1 through 3 do not apply. Let $f(\boldsymbol{p}) = x^*$.

For each $\boldsymbol{p} \in \mathcal{P}^N$, check from Case 1 to Case 4, and determine $f(\boldsymbol{p})$ according to the first case to which $\boldsymbol{p}$ can be applied. Then, the rule f is *anonymous*, *unanimous*, *nonmanipulable*.

Since *anonymity* and *unanimity* of f are clear, we prove *nonmanipulability*. Let $\boldsymbol{p} \in \mathcal{P}^N$. If one of Cases 1, 2, and 3 determines $f(\boldsymbol{p})$, then it is clear that each agent does not have an incentive to lie. Thus, assume that $f(\boldsymbol{p})$ is determined by Case 4. Since the Cases 1 through 3 are not applicable, either $A_1 = A_2 = \emptyset$ or [$A_1 \neq \emptyset$ and $A_2 \neq \emptyset$ and $A_1 \cap A_2 = \emptyset$]. In the former case, manipulation never occurs. Consider the latter case. Consider agent 1. If $x^* \in A_1$, then he has no incentive to lie. Assume $x^* \notin A_1$. To change the social choice, he has to report $p'_1 \in \mathcal{P}$ such that one of Cases 1, 2, and 3 holds. However, in each case, $f(p'_1, p_2) \in A_2$. Since $A_1 \cap A_2 = \emptyset$, such p'_1 is not a profitable misrepresentation. By similar arguments, it can be seen that agent 2 does not have an incentive to lie. Thus, f is *nonmanipulable*. ■

4 Concluding remarks

We analyze a type of manipulation in the preference-approval framework such that each agent i manipulates the social outcome if and only if he can change the social outcome from an unacceptable one to an acceptable one. We show that according to this definition, under some mild domain assumption, there exists an *efficient* and *nonmanipulable* rule under which no agent is decisive. However, when the number of the agents is even, we cannot have an *anonymous*, *efficient*, and *nonmanipulable* rule on each circular domain. For further research, it would be interesting to characterize the set of efficient and *nonmanipulable* rules in preference-approval framework.

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