Price Dispersion and Private Uncertainty

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Abstract

This paper shows that the introduction of an arbitrarily small degree of price dispersion, in an otherwise fully-revealing system of prices, can originate large departures from the perfect-information benchmark. The main result is presented within a fully microfounded model where agents learn from prices and all disturbances are fundamental in nature. When the system of prices is fully revealing the economy has a unique equilibrium. Nevertheless, the introduction of vanishing idiosyncratic disturbances, which blur the informativeness of local prices, generates two equilibrium outcomes. Only one inherits by continuity the properties of the perfect-information benchmark, whereas the other features sizeable heterogeneity of beliefs due to the amplification of private uncertainty through price feedbacks. The two dramatically differ in the impact of shocks at both aggregate and cross-sectional levels. Moreover, when higher-order belief dynamics is used as a selection criterion, the perfect-information limit scenario is discarded whereas the dispersed-information limit outcome prevails.

Keywords: learning from prices, expectational coordination, diverse beliefs.

JEL: D82, D83, E3, J3.

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"The mere fact that there is one price for any commodity - or rather that local prices are connected in a manner determined by the cost of transport, etc. - brings about the solution which (it is just conceptually possible) might have been arrived at by one single mind possessing all the information which is in fact dispersed among all the people involved in the process." Hayek F. A. (1945).

1 Introduction

How do competitive prices transmit information? An old tradition in economics celebrates the competitive price system as the invisible hand which aggregates and disseminates information into the economy (Hayek, 1945). An equally important stream of literature (Grossman and Stiglitz, 1980; Vives, 1988) has challenged this view, arguing that decentralized markets cannot generally ensure a socially efficient acquisition and use of private information.[1]

Nevertheless, no argument seems to made a dent in the widespread conviction that the full-revelation scenario is a robust benchmark, meaning that, any marginal perturbation of a system of fully-revealing prices can only lead to negligible departures away from the perfect-information outcome. The quotation above suggests that the perfect-information scenario well approximates the case where agents deal with local prices which are sufficiently homogeneous to smooth out differences in beliefs about unobservable common factors. According to this view, private uncertainty about aggregate conditions could only have a minor role in the explanation of macro outcomes as long as agents can trade prices which are closely uniform.

This paper draws the limits to this conjecture, emphasizing instead the pervasive effect of endogenous private uncertainty. As sand in the wheels, even a vanishing degree of price dispersion can disrupt the transmission of information through prices, and dramatically alter the economic predictions of a canonical model.

I study an economy where firms have to forecast a sale price which co-moves with an aggregate taste shock, while they observe island-specific prices of their inputs - local capital and labor - which also reflect idiosyncratic fundamentals. The local wage coveys information about an island-specific preference shock, which is nothing else than an *exogenous private signal* of the aggregate one. Local capital instead is produced by intermediaries using an endowment traded across islands. The price of the local capital constitutes an *endogenous private*

[1] Recent studies emphasizing this divide are Cespa and Vives (2012) and Hellwig and Venkateswaran (2011).

signal[2], being equal to the price of the endowment plus an island-specific productivity shock that hits intermediaries. Without productivity disturbances local prices for capital are homogeneous and fully reveal the aggregate unobservable shock to firms. In this case a unique equilibrium exists. With stochastic production instead, firms are confused on whether a change in local prices is due to aggregate or island-specific fluctuations.

The main result of the paper demonstrates that arbitrarily small variance of the productivity shocks, yielding a vanishing dispersion of local prices for capital, originates three rational expectation equilibrium. Nevertheless, only one inherits by continuity all the features of perfect-information benchmark. The other two instead, which characterize the same limit outcome, are featured by a finite precision of the information transmitted by local prices and sizeable cross-sectional variance of beliefs. This radically changes the impact of aggregate and market-specific shocks into the economy.

At the perfect-information limit equilibrium all the average prices move together at the same rate; the aggregate preference shock induces an uniform inflationary effect, leaving average real quantities unchanged. At the dispersed-information limit equilibria instead, prices are distorted by a critical tension between their allocative and informative roles. In response to an aggregate taste shock, the price for consumption overreacts and local wages underact, whereas the price of the endowment remains constant. Moreover, average production and working hours go down and the price for consumption goes up. That is, a positive aggregate preference shock mimics the effects of a negative aggregate supply shock.[3]

At the perfect-information limit equilibrium also local wages are equalized across islands, hence price dispersion is actually null and idiosyncratic taste disturbances only reflect in quantity setting. At the dispersed-information limit equilibria instead, despite local prices for capital are closely uniform, the local wage does react to island-specific preference shocks with an intensity which decreases with their variance. This is the opposite of what implied by the rational inattention approach, as modeled for example by Mackowiack and Wiederholt (2009), where typically agents pay more attention to more volatile disturbances. Learning from prices predicts a different pattern. The higher the cross-sectional heterogeneity of the labor market, the lower is the information transmitted by the local labor market, and so their impact on firms' beliefs. As a conse-

[2]The signal is endogenous to general equilibrium relations, but it is taken as given by the individual agent. This is different from the literature on information acquisition (see for example Sims (2003) and Hellwig and Veldkamp (2008)) where the precision of the information is endogenous to the individual choice.

[3]This result reverses a typical finding of a recent literature on news (see Lorenzoni (2009) and Jaimovich and Rebelo (2009) among others) where, due to informational frictions, a future technological shock shows up as having short-run demand-side effects. A similar channel is emphasized by Benhima (2013) in a unique equilibrium model where a double signal extraction problem generates a boom and bust dynamics.

quence firms' reliance on the information generated by the local capital market increases. However, the simultaneous trade of local capital generates price feedbacks that amplify private uncertainty across islands. Hence, shocks with infinitesimal variance can generate sizeable disagreement across firms, which finally explains a considerable fraction of supply-side cross-sectional moments[4], notably wage dispersion. This would pose serious challenges to an external observer trying to laid out the fundamental or frictional factors driving cross-sectional heterogeneity, as documented for example by Hornstein, Krusell and Violante (2011).

Dispersed-information equilibria exist when price signals are highly reactive to the average expectation. This condition makes the out-of-equilibrium dynamics around the perfect-information equilibrium follow a *local* divergent path. In fact, only when price signals are very precise firms' expectations become highly reactive to them, allowing for a circle of high complementarity. Nevertheless, a second-order effect becomes dominant as the average reaction to local price further increases. As firms put more weight on price signals their informativeness will eventually lower so that dispersed-information limit equilibria emerge. Such mechanism is in play soon as the full-revelation scenario is relaxed by even a small amount.

In the model, the real rigidity introduced by a fix supply of endowment makes its price - and so the prices for local capital - highly reactive to expectations. In particular, independently of the specific calibration of deep parameters, the price of the endowment responds in opposite ways to an aggregate shock in the two extreme scenarios of no information[5] and perfect foresight. In the first case, firms cannot be confident in their price predictions so that, when they see their local wage rising they just demand less labor and, by complementarity, less capital. As a consequence the price for the endowment goes down to clear a fixed supply. Without productivity shocks instead, an increase in the prices of local inputs inform about a rising consumption price and so firms demand more capital. As a consequence the price for the endowment goes up one-for-one with the aggregate shock, as all the other prices. The high elasticity of the price for local capital to the average expectation makes a dispersed-information limit equilibria arise with sufficiently small dispersion of productivity shocks.

To the best of my knowledge this is the first study demonstrating that the continuity of the price correspondence can fail because of the introduction of private signals alone, no matter how small is their dispersion. On the contrary, most of the Global Games literature points on the opposite direction: private uncertainty works against multiplicity. In an influential paper Morris and Shin (1998, 2000) demonstrate that arbitrarily small private uncertainty about the fundamentals leads to a unique equilibrium in an otherwise multiple-equilibrium

[4]The study by Boivin, Giannoni and Mihov (2009) documents that most of price volatility accounts for market-specific rather than for aggregate fluctuations.

[5]That is at the limit of infinite cross-sectional variance of local prices.

model. In fact, this possibility relies on the exogenous nature of the information structure. When market transactions generate sufficiently informative global prices, then the original multiplicity is restored (Angeletos and Werning (2006), and Hellwig, Mukherji and Tsyvinski (2006)). This is a good example of a case where the model predictions drawn under perfect information are a good approximation to the case where the market aggregates private information and makes it public through prices.

In the current setup instead agents observe local prices which are private noisy observations of an underlying global price. The paper demonstrates that private - rather than public - endogenous signals can confuse agents and be a source of multiplicity in a an otherwise unique-equilibrium model. In particular, in a neighborhood of the perfect-information equilibrium, the information dissemination process generated by prices can induce a local divergent path in higher-order beliefs leading to dispersed-information limit equilibria. This means that rational agents, reasoning on the iterated implications of common knowledge, will converge on the dispersed-information limit equilibria and *not* on the perfect-information one. In the present model therefore, private uncertainty creates new equilibria but also provides an out-of-equilibrium argument for selection which discards the perfect-information benchmark.

Unlike sunspot equilibria[6], the presence of dispersed-information limit equilibria does not require any particular condition on the payoff structure, but on the information structure only. Moreover it does not involve any extrinsic noise: all noises are fundamental and transmitted by the price system itself. In addition, rationalizability arguments generally discard sunspot solutions because associated with payoff nonconvexities or indeterminacy. In the current economy instead the equilibrium discarded is the one under perfect-information because of a local out-of-equilibrium divergence which arises, limited to the signal extraction problem, in an otherwise unique-equilibrium model that does not have any nonconvexity in the payoff structure.

This paper contributes to a vast literature on dispersed information by demonstrating the fragility of a competitive price system in the limit of full revelation.[7] Other papers have shown the possibility that multiple equilibria can be sustained by information spillovers away from the limit of perfect-information. The closest paper is Amador and Weill (2010) where a multiplicity originates[8] in the context of a fully microfounded model where agents learn from prices and all

[6]For a comprehensive discussion of sunspots and indeterminacy in macroeconomics see Benhabib and Farmer (1999). For a review of models of sunspots relying on a multiplicity of equilibria see Cooper (1999).

[7]This important stream of litearture was initiated by Lucas (1972) and Frydman and Phelps (1984). More recent key contributions are Albagli, Hellwig and Tsyvinski (2011), Amador and Weill (2010), Angeletos and La'O (2013), Lorenzoni (2009), Nimark (2011), Woodford (2001) and others reviewed for example in Hellwig (2006).

[8]The analysis of multiplicity is presented in the on-line Addendum available on the web page of the authors.

disturbances are fundamental in nature. In their case a multiplicity vanishes as the model approaches the perfect knowledge scenario, so dispersed-information limit equilibria do not exist. In Angeletos, Lorenzoni and Pavan (2010) a multiplicity arises as a by-product of a model where the interaction between the real and the financial sector amplifies non-fundamental noises. Their multiplicity hinges on a two-way beauty-contest motive sustained by a direct payoff externality. When this externality is small enough the model has a unique equilibrium. Direct payoff externalities are instead not necessary for the emergence of a multiplicity in the present framework.

Related is also Benhabib, Wang and Wen (2012) who look at a model in which a partly revealing REE can arise, in addition to a fully revealing one, when agents weight an ad-hoc endogenous signal embodying a non-fundamental component. In contrast to this paper, their multiplicity collapses on the perfect-information one in the limit of no idiosyncratic disturbances. In a follow-up paper Benhabib, Wang and Wen (2013) investigate a setting yielding a fundamental multiplicity that survives even in the case of zero cross-sectional variance of the endogenous signals. Other examples of interest are Ganguli and Yang (2009), Manzano and Vives (2011), and Desgranges and Rochon (2012) all of them find a multiplicity which vanishes for sufficiently small uncertainty (similarly to Amador and Weill (2010)) in asset pricing models. In comparable environments cases of indeterminacy of the equilibrium have been studied by Barlevy and Veronesi (2003).

2 Analysis of the signal extraction problem

This section presents the core analysis in the simplest and most abstract setting for a given information structure. The aim is to provide a sound understanding of the signal extraction problem underlying the main results of the paper. Next section will present a macromodel which microfounds each single element introduced in this section and allows a discussion in terms of macro-outcomes.

2.1 Endogenous signals

Consider a continuum of agents with unit mass. An agent $i \in (0,1)$ chooses an action x_i to solve

$$\min_{x_i \in \Re} \{\mathrm{E}[(x_i - p)^2 \,|\omega_i]\},$$

that is, $x_i = \mathrm{E}^i\,(p)$ where $\mathrm{E}^i\,(p) \equiv \mathrm{E}[p|\omega_i]$ is the expectation of agent i about a global price p, conditional on the information set ω_i available to agent i. Under this specification actions correspond to expectations. The unconstrained first-best is achieved under perfect information, when the variance of the forecast error is zero.

The price p is given by a linear combination of an aggregate shock and the average action

$$p = \varepsilon + \beta\left(\mathrm{E}\left(p\right) - \varepsilon\right), \tag{1}$$

where $\varepsilon \sim N\left(0,1\right)$ is a normally distributed exogenous shock with zero mean and unitary variance and $\mathrm{E}\left(p\right) \equiv \int \mathrm{E}^i\left(p\right)\mathrm{d}i$ is the average price expectation across agents. The impact of the average expectation on the actual outcome is measured by β. With $\beta \neq 0$, the price moves with the average expectation so that direct payoff externalities are involved. In the following I will consider $\beta < 1$ to provide results that are directly applicable to models characterized by direct payoff complementarity ($\beta > 0$) or substitutability ($\beta < 0$).[9] The case $\beta = 0$ implies no direct payoff externalities so that the price is exogenously determined by the process ε.

Suppose agents cannot directly observe (1), but instead they look at a local price which has a similar linear structure but also includes a type-specific disturbance. That is $\omega_i = \{r_i\}$ where

$$r_i = \varepsilon + \varkappa\left(\mathrm{E}\left(p\right) - \varepsilon\right) + \hat{\eta}_i, \tag{2}$$

constitutes a private signal of an underlying aggregate endogenous state - call it $r \equiv \int r_i \mathrm{d}i$ - with $\hat{\eta}_i \sim N\left(0,\hat{\sigma}\right)$ being a private white noise disturbance. If agents were able to directly observe r, the aggregate shock would be perfectly revealed. In such a case $\mathrm{E}^i\left(p\right) = p = \varepsilon$ characterizes the unique perfect information equilibrium. Nevertheless the presence of private uncertainty introduced by $\hat{\eta}_i$ poses a non trivial signal extraction problem. The relative[10] precision of the price signal is endogenous to the average expectation process. In particular, the coefficient $\varkappa$ measures the reactiveness of the private signal to a deviation of the average expectation from the perfect-information outcome ε.

The signal can be rescaled by $\varkappa$ to obtain an equivalent one

$$r_i \propto \mathrm{E}\left(p\right) + \zeta^{-1}\varepsilon + \eta_i. \tag{3}$$

The equivalence obtains defining $\eta_i \equiv \varkappa^{-1}\hat{\eta}_i \sim N\left(0,\zeta^{-2}\sigma\right)$, so that $\sigma = \left(1-\varkappa\right)^{-2}\hat{\sigma}$, and $\zeta \equiv \varkappa/\left(1-\varkappa\right)$. This notation is particularly convenient to directly enlighten two parameters of crucial importance: ζ and σ. The latter measures the variance of the private noise η_i whereas the former the covariance of the fundamental component $\zeta^{-1}\varepsilon$ with the aggregate shock ε, both expressed in terms of the variance of the fundamental component ζ^{-2}. The limit values $\sigma \to \infty$ and $\sigma \to 0$ entail the extreme situations where informational heterogeneity vanishes and agents have respectively no information and perfect information on the fundamental realization.

[9] Examples of two classical macro-models with payoff complementarities or substitutabilities are respectively Lucas (1972) and Muth (1961).

[10] Relative to the precision of the prior which is one.

The optimal forecasting strategy in the case of Gaussian signals is linear. Hence, agent i's forecast is written as

$$\mathrm{E}^{i}\left(p\right)=b_{i}\left(\mathrm{E}\left(p\right)+\zeta^{-1}\varepsilon+\eta_{i}\right), \tag{4}$$

where b_i is a constant coefficient to be determined that weights the signal type i. If all agents use the rule above then *by definition* the aggregate expectation is

$$\mathrm{E}\left(p\right)=\frac{\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon, \tag{5}$$

provided $\mathbf{b}\neq 1$ where $\mathbf{b}\equiv\int b_i \mathrm{d}i$ is the average weight across agents. Therefore an individual expectation can be rewritten as

$$\mathrm{E}^{i}\left(p\right)=b_{i}\left(\frac{1}{1-\mathbf{b}}\zeta^{-1}\varepsilon+\eta_{i}\right), \tag{6}$$

where the signal is now expressed as a function of exogenous shocks depending on the average weight. The private signal r_i provides agents with information about ε of a relative precision $\pi = 1/(1-\mathbf{b})^2\sigma$ which depends non-linearly on the average weight. In particular, it decreases with a $\mathbf{b}$ sufficiently large in absolute value. This is a property of *private* endogenous signals. This feature does not arise in the case of a public endogenous signal, that is with a common η_i instead.[11] In fact, given an homogeneous information set, agents would be able to predict the average forecast and disentangle the endogenous component from the public signal.

Plugging (5) into (1) we can now express the price p as

$$p=\varepsilon+\beta\left(\frac{\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon-\varepsilon\right), \tag{7}$$

being a function of the average weight and the aggregate shock only.

Rational expectations obtain when agents' beliefs are consistent with the actual distribution of p conditional on r_i. In other words, the forecast error of each agent has to be orthogonal to the available information. The orthogonality requirement entails what I will call the *best individual weight function*

$$b_{i}\left(\mathbf{b}\right)=\frac{\zeta(1-\mathbf{b})}{1+\sigma\left(1-\mathbf{b}\right)^{2}}+\frac{\beta\left(\mathbf{b}-\zeta\left(1-\mathbf{b}\right)\right)}{1+\sigma\left(1-\mathbf{b}\right)^{2}}, \tag{8}$$

that is the optimal individual weight given that the average weight is $\mathbf{b}$. $b_i\left(\mathbf{b}\right)$ pins down the unique strictly-dominant action in response to $\mathbf{b}$, a sufficient statistics of the profile of others' actions. A REE is characterized by a profile of weights $b_i\left(\mathbf{b}\right)=\mathbf{b}$ for each $i\in(0,1)$. I can now state a first proposition.

[11] The interested reader can easily go through the previous steps and easily check that in the case of a public signal its relative precision is independent of the average weight.

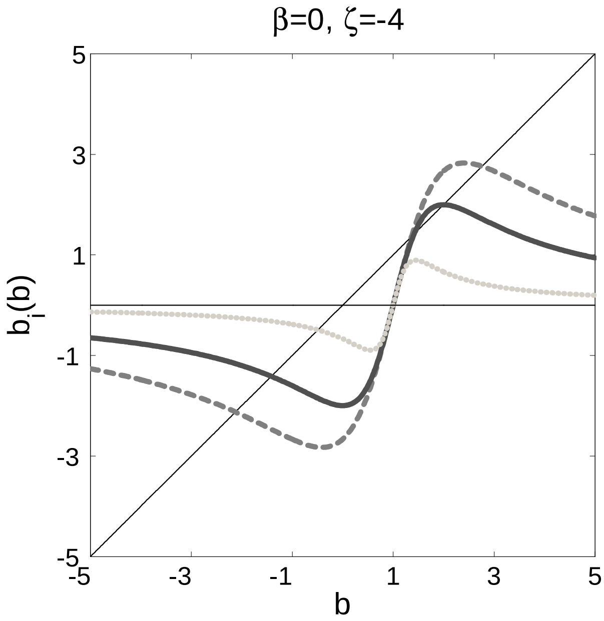

Figure 1: Plot of the best individual weight for $\beta = 0$, $\zeta = -4$ and different values of σ: 0.5 (dashed dark), 1 (solid black) 5 (dotted light).

Proposition 1 *Consider the problem of agents forecasting (1) conditionally on the signal (2). In the perfect information case $\sigma = 0$ a unique equilibrium $\mathbf{b}_* = \varkappa$ exists. For arbitrarily small values of σ, if and only if $\zeta < -1$ (that is $\varkappa > 1$), then three equilibria exist: a unique perfect-information limit equilibrium $\mathbf{b}_\circ$ which lies arbitrarily close to $\mathbf{b}_*$, and two dispersed-information limit equilibria $\mathbf{b}_+ > 0$ and $\mathbf{b}_- < 0$ such that $|\mathbf{b}_\pm| > M$ with M arbitrarily large.*

Proof. Postponed to appendix A.1. ■

A multiplicity of limit equilibria originates due to the composition of two effects. There is a first-order effect, entailed by $\varkappa > 1$, that determines a strong expectational complementarity in a neighborhood the perfect-information equilibrium: if the average weight on the price signal marginally increases then the optimal individual weight must increase even further. Nevertheless, there is a second-order effect which guarantees the emergence of dispersed-information limit equilibria: an increasingly higher average weight on the signal will eventually decrease its informativeness.

Figure 1 illustrates the two effects in the case of no direct payoff externalities. The optimal individual weight function $b_i(\mathbf{b})$ is plotted for $\zeta = -4$ (that is $\varkappa = 4/3$) and $\beta = 0$ at different values of σ namely 0.5 (dashed line), 1 (solid line) and 5 (dotted line). Equilibria lie at the intersection with the bisector. Observe that as $\sigma \to \infty$ the curve approaches the x-axis yielding a unique equilibrium close to the origin. As σ decreases the curve approaches the line $\zeta(1-\mathbf{b})$. Exactly at $\sigma = 0$, a unique equilibrium $\mathbf{b}_* = \varkappa$ exists. For future reference, also notice that the two equilibria arising respectively at the two extreme scenarios $\sigma \to \infty$ and $\sigma = 0$ cannot be obtained by continuously varying σ.

Let us focus now on the first-order effect. Consider a σ strictly positive but sufficiently small. A perfect-information *limit* equilibrium $b_i(\mathbf{b}_\circ) = \mathbf{b}_\circ$ will emerge close to the one under perfect information. The slope of $b_i(\mathbf{b})$ at that point will be $-\zeta$ which is larger than one whenever $\varkappa > 1$. This means that for an average weight $\mathbf{b}$ marginally higher (lower) than $\mathbf{b}_\circ$ the optimal individual weight will lie strictly higher (lower) than $\mathbf{b}$, above (below) the bisector.

Nevertheless a second-order effect will eventually dominate as the average reaction further increases (decreases). For a σ strictly positive, a $\mathbf{b}$ high (or low) enough will eventually decrease the informativeness of the signal so that $b_i(\mathbf{b})$ will converge to zero.

Both effects together imply that by continuity $b_i(\mathbf{b})$ must cross the bisector other two times. In particular, as σ shrinks the two extreme intersections, one positive and one negative, go respectively to $+\infty$ and $-\infty$. I have tagged the equilibria characterized by these values dispersed-information limit equilibria because, as the next proposition will demonstrate, they maintain well-defined dispersed-information features in spite of a vanishing dispersion of price signals.

An interesting observation[12] is that the existence of a multiplicity of limit equilibria does not hinge on the unboundedness of the domain of b_i, but just on the slope of the best individual weight function around the perfect-information limit equilibrium. To see this suppose that we arbitrarily restrict the support of feasible individual weights b_i in a neighborhood of $\mathbf{b}_\circ$, namely $\Im(\mathbf{b}_\circ) \equiv [\underline{\mathbf{b}}, \overline{\mathbf{b}}]$. Whenever $\varkappa > 1$ it is easy to check from a simple inspection of figure 1 that two other equilibria beyond $\mathbf{b}_\circ$ would arise as corner solutions. In fact, if all agents put a weight $\overline{\mathbf{b}}$ ($\underline{\mathbf{b}}$) on the signal then the optimal individual weight would be higher (lower) than it; therefore everybody putting the highest (lowest) feasible weight is an equilibrium.

In the case $\beta \neq 0$ - namely with direct payoff externalities - the same argument can be easily replicated. In this case the slope of the limit line is now given by $-((1-\beta)\zeta - \beta)$ which however is strictly larger than one if and only if $\zeta < -1$, that is $\varkappa > 1$. In the limit of $\sigma \to 0$, $\varkappa > 1$ is still a necessary and sufficient condition for a multiplicity.

Thus, the result stated in proposition 1 is *independent* of the nature - endogenous or exogenous - of the variable to be forecasted; it only relies on a specific condition concerning private endogenous signals, namely $\varkappa > 1$. Put differently, the analysis of the case $\beta = 0$ makes clear that the condition for multiple limit equilibria, $\varkappa > 1$, does not imply the presence of sunspots equilibria as it does not restrict the payoff structure which is instead relevant to them. In fact, in the next section I will investigate a model where typical sunspot do not exist.

[12] I thank George-Marios Angeletos for driving my attention to this point.

The perfect-information limit equilibrium inherits the properties of the equilibrium under perfect information: the precision of private information is infinite, expectations are homogeneous, and, the average expectation, the average signal and the price p all react one-for-one to the aggregate shock. The dispersed-information limit equilibria instead characterize the same limit outcome which implies a dramatically different picture.

Proposition 2 *The dispersed-information limit equilibria featured by* $\mathbf{b}_+$ *and* $\mathbf{b}_-$ *arising in the limit of* $\sigma \to 0$ *characterize the same limit outcome featuring: i) a relative precision of private information about the fundamental shock* ε *equal to*

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} \pi = \frac{\varkappa - 1}{1-\beta},$$

ii) a finite cross-sectional variance of individual expectations

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} \int \left(\mathrm{E}^i\left(p\right) - \mathrm{E}\left(p\right)\right)^2 \mathrm{di} = (1-\beta)\frac{\varkappa - 1}{\varkappa^2},$$

iii) underreaction of the average expectation to the fundamental shock

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} \mathrm{E}\left(p\right) = \frac{\varkappa - 1}{\varkappa}\varepsilon,$$

iv) underreaction with $\beta > 0$ *(overreaction with* $\beta < 0$*) of the price to be forecasted,*

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} p = \frac{\varkappa - \beta}{\varkappa}\varepsilon,$$

v) a vanishing variance of the private and the average signal

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} \int \left(r_i - r\right)^2 \mathrm{di} = 0,$$

where $\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} \bar{r} = 0$.

Proof. Postponed to appendix A.1. ∎

Point v) of proposition 2 sheds further light on the result. Notice that $\varkappa > 1$ implies that the price signals are positively correlated with the aggregate shock ε - and so with the price p - in case of full information ($E\left(p\right) = \varepsilon$ and $r = \varepsilon$), but they are negatively correlated in the opposite extreme case of no information ($E\left(p\right) = 0$ and $r = \left(1 - \varkappa\right)\varepsilon$). That is, the average signal exhibits opposite reactions to the aggregate shock in the two limit scenarios. Therefore, by continuity, it must exist a particular finite value of the relative precision of the private information about the aggregate shock such that the price of the endowment tends to zero.

Such a value is $\pi = \left(\varkappa - 1\right)/\left(1-\beta\right)$, exactly the one featured by the dispersed-information limit equilibria. At this value the rise in the average price expectation makes the average signal r have a vanishing variance. This is the only possibility to fulfill whatever relative precision at the limit of a vanishing dispersion of private signals. This possibility ceases when exactly $\sigma = 0$.

2.2 Endogenous and exogenous signals

Now let us turn attention to the robustness of the previous analysis to the case with both endogenous and exogenous private information. Agents observe (2) plus an exogenous signal $\varepsilon + \hat{\phi}$ where $\hat{\phi} \sim N(0, \sigma_\phi)$. The new information set can be written as

$$\omega_i = \{\mathrm{E}(p) + \zeta^{-1}\varepsilon + \eta_i,\ \zeta^{-1}\varepsilon + \phi_i\} \tag{9}$$

where $\phi_i \equiv \zeta^{-1}\hat{\phi}_i \sim N\left(0, \zeta^{-2}\sigma_\phi\right)$. The relative precision of the exogenous signal about ε, namely σ_ϕ^{-1}, is independent from the average expectation process. Nevertheless the overall precision π of the private information available to agents is still endogenous; it is given by σ_ϕ^{-1} plus the precision of r_i which in turn also depends on σ_ϕ^{-1}. In this case agents have two correlated pieces of information. Their optimal forecasting strategy is written as

$$\mathrm{E}^i(p) = a_i\left(\zeta^{-1}\varepsilon + \phi_i\right) + b_i\left(\mathrm{E}(p) + \zeta^{-1}\varepsilon + \eta_i\right), \tag{10}$$

where a_i and b_i are constants weighting respectively an exogenous and an endogenous signal. A REE is now characterized by a profile of weights such that $(a_i, b_i) = (\mathbf{a}, \mathbf{b})$ for each $i \in (0,1)$, where $\mathbf{a} \equiv \int a_i \mathrm{di}$ and $\mathbf{b} \equiv \int b_i \mathrm{di}$ are the average weight across agents. The following proposition states the result in this case.

Proposition 3 *Consider the problem of agents forecasting (1) conditionally on the information set (9). In the perfect information case $\sigma = 0$ a unique equilibrium $\mathbf{a}_* = 0$, $\mathbf{b}_* = \varkappa$ exists. For arbitrarily small values of σ instead, if and only if $\zeta < -1$ (that is $\varkappa > 1$) and*

$$\sigma_\phi^{-1} < \frac{\varkappa - 1}{1 - \beta}, \tag{11}$$

then three equilibria exist: a unique perfect-information limit equilibrium $(\mathbf{a}_\circ, \mathbf{b}_\circ)$ which lies arbitrarily close to $(\mathbf{a}_*, \mathbf{b}_*)$, and two dispersed-information limit equilibria $(\mathbf{a}_+, \mathbf{b}_+)$ and $(\mathbf{a}_-, \mathbf{b}_-)$ where $\mathbf{a}_\pm = (1-\beta)(1+\zeta)/\sigma_\phi$, $\mathbf{b}_+ > 0$ and $\mathbf{b}_- < 0$ such that $|\mathbf{b}_\pm| > M$ with M arbitrarily large.

Proof. Postponed to appendix A.1. ■

The proposition is linked to the following.

Proposition 4 *Consider the problem of agents forecasting (1) conditionally on the information set (9): Proposition 2 still exactly holds.*

Proof. Postponed to appendix A.1. ■

The intuition for the new condition (11) is simple. As said, the dispersed-information limit equilibria originate in the limit of $\sigma \to 0$ when the precision of the private information π about the exogenous shock ε is $(\varkappa - 1)/(1-\beta)$. The possibility to achieve this value is prevented when $\sigma = 0$, but also when the availability of exogenous signals fixes a lower bound σ_{ϕ}^{-1} to the the precision of private information above such a threshold. In fact the endogenous informational channel can eventually only increase the precision of information, but cannot in any case make agents loose information.

Last, but not least, it is trivial to prove that dispersed-information equilibria are inefficient outcomes when, as for the current specification, the social welfare is given by the variance of the forecast error on p. At the perfect-information limit equilibria such variance is zero, so that this equilibrium represents the *unconstrained* first-best. Dispersed-information limit equilibria instead feature a strictly positive variance of forecast errors and so they are Pareto dominated by the perfect-information outcome. In other words, dispersed information equilibria arise as coordination failures. Agents would be better off if they could coordinate on the perfect-information limit equilibrium. On the contrary, as shown below, common knowledge of rationality implies out-of-equilibrium convergence on the inefficient outcome.

2.3 Out-of-equilibrium selection: rationalizability

The last step before turning to the microfoundations of the information structure is to investigate the out-of-equilibrium properties of the equilibria. This analysis establishes that the local dynamics of higher-order beliefs in a neighborhood of the perfect-information equilibrium is locally unstable, whereas is locally stable around the dispersed information equilibria. Therefore rational agents reasoning on the iterative implication of common knowledge will converge on the dispersed-information limit outcome and *not* on the perfect-information one.

Importantly notice that the law of motions for the global price, signals and expectations, (4)-(7), have been obtained without guessing any a-priori form, but just using definitions and temporary equilibrium conditions. Hence these relations are still valid to describe *disequilibrium* processes, that is, they entail the course of the global price, signals and expectations, given a possibly non-optimal profile of weights. This allows to inquire how agents can possibly coordinate in higher-order beliefs on a particular outcome.

Here I provide a formal analysis which is inspired by the work on Eductive Learning by Guesnerie (1992, 2005) and has connections with the usual rationalizability argument used in the Global Games literature (Carlsson and Van Damme, 1993; Morris and Shin 1998). Eductive Learning assesses whether or not rational expectation equilibria can be selected as locally unique rationalizable outcome according to the original criterion formulated by Bernheim (1984) and Pearce (1984). The difference is that here I deal with a well-defined probabilistic structure. I consider beliefs on the average weights $(\mathbf{a}, \mathbf{b})$ characterizing the equilibria, rather than beliefs directly specified in terms of price forecasts, as generally assumed by Guesnerie in settings of perfect information.

Suppose that it is common knowledge that all the individual weights put on the signals lie in a neighborhood $\Im(\hat{\mathbf{a}}, \hat{\mathbf{b}})$ of the equilibrium characterized by $(\hat{\mathbf{a}}, \hat{\mathbf{b}})$. *Is this a sufficient condition for convergence in higher-order beliefs to $(\hat{a}, \hat{b})$?* The process of iterated deletion of never-best replies works as follows. Let index by τ the iterative round of deletion. Common knowledge of $(a_{i,0}, b_{i,0}) \in \Im(\hat{\mathbf{a}}, \hat{\mathbf{b}})$ for each i implies that the average weights $(\mathbf{a}_0, \mathbf{b}_0)$ belong to $\Im(\hat{\mathbf{a}}, \hat{\mathbf{b}})$. But since the map[13] $B : (\hat{\mathbf{a}}, \hat{\mathbf{b}}) \to (a_i, b_i)$ is also common knowledge then this implies that a second-order belief is rationally justified for which $(a_{i,1}, b_{i,1}) = B(\mathbf{a}_0, \mathbf{b}_0)$ for each i, so that $(a_{i,1}, b_{i,1}) \in B(\Im(\hat{\mathbf{a}}, \hat{\mathbf{b}}))$ and as a consequence $(\mathbf{a}_1, \mathbf{b}_1) \in B(\Im(\hat{\mathbf{a}}, \hat{\mathbf{b}}))$. Iterating the argument we have that $(\mathbf{a}_\tau, \mathbf{b}_\tau) \in B^\tau(\Im(\hat{\mathbf{a}}, \hat{\mathbf{b}}))$.

Definition 1 *A REE $(\hat{a}, \hat{b})$ is a locally unique rationalizable outcome if and only if*

$$\lim_{\tau \to \infty} B^\tau(\Im(\hat{\mathbf{a}}, \hat{\mathbf{b}})) = (\hat{\mathbf{a}}, \hat{\mathbf{b}}).$$

From an operational point of view local uniqueness requires that

$$||J(B)_{(\hat{\mathbf{a}}, \hat{\mathbf{b}})}|| < 1$$

where $J(B)_{(\hat{\mathbf{a}}, \hat{\mathbf{b}})}$ is the Jacobian of the map B calculated at the equilibrium values $(\hat{\mathbf{a}}, \hat{\mathbf{b}})$. The unidimensional case obtains replacing B with $b_i(\mathbf{b})$ so that the condition for local rationalizability becomes simply $|b_i'(\hat{\mathbf{b}})| < 1$. The following proposition states a result which holds for all the cases investigated in section 3.

Proposition 5 *The perfect-information limit equilibria is not a locally unique rationalizable outcome whereas the dispersed-information limit equilibria are.*

To get an intuition for the finding it is instructive to look back at figure 1. It is easy to see that the perfect-information limit equilibrium is not a locally unique rationalizable outcome. Remember that at the perfect-information limit equilibrium agents weigh more than one-for-one ($\mathbf{b}_\circ = \varkappa > 1$) a signal of the average expectation. This entails a local divergence process in beliefs. If agents expect that the average weight on the endogenous signal is in a neighborhood of that equilibrium then their best individual weight must be further away from the equilibrium. But since this is common knowledge, a second-order rational belief on the average weight must equally lie further away from the equilibrium, etc. In other words, this equilibrium cannot be obtained as a singleton from a rationalizability process. Nevertheless, given that an increase in the average weight will eventually decrease the informativeness of the signal, then, for a high enough average reaction, the sequence of higher-order beliefs will enter in a contracting dynamics converging to dispersed-information equilibria. In the case of a fictitious restriction to the support of the feasible weights $\Im(\mathbf{b}_\circ) \equiv$

[13]Given by (36)-(37), see appendix A.1.

$[\underline{\mathbf{b}}, \overline{\mathbf{b}}]$ such convergence will lead to the corner equilibria no matter how small $\Im(\mathbf{b}_\circ)$ is.

3 Microfoundations and macro-outcomes

This section presents a fully microfounded macro-model which features the signal extraction problem studied above. I will present a standard RBC economy where agents learn from equilibrium prices and all disturbances are fundamental in nature. In this economy a system of prices generates a signal extraction problem which renews at each period.

A key element is the assumption that the supply of capital is fix in aggregate. This real rigidity entails a large elasticity of the local prices for capital, which are used by firms as predictors, to the average price expectation, so that the conditions for a multiplicity of limit equilibria are met. At the end of this section I will discuss the implications of dispersed-information limit equilibria for the aggregate and cross-sectional dynamics, with a special focus on wage dispersion.

Preferences and technology

Consider an economy composed of a continuum of islands with unit mass. Each island $i \in (0, 1)$ is inhabited by a representative consumer and a representative producer. The utility of the representative consumer in island i is

$$U_{i,t} \equiv \sum_{t=0}^{\infty} \delta^t \left(\Phi_{i,t} \left(\frac{C_{i,t}^{1-\psi}}{1-\psi} - \frac{\left(L_{i,t}^s\right)^{1+\gamma}}{1+\gamma} \right) + \log \frac{M_{i,t-1}}{P_t} \right), \tag{12}$$

subject to a budget constraint for each period

$$\frac{R_t}{P_t} + \frac{W_{i,t}}{P_t} L_{i,t}^s + \frac{M_{i,t-1}}{P_t} = C_{i,t} + \frac{M_{i,t}}{P_t} + \frac{T_{i,t}}{P_t}, \tag{13}$$

where $\psi > 0$ is the inverse of the elasticity of intertemporal substitution and $\gamma > 0$ is the inverse of Frisch elasticity of labor supply, R is the return on one unit of an endowment that expires in one period and is renewed each time in each island, W_i is an island-specific wage, L_i^s is the supply of island-specific (local) labor, C_i is the consumption of the final good whose price is P and M_i is the money demand on island i.[14] T_i is a redistributive nominal transfer such that[15] $\int T_i \mathrm{di} = 0$.

[14] The utility function is the same used by Amador and Weill (2010). The main advantage of this specification for the purpose of this paper is that it delivers all multiplicative first-order conditions so that the model can be expressed in logs without any approximation.

[15] The only scope of the transfer is ensuring that in equilibrium the budget constraint holds for each i.

The endowment is acquired in an inter-island market to be transformed in island-specific capital K_i. The transformation is operated by competitive intermediate producers maximizing profits

$$R_{i,t}K_{i,t}^{s} - R_t Z_{i,t}, \tag{14}$$

under the constraint of the following linear technology

$$K_{i,t}^{s} \equiv e^{-\hat{\eta}_{i,t}} Z_{i,t}, \tag{15}$$

where $e^{-\hat{\eta}_{i,t}}$ is a island-specific productivity factor. The local capital K_i^s is produced using Z_i units of the endowment which are acquired in a global market at a price R.

Local capital and local labor are used by the representative final producer in island i to produce an homogeneous consumption good that is consumed across islands. Competitive firms maximizes profits

$$P_t Y_{i,t} - W_{i,t}L_{i,t} - R_{i,t}K_{i,t}, \tag{16}$$

under the constraint of a Cobb-Douglas technology with constant return to scale

$$Y_{i,t}\left(K_{i,t}, L_{i,t}\right) \equiv K_{i,t}^{1-\alpha} L_{i,t}^{\alpha}, \tag{17}$$

with $\alpha \in (0,1)$, where K_i, L_i, and Y_i denote respectively the demand for local capital, the demand for local labor and the produced quantity of the consumption good, generated in island i. Notice that final production is island-specific, that is, each representative producer hires labor and capital from his own island only. Input markets are segmented and there is one price for each input on each island.

Shocks

At each date the economy is hit by i.i.d. aggregate and island-specific disturbances. The productivity of the intermediate sector is affected by the stochastic noise

$$\hat{\eta}_{i,t} \sim N\left(0, \hat{\sigma}\right), \tag{18}$$

where $\hat{\eta}_i$ is an island-specific realization distributed independently across the islands. A second source of randomness concerns the utility of consumption and leisure in each island. It is determined by a shock

$$\log \Phi_{i,t} = \varepsilon_t + \hat{\phi}_{i,t}, \tag{19}$$

composed by an aggregate component $\varepsilon_t \sim N(0,1)$ drawn from a white noise distribution, and an island-specific component $\hat{\phi}_{i,t} \sim N(0, \sigma_\phi)$ which is a white noise disturbance independently distributed across the islands.

Equilibrium

Each period consists of two stages. In stage one the shocks hit and all input markets - two local for labor and capital on each island, and one global for the endowment - open and clear simultaneously. The production of the consumption good is implemented at the end of the first stage. In the second stage, the final market operates so that the consumption good clears across the islands and its price emerges. All agents in the economy have the same unbiased prior belief about the distribution of the shocks and acquire information through the equilibrium prices with which they deal. This means that in equilibrium each agent must take actions that are consistent with the information content of the prices she observes at the stage of the action.

As usual in the literature on noisy rational expectations from Grossman (1975) and Hellwig (1980) onward, I restrict attention to equilibria with a log-linear representation which, as we will see, is obtained without approximations for the model at hand. A formal definition of an equilibrium follows.

Definition 2 *A log-linear rational expectation equilibrium is a distribution of local prices* $\{R_{i,t}, W_{i,t}\}_{(0,1)}$, *global prices* (P_t, R_t) *and relative individual and aggregate quantities, contingent on the stochastic realizations* $(\varepsilon_t, \{\hat{\phi}_{i,t}, \hat{\eta}_{i,t}\}_{(0,1)})$, *such that:*

- *(*optimality*) agents optimize their actions conditional to the prices they observe;*

- *(*market clearing*) demand and supply in local markets match,* $L_{i,t} = L^s_{i,t}$ *and* $K_{i,t} = K^s_{i,t}$*; the money market and the endowment market clear, respectively* $M^d_{i,t} = \bar{M}^s_i$ *and* $\int Z_{i,t}di = 1$*;*

- *(*log-linearity*) and log-deviations of individual actions from their equilibrium steady state are linear functions of the shocks.*

The first condition requires agents' actions to be optimal conditional on the information agents infer from the equilibrium prices that they observe. Concerning market clearing conditions, notice that I assume that there is a constant amount of island-specific money $\bar{M}^s_i$ available in each island. This implies that the market condition for the consumption good $\int Y_i di = \int C_i di$ is obtained from the aggregation of the budget constraints (13). The requirement of a log-linear equilibrium allows the tractability of aggregate relations and more importantly, ensures that deviations of global prices from the equilibrium steady state are one-to-one functions of the aggregate shock only. Therefore observing a global price is informationally equivalent to observing the aggregate shock.

Learning from prices

Now let us spell out what each type of agent can learn from the equilibrium prices that she observes. At the first stage the consumer-workers are able to point-wise predict the price of the consumption good, which is not observable yet. In fact, the consumer-workers and the intermediate producers trade the endowment during the first stage on a global market, so they are able to infer the only aggregate shock perturbing global prices.

In the first stage final producers do not trade on any global market. Thus, they will be uncertain about the consumption price at the time of planning production[16]. In particular, a firm type i acquires input quantities $K_{i,t}(W_{i,t}, R_{i,t}, \mathrm{E}_t^i(P_t))$ and $L_{i,t}(W_{i,t}, R_{i,t}, \mathrm{E}_t^i(P_t))$ and produces a quantity $Y_{i,t}$ depending on the realizations of local prices. The latter have a twofold effect on firms' decision. They have a direct *allocational* effect, that is, increasing local prices discourage input demand. But also they have an indirect *informational* effect as a price expectation

$$\mathrm{E}_t^i(P_t) \equiv \mathrm{E}[P_t|\omega_{i,t}],$$

is conditional on

$$\omega_{i,t} = \{W_{i,t}, R_{i,t}\}, \tag{20}$$

namely, the local prices they observe in the transactions they carry out during the first stage. In particular, an increase in local prices could inform about a rising sale price boosting demand for inputs. The strength of the informational effect depends on the correlation of local and global prices which is a general equilibrium outcome.

Producers' uncertainty is solved at the end of the second stage. Once they sell the quantity of consumption good produced in the first stage, the price P_t is finally observed. Therefore, at the end of the second stage all agents have the same information, so each period is informationally independent. Moreover, given that shocks are i.i.d. and the supply of money is fixed in each island, consumers expect the unique stochastic steady state at each future date. Hence, as in Amador and Weill (2010) the only intertemporal first-order condition - the one for money - collapses to the one-period equilibrium relation

$$\frac{\Lambda_{i,t}}{P_t} = \delta \mathbf{E}_t\left[\frac{\Lambda_{t+1,i}}{P_{t+1}}\right] + \delta\frac{1}{\bar{M}_i^s} = \frac{\delta}{1-\delta}\frac{1}{\bar{M}_i^s} = 1 \tag{21}$$

where I substituted the market clearing condition $M_{i,t}^d = \bar{M}_i^s$ and normalized to one without loss of generality.[17] This means that firms actually need to anticipate the current price of consumption only, each period. In other words, a repeated static signal extraction problem is embedded in a dynamic macro model by the only mean of a price system. This avoids less natural assumptions usually made in the literature: the adoption of permanent shocks as in Amador and Weill (2010) or the worker-shopper metaphor inspired by Lucas

[16]The consumption price does not reveal simultaneously to the production choice. Lack of simultaneity is what makes informational frictions matter. For a deep analysis of the issue see Hellwig and Venkateswaran (2011).

[17]I use $\mathbf{E}_t(\cdot)$ to index the expectation operator conditional to all available information up to time t.

(1980). From here onward I will omit time indices as the following relations are all simultaneous.

Part of the information that consumers and intermediate producers hold is transmitted to final producers through local market transactions. The optimal supply of local labor moves with the preference shock and the nominal wage to satisfy

$$W_i = \Phi_i \left(L_i^s \right)^{\gamma} \tag{22}$$

where the real value of the island-specific multiplier $\Lambda_i / P = 1$ is fixed by (21). Hence, in equilibrium the wage observed by firms type i hiring $L_i = L_i^s$ reveals the preference shock Φ_i affecting the consumer-worker type i. That is, the local wage conveys a *private exogenous signal* of the aggregate shock. Notice that a quantity arising in the local markets type i can be expressed as a function of $E^i(P), \Phi_i$ and R_i.[18] Specifically, all the observables are measurable with respect to Φ_i and R_i that constitute therefore the finest available information set. The two pieces of information are different in nature. In particular, firm i hires local capital at the equilibrium price

$$R_i = R e^{\hat{\eta}_i} \tag{23}$$

which is a noisy signal of the price for the endowment.[19] In contrast to Φ_i, the price of local capital transmits a *private endogenous signal,* that is a noisy island-specific observation of the price for the endowment which embodies untangled information about the aggregate shock and producers' expectations. Figure 2 summarizes the flows of information in the economy.

Characterization of an equilibrium

All first order conditions in the model have a multiplicative form, so they can be log-linearized and solved without any approximation. In particular, the requirement of a log-rational equilibrium implies that the price (as any other variable in the model) is distributed lognormally according to

$$P = \bar{P} e^{p - \sigma(p)/2}$$

where $\sigma(\cdot)$ denotes the variance operator and $p \sim N(0, \sigma(p))$ is the stochastic log-component of a deviation of P from its stochastic steady state $\bar{P}$, obtained as a linear combination of all the shocks. Let me state the following proposition which enables a characterization of the equilibrium in terms of a profile of firms' expectations about p.

[18] To see this one can work with (42a), (42b),(42d) and (42f) in appendix.

[19] Any quantity of the island-specific capital is supplied at a price equal to (or more precisely, at the minimum price not smaller than) the global price of the endowment augmented for an i.i.d. productivity disturbance.

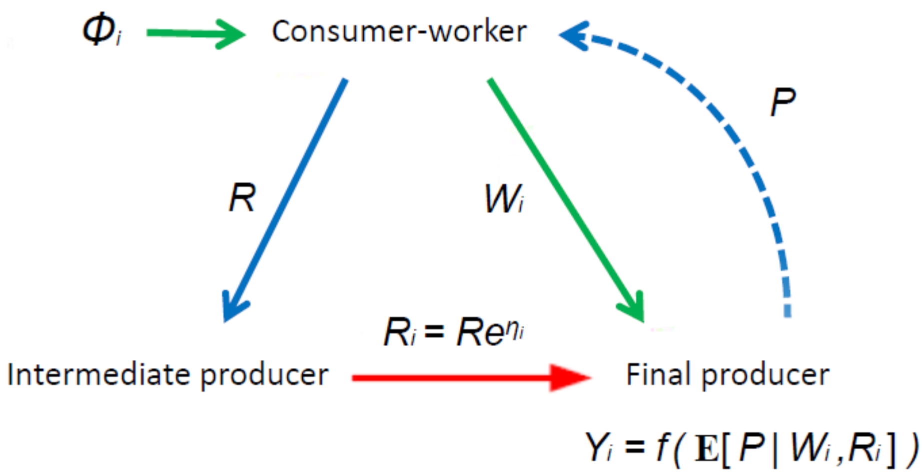

Figure 2: Flows of information in the economy. Green arrows denote exogenous private signals, blue are for public endogenous and red for private endogenous. Solid is information at stage 1, dashed is information at stage 2.

Proposition 6 *Given a profile of weights* $\{\mathrm{e}_\varepsilon, \mathrm{e}_\phi, \mathrm{e}_\eta\}$ *such that log-linear expectations for final producers are described by*

$$\mathrm{E}^i(P) = \bar{P} e^{E^i(p) - \frac{1}{2}\sigma\left(E^i(p)\right)} \tag{24}$$

with[20]

$$\mathrm{E}^i(p) = \mathrm{e}_\varepsilon \varepsilon + \mathrm{e}_\phi \hat{\phi}_i + \mathrm{e}_\eta \hat{\eta}_i, \tag{25}$$

then there exists a unique log-linear conditional deviation and a unique steady state for each variable in the model.

Proof. Appendix A.2. ■

The characterization of an equilibrium follows straightway once the requirement of rational expectations is imposed.

Definition 3 *A log-linear rational expectation equilibrium is characterized by a profile of weights* $\{\mathrm{e}_\varepsilon, \mathrm{e}_\phi, \mathrm{e}_\eta\}$ *such that (24) are rational expectations conditional on the information set (20).*

---

[20] An other way to see (24) is to start from a log-normal price distribution $P = P^m e^p$ where $p \sim N(0, \sigma(p))$ is normally distributed and P^m is the unconditional median. Then the correct conditional expectation is

$$\mathbf{E}(P \mid \omega_i) = P^m e^{\mathbf{E}(p|\omega_i) + \frac{\mathbf{E}(\sigma(p)|\omega_i)}{2}},$$

which can be rewritten as (24) using the law of total variance $\mathbf{E}(\sigma(p) \mid \omega_i) = \sigma(p) - \sigma(\mathbf{E}(p|\omega_i))$ where remeber $E^i(\cdot) = \mathbf{E}(\cdot|\omega_i)$ and the expression for the steady state is $\bar{P} = P^m e^{\sigma(p)/2}$.

In practice, an equilibrium is characterized by a distribution of firms' expectations about p, the stochastic component of the consumption price P. Each individual price expectation type i is conditioned on the observation of $\log \Phi_i$ and r_i, denoting the stochastic log-components of respectively Φ_i and R_i observed in the input markets. Both are log-linear functions of the shocks. Hence, a profile of optimal weights given to these two pieces of information maps into a profile of weights $\{\mathrm{e}_\varepsilon, \mathrm{e}_\phi, \mathrm{e}_\eta\}$. That is, the number of equilibria of the model corresponds to the number of solutions to the signal extraction problem.

The information structure

Before looking at how the two endogenous elements of the inference problem p and r_i move, it is useful to observe that under perfect information there exists a unique equilibrium where the aggregate shock has a pure inflationary effect. This is due to the fact that the aggregate shock alters the ratio between the marginal utility of consumption and money holdings, but not the one between consumption and leisure. When a positive aggregate shock hits, the price increases so that the real value of money decreases and the marginal utility of cash holding matches the increased marginal utility of consumption and leisure.

The stochastic log-linear component of the consumption price is given by

$$p = \varepsilon + \beta \left(\mathrm{E}\left(p\right) - \varepsilon \right), \tag{26}$$

with

$$\beta \equiv -\frac{\alpha\psi}{1+\gamma-\alpha} < 0$$

measuring the impact of the aggregate expectation on the consumption price (for details see appendix A.2); (26) is the same as (1). Under perfect information $\mathrm{E}^i\left(p\right) = p = \varepsilon$, that is the consumption price reacts one-for-one to an aggregate demand shock. In the opposite case of no information, that is with $\mathrm{E}^i\left(p\right) = 0$, labor supply shrinks, but the demand for local labor does not rise since final producers do not foresee any increase in the value of production. Therefore $p = \left(1-\beta\right)\varepsilon$ overreacts to the aggregate shock to clear a suboptimal production. Nevertheless, as stressed in section 2, the main results do not hinge on the specific relation between p and $\mathrm{E}\left(p\right)$ governed by β.

What matters is instead the restriction on the endogenous information discussed here. In analogy with the consumption price, one can express a stochastic log-deviation r_i of the price for local capital R_i from its steady state as

$$r_i = \varepsilon + \varkappa \left(\mathrm{E}\left(p\right) - \varepsilon \right) + \hat{\eta}_i \tag{27}$$

with

$$\varkappa \equiv \frac{1+\gamma}{1+\gamma-\alpha} > 1$$

(for details see appendix A.2); (27) is the same as (2). Moreover, it also satisfies the condition for multiplicity $\varkappa > 1$: the average signal goes down with ε and goes up with $\mathrm{E}(p)$ which in turn co-moves with ε depending on the precision of available information.

The price for the local endowment exhibits opposite reactions in the limit cases of perfect knowledge ($r = \varepsilon$) or no information ($r = (1-\varkappa)\,\varepsilon$). The underlying economic intuition is simple. When the aggregate shock is perfectly observed, as said above, it produces a neutral inflationary effect: moves all global prices at the same rate whereas leaves quantities unchanged. If instead final producers do not expect a positive shock to occur, labor supply shrinks in front of an unchanged labor demand as producers do not foresee variations in the consumption price. As a consequence, in equilibrium final producers hire less labour and the local wage increases less quickly than under perfect knowledge. This also determines a fall in the productivity of local capital and so a reduction in the market-clearing price for the endowment provided in fix supply.

Equilibria

Note that the cases of no information and full information emerge respectively with infinite volatility of both types of island-specific shocks and null volatility of at least one type of island-specific shocks. In the two scenarios the economy has a unique equilibrium. The following proposition states a multiplicity result at the limit of a vanishing dispersion of productivity shocks, that is, at the limit of a vanishing dispersion of local prices for capital.

Proposition 7 *Consider the problem of agents forecasting (26) conditionally on the information set (20). If the variance of demand shocks σ_ϕ satisfies*

$$\sigma_\phi > \pi^{-1} = \frac{1+\gamma}{\alpha} - (1-\psi)\,, \tag{28}$$

then in the limit of no productivity shocks, that is for $\hat{\sigma} \to 0$, there exists a unique perfect-information limit equilibrium which lies arbitrarily close to the perfect-information one characterized by

$$\mathrm{e}_{\varepsilon,\circ} = 1, \mathrm{e}_{\phi,\circ} = 0, \mathrm{e}_{\eta,\circ} = 1 \tag{29}$$

and two dispersed-information limit equilibria obtained for

$$\mathrm{e}_{\varepsilon,\pm} = \frac{\alpha}{1+\gamma},\ \mathrm{e}_{\phi,\pm} = \frac{1+\gamma-(1-\psi)\,\alpha}{1+\gamma}\sigma_\phi^{-1},\ \mathrm{e}_{\eta,\pm} = \frac{1+\gamma-\alpha}{1+\gamma}\mathbf{b}_\pm \tag{30}$$

where $\lim_{\sigma\to 0,\mathbf{b}\to\mathbf{b}_\pm} \mathbf{b}^2\hat{\sigma} = \pi^{-1}\left(\sigma_\phi - \pi^{-1}\right)\sigma_\phi^{-1}$, with π being the precision of the overall private information available about ε at the dispersed-information outcome.

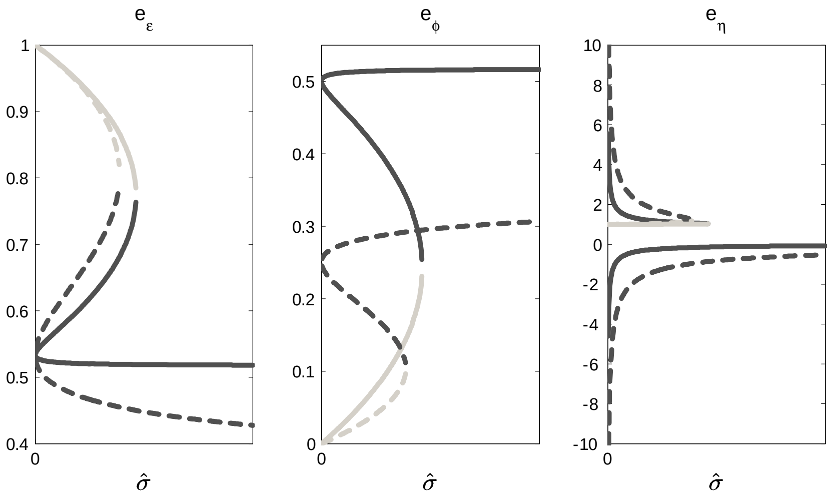

Figure 3: The equilibrium weights $\{\mathrm{e}_\varepsilon, \mathrm{e}_\phi, \mathrm{e}_\eta\}$ as functions of the dispersion of local prices for capital ($\hat{\sigma}$) for $\psi = 1$, $\gamma = 0.5$, $\alpha = 0.8$ and two different values of σ_ϕ: 2 (solid) and 4 (dotted).

The proof directly follows from proposition 3 and 4. The fix point equation of the signal extraction problem on (26) when observing (19) and (27) has three distinct real solutions which pin down three different triples - $\mathrm{e}_\varepsilon = (\mathbf{a} + \mathbf{b})(1 - \varkappa)/(\varkappa - \mathbf{b}\varkappa)$, $\mathrm{e}_\phi = \mathbf{a}(1-\varkappa)/\varkappa$, $\mathrm{e}_\eta = \mathbf{b}/\varkappa$ - for each i, where $\mathbf{a}$ and $\mathbf{b}$ are rescaled optimal weights put respectively on (19) and (27) yielding rational expectations of (26).

The proposition enlightens the role of preference and technological parameters. Notice that (28) is the transposition of (19) which represents a lower bound to the cross-sectional variance of the exogenous information. Such threshold is defined by a simple combination of the CES parameters. In particular, it is an index of the convexity of the problem: it is zero with $\alpha = 1$ and $\psi = 0$ and $\gamma = 0$, whereas it approaches infinity when $\alpha = 0$, or $\psi \to \infty$ or $\gamma \to \infty$. This implies that, ceteris paribus, the lower is the elasticity of intertemporal substitution, supply of labor and output to labor, the tighter the requirement for the existence of dispersed-information limit equilibria. Intuitively, as the model approaches linearity the coefficient $\varkappa$ increases, so that, in the limit of no information the price for the endowment has to drop more to clear a fix supply. In this case a higher precision of information is needed for the endowment price to approach a zero variance which is the condition for the emergence of a multiplicity in the limit.

Figure 3 illustrates the profile of weights $\{\mathrm{e}_\varepsilon, \mathrm{e}_\phi, \mathrm{e}_\eta\}$ as functions of $\hat{\sigma}$ for $\psi = 1$, $\gamma = 0.5$ and[21] $\alpha = 0.8$, and two different values of σ_ϕ, namely 2 (solid) and 4 (dotted). These can be interpreted as the responses of an individual price

[21] The calibration of the elasticity of intertemporal substitution and the Frisch elasticity of labor supply are consistent with the estimates of respectively Vissing-Jorgensen and Attanasio (2003), and Mulligan (1998).

expectation to an unitary realization of the three types of shocks. The limit values (29) and (30) are at the intersections of the curves with the y-axis. In the first two panels dispersed-information limit equilibria take the same value, which characterizes a unique limit outcome and lies away from the perfect-information one. The third panel shows that $\mathrm{e}_{\eta,\pm}$ go respectively to plus and minus infinity, but keep in mind that this is a reaction to an unitary realization of a shock that has in fact infinitesimal variance.

The picture also explores the evolution of the equilibria away from the limit $\hat{\sigma} \to 0$. In light (dark) are denoted the values that are obtained by continuity from the perfect-information (dispersed-information) limit equilibrium for larger values of $\hat{\sigma}$. Three remarks are in order here.

First, in analogy with the insights of figure 1, only one equilibrium, out of three, survives in the whole range spanned by a strictly positive value of $\hat{\sigma}$. This equilibrium entails dispersed-information at the limit $\hat{\sigma} \to 0$ and disappears instead for $\hat{\sigma} = 0$. Hence, the unique perfect-information equilibrium cannot be obtained by continuity from the unique dispersed-information equilibrium arising for a sufficiently high $\hat{\sigma}$.

Second, at the dispersed-information limit outcome, changes in σ_ϕ do not affect firms' reaction to an aggregate shock as long as (28) holds, as noted by proposition 4. That is, the precision of private information about the unobserved common fundamental ε does not vary with a (sufficiently low) precision of exogenous information σ_ϕ^{-1}. Nevertheless, a higher σ_ϕ decreases the parameter region where a multiplicity exists for strictly positive $\hat{\sigma}$-values. In other words, for a given $\hat{\sigma}$ a multiplicity disappears for large enough values of σ_ϕ.

Third, at the dispersed-information limit outcome, an individual expectation reacts to a type-specific preference shock $\hat{\phi}_i$ by an amount that is inversely proportional to the volatility of the same. The central panel shows that for a higher value of σ_ϕ the corresponding reaction move downwards. In particular, for the specific calibration we are considering, which implies logaritmic utility for consumption, it is $\mathrm{e}_\phi = \sigma_\phi^{-1}$. This effect occurs because as long as σ_ϕ increases the information transmitted by the local wage becomes looser, and so, it reflects less in firms' expectations. This is a typical implication of learning from prices.

Aggregate and cross-sectional dynamics

How heterogeneous firms' expectations map into observables of the model, and how these relate to deep parameters? In the following I will show that a larger Frisch elasticity of labor γ^{-1} and a higher elasticity of output to labor α make easier the emergence of dispersed-information equilibria and also magnify the reaction of expectations to aggregate and market-specific shocks, although this does not automatically reflects in a higher wage dispersion.

Aggregate dynamics. Let us first look at the aggregate dynamics. As said, at the perfect-information equilibrium all prices react one-for-one to the aggregate demand shock whereas average real quantities remain at the steady state (that is $p = w = r = \varepsilon$ and $l = y = 0$). At the dispersed-information outcome instead the aggregate demand shock transmits to the economy in the form of a supply-side shock: the consumption price goes up, whereas production and worked hours go down, respectively $y = -\alpha\varepsilon / (1+\gamma)$ and $l = -\varepsilon / (1+\gamma)$. The supply-side effect obtains because, as the informativeness of input prices dampens, firms cannot correctly anticipate changes in the consumption price and so they produce a suboptimal quantity.

Dispersed information also alters the volatility of prices yielding a full range of cases from excess volatility to perfect stickiness: the consumption price overreacts $p = (1 + \alpha\psi / (1+\gamma))\,\varepsilon$, the wage underreacts $w = \varepsilon / (1+\gamma)$, whereas the price for the endowment does not move $r = 0$.[22] This is illustrated by figure 4 where, in analogy to figure 3, the analysis is extended to values of $\hat{\sigma}$ strictly above zero. In particular, the role of a change in the elasticity of labor supply is explored. The plot is obtained for $\psi = 1$, $\alpha = 0.8$ and $\sigma_\phi = 2$ and two different values of γ, namely 0.5 (solid) and 0.55 (dotted). At the dispersed-information equilibria a higher γ makes consumption price, output and wages less reactive to the aggregate shock whereas leaves, as expected, the price for the endowment at steady state. An higher γ also increases the parameter region where a multiplicity exists for strictly positive $\hat{\sigma}$-values. Nevertheless, at the same time, it imposes a higher upper bound on σ_ϕ according to (28). The evolution of the three equilibria for strictly-positive values of $\hat{\sigma}$ is qualitatively the same shown in figure 3.

Wage dispersion. At the perfect-information limit equilibria firms have homogeneous price expectations. Dispersed-information equilibria instead feature a sizeable cross-sectional variance of individual expectations

$$\int \left(\mathrm{E}^i\left(p\right) - \mathrm{E}\left(p\right)\right)^2 \mathrm{di} = \frac{\alpha\left(1 + \gamma - (1-\psi)\alpha\right)}{\left(1+\gamma\right)^2}, \tag{31}$$

that reflects in the cross-sectional variance of all the market-specific variables in the model, except for the price of local capital which maintains by construction a negligible dispersion. Such an increase in dispersion is due to the amplification of private uncertainty introduced by vanishing productivity shocks.

In this respect, the cross-sectional dynamics of local wages is instructive. At the perfect-information limit outcome the local wage does not react to the local shocks, that is, local wages are equalized across islands.[23] At the dispersed-

[22] These value can be obtained using proposition 4 and (56) in appendix A.2.

[23] To see this notice that when close to perfect information firms correctly anticipate the price for consumption, so their price expectations are homogeneous and do not move with local shocks. The first order condition (42b) implies that, if firms have the same price expectation and face the same cost for local price then market-specific quantities of labor and capital must either both stay constant or both vary at the same rate. Condition (42e) will then pin down the variation of respectively labor and capital According to (42a), these joint quantity movements

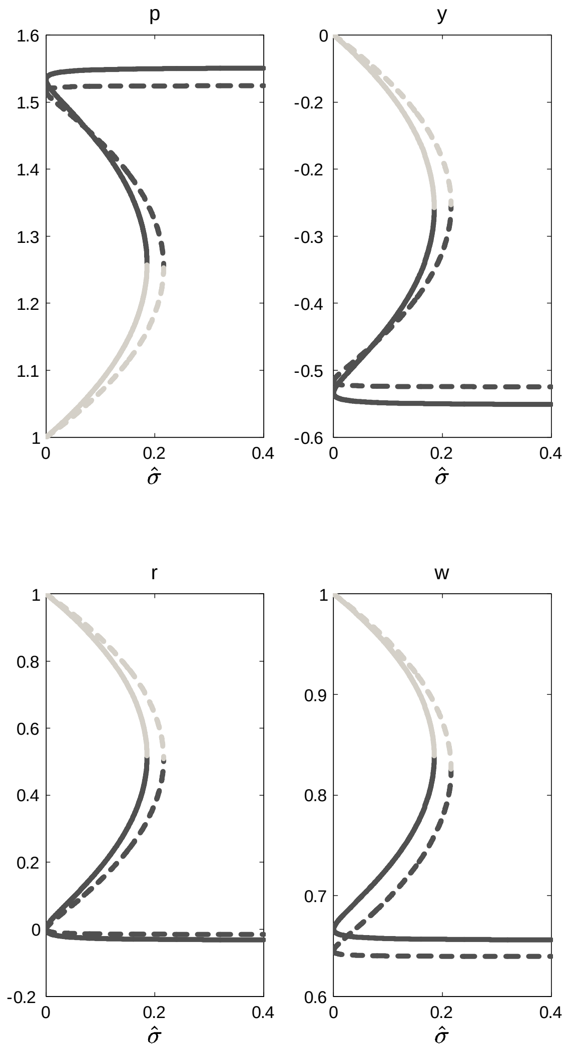

Figure 4: Four aggregate variables $\{p, y, r, w\}$ as functions of the dispersion of local prices for capital ($\hat{\sigma}$), for $\sigma_\phi = 2$, $\alpha = 0.8$ and $\psi = 1$ and two different values of γ: 0.5 (solid) and 0.55 (dotted).

information one instead the local wage does react to both local shocks as firms' expectations do.

Figure 5 plots the dispersion of local wages as a function of the dispersion of local prices for capital $\hat{\sigma}$ with a solid line. The dotted line denotes the fraction of total cross-sectional variance explained by island-specific preference shocks only. The three panels are obtained for $\psi = 1, \alpha = 0.8$ and different calibrations of σ_ϕ and γ: $\gamma = 0.5$ and $\sigma_\phi = 2$ (left), $\gamma = 0.5$ and $\sigma_\phi = 4$ (centre) and $\gamma = 0.55$ and $\sigma_\phi = 4$ (right).

At the perfect-information limit equilibrium wages are homogenous, whereas at the dispersed-information limit equilibria wages are dispersed as they reflect both preference and productivity island-specific shocks. Although the dispersion of the local capital price is taken infinitesimal, expectations exhibit sizeable heterogeneity which transmits to the cross-sectional distribution of local wages. In particular, as σ_ϕ increases, the total cross-sectional volatility decreases, and the fraction of cross-sectional volatility explained by $\hat{\phi}$ decreases even further. This is, again, a consequence of learning from prices: as the information transmitted by the local wage decreases, labor disturbances have a smaller impact on expectations and so on wage dispersion. An external observer looking for the fundamental or frictional factors driving such a wide distribution of wages will hardly detect the infinitesimal fundamental origins of firms' disagreement, which actually comes from the amplification of private uncertainty through price feedbacks. Hornstein, Krusell and Violante (2011) document this difficulty.

Nevertheless the transmission of heterogeneity of firms' beliefs to wage dispersion does not always follow the same direction. The last panel makes the point. It shows that a higher γ, that is a lower Frisch elasticity of labor supply, amplifies the cross-sectional volatility of wages but does not affect the amount of wage dispersion that can be explained by island-specific preference shocks only, at least for the case explored here (logaritmic utility for consumption). In fact, a higher γ decreases the dispersion of price expectations (see (31)), but also lowers the elasticity of labor supply, so that, ceteris paribus, the local wage has to react more to clear the market. The numerical exploration demonstrates that the latter effect can dominate. In sum, wage dispersion is originated by the cross-sectional variance of price expectations, nevertheless a variation in the latter can affect in either way the former depending on the microfoundations of the change.

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imply that the local wage will not react to local shocks too, or more precisely, its cross-sectional variance will be of the same order of magnitude of the variance of productivity shocks which is negligible at the limit.

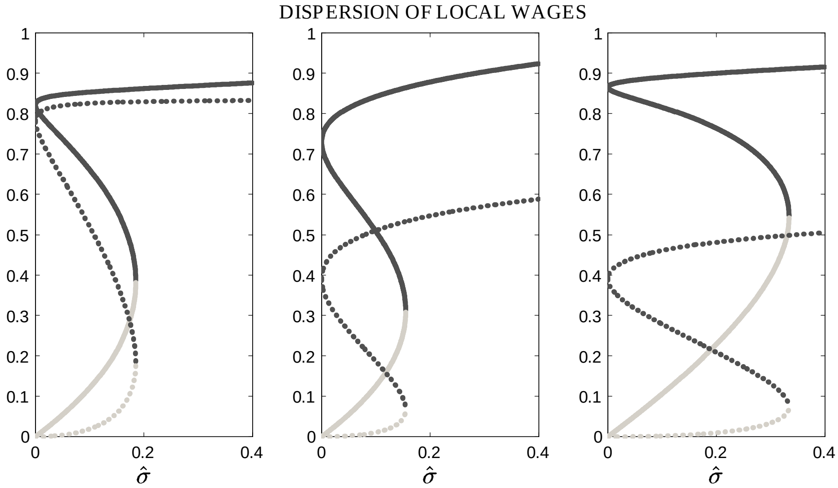

Figure 5: The dispersion of local wages as a function of the dispersion of local prices for capital ($\hat{\sigma}$), for $\psi = 1$, $\alpha = 0.8$ and different values of γ and σ_ϕ: $\gamma = 0.5$ and $\sigma_\phi = 2$ (left), $\gamma = 0.5$ and $\sigma_\phi = 4$ (centre) and $\gamma = 0.55$ and $\sigma_\phi = 4$ (right). Dotted lines indicate wage heterogeneity explained by preference shocks only.

4 Conclusions

This paper has laid out the general conditions under which the introduction of an arbitrarily small degree of price dispersion, in an otherwise fully-revealing system of prices, determines three rational expectation equilibria in models that have a unique equilibrium under perfect knowledge or absence of endogenous signals. This occurs when agents are privately uncertain about a global price that exhibits opposite reactions to an aggregate shock in the limit scenarios of no information and perfect foresight. In particular, in the limit of no private uncertainty only one equilibrium lies arbitrarily close to the one under perfect information whereas the other two maintain well-defined dispersed information. Dispersed-information limit equilibria feature the same limit outcome that entails large departures from the predictions of the perfect information benchmark, both at an aggregate and cross sectional level.

A theoretical questions is left for future research. The model I presented is static in nature, as the realization of the shock is not informative about the future course of the economy. When this is not the case agents cumulate correlated information through time. As showed by Angeletos, Hellwig and Pavan (2007) the dynamic interaction between exogenous and endogenous information can still sustain a multiplicity of equilibria in the context of coordination games encompassing the currency attack model. How this can survive in a microfounded macromodel with a unique equilibrium under complete information is a question that hopefully the analysis in this paper can help to address.

Appendix A

A.1 Analysis of the signal extraction problem

Proof. of proposition 1. With $\sigma = 0$, $b_i(\mathbf{b})$ is linear, so it is trivial to prove $\mathbf{b}_* = \zeta/(1+\zeta)$ to be the unique solution of the fix point equation $b_i(\mathbf{b}) = \mathbf{b}$. No matter how small is σ, the fix point equation $b_i(\mathbf{b}) = \mathbf{b}$ is instead a cubic which can be satisfied at most by three real roots. By continuity there must exist a positive solution $\mathbf{b}_\circ$ such that $\lim_{\sigma\to 0} b_i(\mathbf{b}_\circ) = \mathbf{b}_\circ = \mathbf{b}_*$. The condition $\zeta < -1$ is necessary and sufficient to have

$$\lim_{\sigma\to 0} b_i'(\mathbf{b}_*) = -\left((1-\beta)\zeta - \beta\right) > 1$$

for any $\beta < 1$. In such a case, given that

$$\lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) = \lim_{\sigma\to 0} b_i'(\mathbf{b}_*)$$

and that for any non-null σ we have $\lim_{\mathbf{b}\to+\infty} b_i(\mathbf{b}) = 0$, then by continuity at least an other intersection of $b_i(\mathbf{b})$ with the bisector at a positive value $\mathbf{b}_+$ must exist too. Moreover, $\lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) > 1$ also implies $b_i(0) < 0$. This, jointly with the fact that for $\sigma \neq 0$ it is $\lim_{\mathbf{b}\to-\infty} b_i(\mathbf{b}) = 0$, implies that by continuity at least an intersection of $b_i(\mathbf{b})$ with the bisector at a negative value $\mathbf{b}_-$ also exists. Therefore $(\mathbf{b}_-\mathbf{b}_\circ, \mathbf{b}_+)$ are the three solutions we were looking for.

Now suppose that $\mathbf{b}_-$ and $\mathbf{b}_+$ take finite values - that is $|\mathbf{b}_\pm| \leq M$ with M being an arbitrarily large finite number - then $\lim_{\sigma\to 0} \sigma(1-\mathbf{b}_\pm)^2 = 0$ and so necessarily $\mathbf{b}_\pm = \mathbf{b}_\circ > 0$. Nevertheless $\mathbf{b}_- < 0$ so a first contradiction arises. Moreover if $\mathbf{b}_+$ is arbitrarily close to $\mathbf{b}_\circ$ then by continuity $\lim_{\sigma\to 0} b_i'(\mathbf{b}_+) > 1$ which, for the same argument above, would imply the existence of a fourth root; a second contradiction arises. Hence we can conclude $|\mathbf{b}_\pm| > M$ with M arbitrarily large.

Finally let me prove that a multiplicity arises for a σ small enough if and only if $\zeta < -1$. For $\zeta > -1$, two cases are possible, either $0 < \lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) < 1$ or $\lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) < 0$. The case $0 < \lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) < 1$ provides, for a σ small enough, at least one intersection of $b_i(\mathbf{b})$ with the bisector must exist by continuity. Nevertheless in this case such intersection is unique as the existence of a second one would require $\max b_i'(\mathbf{b}) > 1$ given that $\lim_{\mathbf{b}\to\pm\infty} b_i(\mathbf{b}) = 0_\pm$ but $\sup_{\mathbf{b}} \lim_{\sigma\to 0} b_i'(\mathbf{b}) = \lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ)$. With $\lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) < 0$ instead - for a σ small enough - the curve $b_i(\mathbf{b})$ either is strictly decreasing in the first quadrant ($b_i(\mathbf{b}) > 0, \mathbf{b} > 0$) and never lies in the fourth quadrant ($b_i(\mathbf{b}) < 0, \mathbf{b} < 0$) or is strictly decreasing in the fourth quadrant and never lies in the first quadrant. Hence $\lim_{\sigma\to 0} b_i(\mathbf{b})$ can only have one intersection with the bisector. ∎

Proof. of proposition 2. i) Remember $\pi = (1-\mathbf{b})^{-2}\sigma^{-1}$. Using (8) we get

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm}(1-\mathbf{b})^{-2}\sigma^{-1} = $$

$$= \left(\frac{(1-\beta)\zeta - ((1-\beta)\zeta-\beta)\,\mathbf{b}_\pm}{\mathbf{b}_\pm} - 1\right)^{-1} =$$

$$= -\frac{1}{(1-\beta)(1+\zeta)} = \frac{\varkappa-1}{1-\beta}$$

which is positive provided $\varkappa > 1$. ii) The cross sectional variance of individual expectations is given by $\mathbf{b}^2\zeta^{-2}\sigma$. Notice

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm}(1-\mathbf{b})^2\sigma = \lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm}\mathbf{b}^2\sigma.$$

so that, as shown above,

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm}\mathbf{b}^2\zeta^{-2}\sigma = -\zeta^{-2}(1-\beta)(1+\zeta) = (1-\beta)\frac{\varkappa-1}{\varkappa^2}$$

which is positive provided $\varkappa > 1$. iii) Given (5) it is easy to show that

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm}\mathrm{E}(\varepsilon) = \frac{\mathbf{b}_\pm}{1-\mathbf{b}_\pm}\zeta^{-1}\varepsilon = -\zeta^{-1}\varepsilon = \frac{\varkappa-1}{\varkappa}\varepsilon.$$

iv) Substituting the expression above into (1) we get an expression for the price to be forecasted. v) The individual signal becomes

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} r_i = \varepsilon + \varkappa\left(\frac{\varkappa-1}{\varkappa}\varepsilon - \varepsilon\right) + \hat{\eta}_i = 0.$$

■

Proof. of proposition 3. Since all agents use the rule (10) then by definition

$$\mathrm{E}(p) = \frac{\mathbf{a}+\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon. \tag{33}$$

An individual expectation can be rewritten as

$$\mathrm{E}^i(p) = a_i\left(\zeta^{-1}\varepsilon + \phi_i\right) + b_i\left(\frac{\mathbf{a}+1}{1-\mathbf{b}}\zeta^{-1}\varepsilon + \eta_i\right). \tag{34}$$

The actual law of motion of the market price (26) is given by

$$p = (1-\beta)\varepsilon + \beta\frac{\mathbf{a}+\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon, \tag{35}$$

as functions of weights and exogenous shocks only. Now we have a bidimensional individual best weight function written as

$$a_i(\mathbf{a},\mathbf{b}) = \frac{(1-\beta)\,\zeta + \beta\frac{\mathbf{a}+\mathbf{b}}{1-\mathbf{b}} - \mathbf{b}\frac{\mathbf{a}+1}{1-\mathbf{b}}}{1+\sigma_\phi} \tag{36}$$

$$b_i(\mathbf{a},\mathbf{b}) = \frac{(1-\beta)\,\frac{1-\mathbf{b}}{1+\mathbf{a}}\zeta + \beta\frac{\mathbf{a}+\mathbf{b}}{(1+\mathbf{a}} - \mathbf{a}\frac{1-\mathbf{b}}{1+\mathbf{a}}}{1+\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma} \tag{37}$$

provided $\mathbf{b} \neq 1$.

From (36) we can recover the expression for the equilibrium $\mathbf{a} = a_i(\mathbf{a},\mathbf{b})$ which is

$$\mathbf{a} = -\frac{((1+\zeta)\mathbf{b}-\zeta)\,(1-\beta)}{(1-\beta)+(1-\mathbf{b})\,\sigma_\phi}. \tag{38a}$$

Notice that, with $\sigma_\phi \neq 0$, $\mathbf{a}$ takes finite values for any $\mathbf{b}$. Plugging the expression above at the numerator or (37) we obtain a fix point equation in $\mathbf{b}$

$$b_i(\mathbf{b}) = \frac{\mathbf{b}\frac{(1-\beta)(1+\zeta)-((1-\beta)\zeta-\beta)\sigma_\phi}{(1-\beta)(1+\zeta)+\sigma_\phi} + \frac{(1-\beta)\zeta\sigma_\phi}{(1-\beta)(1+\zeta)+\sigma_\phi}}{1+\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma} \tag{38b}$$

which is a cubic continuous function having the same structure delivered by (8). With $\sigma = 0$, $b_i(\mathbf{b})$ is linear, so it is trivial to prove $(\mathbf{a}_*,\mathbf{b}_*) = (0, \zeta/(1+\zeta))$ to be the unique solution of the fix point equation (38b). No matter how small is σ, the fix point equation (38b) is instead a cubic which can be satisfied at most by three real roots. By continuity there must exist a positive solution $\mathbf{b}_\circ$ such that $\lim_{\sigma\to 0} b_i(\mathbf{b}_\circ) = \mathbf{b}_\circ = \mathbf{b}_*$. The condition $\sigma_\phi > -(1-\beta)(1+\zeta)$ with $\zeta < -1$ is necessary and sufficient to have

$$\lim_{\sigma\to 0} b_i'(\mathbf{b}_*) = \frac{(1-\beta)(1+\zeta) - ((1-\beta)\zeta-\beta)\,\sigma_\phi}{(1-\beta)(1+\zeta)+\sigma_\phi} > 1$$

for any $\beta < 1$. In such a case given that

$$\lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) = \lim_{\sigma\to 0} b_i'(\mathbf{b}_*)$$

and for $\sigma \neq 0$ we have $\lim_{\mathbf{b}\to+\infty} b_i(\mathbf{b}) = 0$, then by continuity at least an other intersection of $b_i(\mathbf{b})$ with the bisector at a positive value $\mathbf{b}_+$ must exist too. Moreover, $\lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) > 1$ also implies $b_i(0) < 0$. This, jointly with the fact that for $\sigma \neq 0$ it is $\lim_{\mathbf{b}\to-\infty} b_i(\mathbf{b}) = 0$, implies that by continuity at least an intersection of $b_i(\mathbf{b})$ with the bisector at a negative value $\mathbf{b}_-$ also exists. Therefore $(\mathbf{b}_-\mathbf{b}_\circ, \mathbf{b}_+)$ are the three solutions we were looking for.

Now suppose that $\mathbf{b}_-$ and $\mathbf{b}_+$ take finite values- that is $|\mathbf{b}_\pm| \leq M$ with M being an arbitrarily large finite number - then, since $\mathbf{a}$ is finite too, $\lim_{\sigma\to 0}$ $\sigma\left((1-\mathbf{b}_\pm)/(1+\mathbf{a})\right)^2 = 0$ and so necessarily $\mathbf{b}_\pm = \mathbf{b}_\circ > 0$. Nevertheless

$\mathbf{b}_- < 0$ so a first contradiction arises. Moreover if $\mathbf{b}_+$ is arbitrarily close to $\mathbf{b}_\circ$ then by continuity $\lim_{\sigma\to 0} b_i'(\mathbf{b}_+) > 1$ which, for the same argument above, would imply the existence of a fourth root; a second contradiction arises. Hence we can conclude $|\mathbf{b}_\pm| > M$ with M arbitrarily large.

Finally let me prove that a multiplicity arises for a σ small enough if and only if $\sigma_\phi > -(1-\beta)(1+\zeta)$ with $\zeta < -1$. As before, out of these conditions two cases are possible, either $0 < \lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) < 1$ or $\lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) < 0$. The case $0 < \lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) < 1$ provides, for a σ small enough, at least one intersection of $b_i(\mathbf{b})$ with the bisector must exist by continuity. Nevertheless in this case such intersection is unique as the existence of a second one would require $\sup_{\mathbf{b}} \lim_{\sigma\to 0} b_i'(\mathbf{b}) > 1$ given that $\lim_{\mathbf{b}\to\pm\infty} b_i(\mathbf{b}) = 0_\pm$ but $\sup_{\mathbf{b}} \lim_{\sigma\to 0} b_i'(\mathbf{b}) = \lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ)$. With $\lim_{\sigma\to 0} b_i'(\mathbf{b}_\circ) < 0$ instead - for a σ small enough - the curve $b_i(\mathbf{b})$ either is strictly decreasing in the first quadrant ($b_i(\mathbf{b}) > 0, \mathbf{b} > 0$) and never lies in the fourth quadrant ($b_i(\mathbf{b}) < 0, \mathbf{b} < 0$), or is strictly decreasing in the fourth quadrant and never lies in the first quadrant. Hence $\lim_{\sigma\to 0} b_i(\mathbf{b})$ can only have one intersection with the bisector. ■

Proof. of proposition 4. i) From (38a) we get that

$$\lim_{\mathbf{b}\to\pm\infty} \mathbf{a} = \frac{(1-\beta)(1+\zeta)}{\sigma_\phi}.$$

Using (38b) we have that

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} \left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2 \sigma = -\frac{(1-\beta)(\zeta+1)\sigma_\phi}{(1-\beta)(\zeta+1)+\sigma_\phi}. \tag{40}$$

The total relative precision of private information about ε is given by

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} \pi = \lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} \left(\sigma_\phi^{-1} + \left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^{-2} \sigma^{-1}\right) =$$
$$= -\frac{1}{(1-\beta)(1+\zeta)} = \frac{\varkappa-1}{1-\beta}.$$

which is positive provided $\varkappa > 1$. ii) The cross sectional variance of individual expectations is given by $\zeta^{-2}(\mathbf{a}\sigma_\phi + \mathbf{b}^2\sigma)$. Note that

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} \left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2 (1+\mathbf{a})^2\sigma = \lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} (1-\mathbf{b})^2\sigma =$$
$$= \frac{-(1-\beta)(\zeta+1)((1-\beta)(1+\zeta)+\sigma_\phi)}{\sigma_\phi}.$$

As before,

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} (1-\mathbf{b})^2\sigma = \lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} \mathbf{b}^2\sigma.$$

Using the relations above we have

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm}\zeta^{-2}\left(\mathbf{a}^2\sigma_\phi+\mathbf{b}^2\sigma\right)=-\zeta^{-2}\left(1-\beta\right)\left(1+\zeta\right)=\left(1-\beta\right)\frac{\varkappa-1}{\varkappa^2}$$

which is positive provided $\varkappa>1$. Points iii), iv) and v) are as in the proof. of proposition 3. ■

Proof. of proposition 5. To check rationalizability in the most general case I need to build up the Jacobian computed around the equilibria. It is

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm}J=\begin{pmatrix}\frac{\partial a_i}{\partial\mathbf{a}} & \frac{\partial a_i}{\partial\mathbf{b}}\\ \frac{\partial b_i}{\partial\mathbf{a}} & \frac{\partial b_i}{\partial\mathbf{b}}\end{pmatrix}$$

with

$$\begin{aligned}
J_{11} &= \frac{\beta\frac{\mathbf{1}}{1-\mathbf{b}}-\frac{\mathbf{b}}{1-\mathbf{b}}}{1+\sigma_\phi}\\
J_{12} &= \frac{\beta\frac{\mathbf{a}+\mathbf{1}}{(1-\mathbf{b})^2}-\frac{\mathbf{a}+1}{(1-\mathbf{b})^2}}{1+\sigma_\phi}\\
J_{21} &= \frac{-\frac{(1-\beta)(1+\zeta)(1-\mathbf{b})}{(1+\mathbf{a})^2}}{1+\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma}+\frac{2\left(\left(1-\beta\right)\frac{1-\mathbf{b}}{1+\mathbf{a}}\zeta+\beta\frac{\mathbf{a}+\mathbf{b}}{1+\mathbf{a}}-\mathbf{a}\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)\frac{(1-\mathbf{b})^2\sigma}{(1+\mathbf{a})^3}}{\left(1+\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma\right)^2}\\
J_{22} &= \frac{\frac{\beta(1+\zeta)-\zeta+\mathbf{a}}{1+\mathbf{a}}}{1+\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma}+\frac{2\left(\left(1-\beta\right)\frac{1-\mathbf{b}}{1+\mathbf{a}}\zeta+\beta\frac{\mathbf{a}+\mathbf{b}}{(1+\mathbf{a}}-\mathbf{a}\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)\frac{(1-\mathbf{b})\sigma}{(1+\mathbf{a})^2}}{\left(1+\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma\right)^2}.
\end{aligned}$$

The latter terms become

$$\begin{aligned}
J_{21} &= \frac{-\frac{(1-\beta)(1+\zeta)(1-\mathbf{b})}{(1+\mathbf{a})^2}\mathbf{b}}{\left(1-\beta\right)\frac{1-\mathbf{b}}{1+\mathbf{a}}\zeta+\beta\frac{\mathbf{a}+\mathbf{b}}{1+\mathbf{a}}-\mathbf{a}\frac{1-\mathbf{b}}{1+\mathbf{a}}}+2\frac{\frac{\mathbf{b}}{1+\mathbf{a}}\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma}{1+\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma}\\
J_{22} &= \frac{\frac{\beta(1+\zeta)-\zeta+\mathbf{a}}{1+\mathbf{a}}\mathbf{b}}{\left(1-\beta\right)\frac{1-\mathbf{b}}{1+\mathbf{a}}\zeta+\beta\frac{\mathbf{a}+\mathbf{b}}{1+\mathbf{a}}-\mathbf{a}\frac{1-\mathbf{b}}{1+\mathbf{a}}}+2\frac{\frac{\mathbf{b}}{1-\mathbf{b}}\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma}{1+\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma}
\end{aligned}$$

after substituting for (37). Notice that

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm}J_{21}J_{12}=0$$

since $\mathbf{a}$ is always finite, J_{21} goes to infinity of order $\mathbf{b}_\pm$ whereas J_{12} goes to an infinitesimal of order $\mathbf{b}_\pm^{-2}$. Therefore the eigenvalues of the Jacobian are

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm}J_{11}=\frac{1}{1+\sigma_\phi}\in(0,1)$$

and

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm} J_{22} = 1-2\frac{\Gamma}{1+\Gamma}\in(-1,1)$$

where (see also (40))

$$\Gamma\equiv\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_\pm}\left(\frac{1-\mathbf{b}}{1+\mathbf{a}}\right)^2\sigma = -\frac{(1-\beta)(\zeta+1)\,\sigma_\phi}{(1-\beta)(\zeta+1)+\sigma_\phi}>0$$

if and only if $(1-\beta)(1+\zeta)+\sigma_\phi>0$ and $\zeta<-1$, which are the conditions for dispersed-information limit equilibria.

At the perfect-information limit equilibrium instead the Jacobian is given by

$$\lim_{\sigma\to 0}\lim_{\mathbf{b}\to\mathbf{b}_*} J = \begin{pmatrix}\frac{\beta(1+\zeta)-\zeta}{1+\sigma_\phi} & -\frac{(1-\beta)(\zeta+1)^2}{1+\sigma_\phi}\\ -(1-\beta) & \beta(1+\zeta)-\zeta\end{pmatrix}.$$

The product of its eigenvalues (the determinant of J) is

$$\Delta(J)=\frac{1-2(1-\beta)(1+\zeta)}{1+\sigma_\phi},$$

and their sum (the trace of J) is

$$\mathrm{Tr}(J)=\frac{2+\sigma_\phi}{1+\sigma_\phi}(\beta(1+\zeta)-\zeta).$$

Provided $\mathrm{Tr}(J)^2>4\Delta(J)$, the largest real eigenvalue is greater than one whenever

$$\frac{1}{2}\mathrm{Tr}(J)+\frac{1}{2}\sqrt{\mathrm{Tr}(J)^2-4\Delta(J)}>1$$

that is $\mathrm{Tr}(J)>\min\{1+\Delta(J),2\}$. After simple manipulations it is easy to show that $\mathrm{Tr}(J)>1+\Delta(J)$ requires

$$\beta(1+\zeta)-\zeta>1$$

which is true with $\zeta<-1$ for any $\beta<1$ (this also implies $\Delta(J)>0$). Let us prove now $\mathrm{Tr}(J)^2>4\Delta(J)$, that is

$$\frac{(2+\sigma_\phi)^2}{4(1+\sigma_\phi)}k^2-2k+1>0$$

where $k\equiv\beta(1+\zeta)-\zeta$. Notice that it is always $(2+\sigma_\phi)^2>4+4\sigma_\phi$, so we can easily conclude that

$$\frac{(2+\sigma_\phi)^2}{4(1+\sigma_\phi)}k^2-2k+1>(k-1)^2>0.$$

Finally, notice that J_{22} alone characterizes the univariate case putting $\mathbf{a}=0$ which obtains at the limit $\sigma_\phi\to\infty$. ■

A.2 Relations in the model

First-order conditions and the deterministic steady state

The whole list of first-order conditions of the model are

$$
\begin{aligned}
W_i &= \alpha E^i (P) L_i^{\alpha-1} K_i^{1-\alpha} && \text{(42a)}\\
R_i &= (1-\alpha) E^i (P) L_i^{\alpha} K_i^{-\alpha} && \text{(42b)}\\
Y_i &= L_i^{\alpha} K_i^{1-\alpha} && \text{(42c)}\\
\Phi_i (L_i^s)^{\gamma} &= W_i \frac{\Lambda_i}{P} && \text{(42d)}\\
\Phi_i C_i^{-\psi} &= \Lambda_i && \text{(42e)}\\
\frac{\Lambda_i}{P} &= \frac{\delta}{1-\delta} \frac{1}{\bar{M}_i} = 1 && \text{(42f)}\\
R &= e^{-\hat{\eta}_i} R_i && \text{(42g)}\\
K_i &= e^{-\hat{\eta}_i} Z_i && \text{(42h)}
\end{aligned}
$$

where the first three refer to the problem of final producers, the last two to the problem of intermediate producers and the rest to the consumers' problem.

The unique deterministic price and aggregate production obtain respectively as $P^* = \alpha^{\frac{-\alpha\psi}{1+\gamma-\alpha+\alpha\psi}}$ and $Y^* = \alpha^{\frac{\alpha}{1+\gamma-\alpha+\alpha\psi}}$ after solving the system for $\varepsilon = 0$, $\hat{\phi} = 0$, $\hat{\eta}_i = 0$ and constant actions across agents.

Proof. proposition 6

Notation. The requirement that the equilibrium has a symmetric log-linear representation means that any variable X_i in the model has the form

$$X_i = \bar{X} e^{x_i - \frac{1}{2}\sigma(x_i)}$$

where $\bar{X}$ is the steady state and $\sigma(x_i)$ is the variance of x_i (denoted in *italics*) which is a stochastic log-deviation from the steady state

$$x_i = \mathrm{x}_{\varepsilon}\varepsilon + \mathrm{x}_{\phi}\hat{\phi}_i + \mathrm{x}_{\eta}\hat{\eta}_i$$

expressed as a linear combination of the shocks (whose weights are denoted in `roman`). The aim of this section to show that for each variable in the model there exists a unique steady state and a unique log-linear deviation implied by fixing a profile of coefficients $(\mathrm{e}_{i,\varepsilon}, \mathrm{e}_{i,\phi}, \mathrm{e}_{i,\eta})$ in (25). For the sake of notational convenience, I analyze symmetric equilibria, that is ones in which individual weights $(\mathrm{e}_{i,\varepsilon}, \mathrm{e}_{i,\phi}, \mathrm{e}_{i,\eta})$ for each i are equal to the average ones $(\mathrm{e}_{\varepsilon}, \mathrm{e}_{\phi}, \mathrm{e}_{\eta})$. This choice is without loss of generality as the average weights are sufficient statistics of a profile of individual weights.

Production side. The aggregate demand for the endowment

$$\int Z_i \mathtt{di} = \int \bar{Z} e^{\mathtt{z}_\varepsilon \varepsilon + \mathtt{z}_\phi \hat{\phi}_i + \mathtt{z}_\eta \hat{\eta}_i - \frac{1}{2}\sigma(z_i)} \mathtt{di} = \bar{Z} e^{\mathtt{z}_\varepsilon \varepsilon - \frac{1}{2}\sigma(\mathtt{z}_\varepsilon \varepsilon)},$$

satisfies the market clearing condition $\int Z_i \mathtt{di} = 1$ for any ε so that necessarily $\mathtt{z}_\varepsilon = 0$ and $\bar{Z} = 1$. Using (42h) and the relation above we obtain

$$K_i = \bar{K} e^{\mathtt{k}_\varepsilon \varepsilon + \mathtt{k}_\phi \hat{\phi}_i + \mathtt{k}_\eta \hat{\eta}_i - \frac{1}{2}\sigma(k_i)} = e^{-\hat{\eta}_i} Z_i = e^{\mathtt{z}_\phi \hat{\phi}_i + (\mathtt{z}_\eta - 1)\hat{\eta}_i - \frac{1}{2}\sigma(z_i)},$$

where therefore $\mathrm{k}_\varepsilon = \mathrm{z}_\varepsilon = 0$, $\mathrm{k}_\phi = \mathrm{z}_\phi$, $\mathrm{k}_\eta = \mathrm{z}_\eta - 1$ and $\bar{K} = \bar{Z} = 1$ is the only possibility for the equality to hold for any $(\varepsilon, \hat{\phi}_i, \hat{\eta}_i)$ realization. According to (42g),

$$R = e^{-\hat{\eta}_i} R_i = \bar{R} e^{\mathtt{r}_\varepsilon \varepsilon + \mathtt{r}_\phi \hat{\phi}_i + (\mathtt{r}_\eta - 1)\hat{\eta}_i - \frac{1}{2}\sigma(r_i)},$$

but also integrating both sides across islands we have,

$$R = \bar{R} e^{\mathtt{r}_\varepsilon \varepsilon + \frac{\mathtt{r}_\phi^2 \sigma_\phi + (\mathtt{r}_\eta - 1)^2 \hat{\sigma}_i}{2} - \frac{1}{2}\sigma(r_i)}, \tag{43}$$

so that $\bar{R} = R$, $\mathtt{r}_\phi = 0$ and $\mathtt{r}_\eta = 1$ and only $\mathtt{r}_\varepsilon$ is left to be determined. Plugging (42f) in (42d) and the resulting in (42a) we get

$$L_i = \alpha^{\frac{1}{1-\alpha+\gamma}} e^{-\frac{\varepsilon + \hat{\phi}_i}{1-\alpha+\gamma}} \mathtt{E}^i (P)^{\frac{1}{1-\alpha+\gamma}} K_i^{\frac{1-\alpha}{1-\alpha+\gamma}} \tag{44}$$

and substituting the expression above in (42b) we have

$$R_i = (1-\alpha)\, \alpha^{\frac{\alpha}{1-\alpha+\gamma}} e^{-\frac{\alpha\varepsilon + \alpha\hat{\phi}_i}{1-\alpha+\gamma}} \mathtt{E}^i (P)^{\frac{1+\gamma}{1-\alpha+\gamma}} K_i^{-\frac{\alpha\gamma}{1-\alpha+\gamma}}$$

where $R_i = \bar{R} e^{\mathtt{r}_\varepsilon \varepsilon + \hat{\eta}_i - \frac{1}{2}\sigma(r_i)}$ according to (42g). Hence the following restrictions on log-deviations must hold for any $(\varepsilon, \hat{\phi}_i, \hat{\eta}_i)$ realization

$$\mathtt{r}_\varepsilon = \frac{1+\gamma}{1-\alpha+\gamma}\mathtt{e}_\varepsilon - \frac{\alpha}{1-\alpha+\gamma}, \tag{45}$$

$$0 = \frac{1+\gamma}{1-\alpha+\gamma}\mathtt{e}_\phi - \frac{\alpha\gamma}{1-\alpha+\gamma}\mathtt{k}_\phi - \frac{\alpha}{1-\alpha+\gamma}, \tag{46}$$

$$1 = \frac{1+\gamma}{1-\alpha+\gamma}\mathtt{e}_\eta - \frac{\alpha\gamma}{1-\alpha+\gamma}\mathtt{k}_\eta \tag{47}$$

which pin down $\mathtt{k}_\phi$ and $\mathtt{k}_\phi$ as functions of $\mathtt{e}_\varepsilon$, $\mathtt{e}_\phi$ and $\mathtt{e}_\eta$. Concerning the steady state of $\bar{R}$ it is related to $\bar{P}$ by

$$\bar{R} = (1-\alpha)\, \alpha^{\frac{\alpha}{1-\alpha+\gamma}} \bar{P}^{\frac{1+\gamma}{1-\alpha+\gamma}} e^{\Psi_{\bar{R}}}$$

where $\Psi_{\bar{R}}$ is a constant term depending on the variance-covariance of the log-stochastic deviations of $\mathtt{E}^i(P)$, K_i and Φ_i; these are zero in the deterministic case when the median and the average action coincide. Therefore for *given* $\mathtt{e}_\varepsilon$,

$\mathtt{e}_\phi$ and $\mathtt{e}_\eta$ there exists unique steady state value of R, R_i, K_i, Z_i and unique relative deviations defined by the relations above. Once K_i is determined then also L_i is according to

$$\begin{aligned} \mathtt{l}_\varepsilon &= \frac{1}{1-\alpha+\gamma}\mathtt{e}_\varepsilon - \frac{1}{1-\alpha+\gamma}, && (48a) \\ \mathtt{l}_\phi &= \frac{1}{1-\alpha+\gamma}\mathtt{e}_\phi + \frac{1-\alpha}{1-\alpha+\gamma}\mathtt{k}_\phi - \frac{1}{1-\alpha+\gamma}, && (48b) \\ \mathtt{l}_\eta &= \frac{1}{1-\alpha+\gamma}\mathtt{e}_\eta + \frac{1-\alpha}{1-\alpha+\gamma}\mathtt{k}_\eta && (48c) \end{aligned}$$

with steady state

$$\bar{L} = \alpha^{\frac{1}{1-\alpha+\gamma}} \bar{P}^{\frac{1}{1-\alpha+\gamma}} e^{\Psi_{\bar{L}}},$$

where again where $\Psi_{\bar{L}}$ is a constant term depending on the variance-covariance of the log-stochastic deviations of $\mathtt{E}^i(P)$, K_i and Φ_i. Analogously we can find the unique implied steady state and log-deviation of W_i and Y_i. From (42f) in (42d) we have the restrictions

$$\begin{aligned} \mathtt{w}_\varepsilon &= \gamma \mathtt{l}_\varepsilon + 1 && (49a) \\ \mathtt{w}_\phi &= \gamma \mathtt{l}_\phi + 1 && (49b) \\ \mathtt{w}_\eta &= \gamma \mathtt{l}_\eta && (49c) \end{aligned}$$

and $\bar{W} = \bar{L}^\gamma e^{\Psi_{\bar{W}}}$ where $\Psi_{\bar{W}}$ is a constant term depending on the variance-covariance of the log-stochastic deviations L_i and Φ_i. Plugging (44) into (42c) we have

$$Y_i = \alpha^{\frac{\alpha}{1-\alpha+\gamma}} e^{\frac{-\alpha\left(\varepsilon+\hat{\phi}_i\right)}{1-\alpha+\gamma}} \mathtt{E}^i(P)^{\frac{\alpha}{1-\alpha+\gamma}} K_i^{\frac{(1-\alpha)(1+\gamma)}{1-\alpha+\gamma}}$$

which implies

$$\begin{aligned} \mathtt{y}_\varepsilon &= \frac{\alpha}{1-\alpha+\gamma}\mathtt{e}_\varepsilon - \frac{\alpha}{1-\alpha+\gamma}, && (50) \\ \mathtt{y}_\phi &= \frac{\alpha}{1-\alpha+\gamma}\mathtt{e}_\phi + \frac{(1-\alpha)(1+\gamma)}{1-\alpha+\gamma}\mathtt{k}_\phi - \frac{\alpha}{1-\alpha+\gamma}, && (51) \\ \mathtt{y}_\eta &= \frac{\alpha}{1-\alpha+\gamma}\mathtt{e}_\eta + \frac{(1-\alpha)(1+\gamma)}{1-\alpha+\gamma}\mathtt{k}_\eta, && (52) \end{aligned}$$

and steady state

$$\bar{Y} = \alpha^{\frac{\alpha}{1-\alpha+\gamma}} \bar{P}^{\frac{\alpha}{1-\alpha+\gamma}} e^{\Psi_{\bar{Y}}},$$

where $\Psi_{\bar{Y}}$ is a constant term depending on the variance-covariance of the log-stochastic deviations of $E^i(P)$, K_i and Φ_i. This concludes the description of the supply side which is completely determined at stage 1 for a given profile of weights $\mathtt{e}_\varepsilon$, $\mathtt{e}_\phi$ and $\mathtt{e}_\eta$.

Demand side. From (42e) we have

$$C_i = P^{-\frac{1}{\psi}} e^{\frac{1}{\psi}\left(\varepsilon+\hat{\phi}_i\right)}$$

using (42f) after substituting for (42e). The relation above gives the restrictions

$$\mathtt{c}_1 = -\frac{1}{\psi}(\mathtt{p}_1 - 1) \tag{53}$$

$$\mathtt{c}_2 = \frac{1}{\psi} \tag{54}$$

$$\mathtt{c}_3 = 0 \tag{55}$$

and steady state $\bar{C} = \bar{P}^{-\frac{1}{\psi}} e^{\frac{1}{2\psi}\sigma(p)}$. The clearing condition for the consumption market is therefore

$$\int Y_i \mathtt{di} = \bar{Y} e^{\mathtt{y}_\varepsilon \varepsilon - \frac{1}{2}\sigma(\mathtt{y}_\varepsilon \varepsilon)} = \int C_i \mathtt{di} = \bar{P}^{-\frac{1}{\psi}} e^{-\frac{1}{\psi}(\mathtt{p}_\varepsilon - 1)\varepsilon + \frac{1}{2\psi}\sigma(p)}$$

from which one can determine the unique price process

$$P = \bar{P} e^{\mathtt{p}_\varepsilon \varepsilon - \frac{1}{2}\sigma(p)}$$

such that the stochastic price deviation is pinned down by the relation $-\frac{1}{\psi}(\mathtt{p}_\varepsilon - 1) = \mathtt{y}_\varepsilon$ yielding

$$\mathtt{p}_\varepsilon = -\frac{\alpha\psi}{1-\alpha+\gamma}\mathtt{e}_\varepsilon + \frac{1-\alpha+\gamma+\alpha\psi}{1-\alpha+\gamma}$$

and a steady state

$$\bar{P} = \alpha^{\frac{-\alpha\psi}{1-\alpha+\gamma+\alpha\psi}} e^{\Psi_{\bar{P}}}$$

where $\Psi_{\bar{P}}$ is a constant term depending on $\Psi_{\bar{Y}}$ the variance of p. Notice $\bar{P} = P^*$ in the deterministic case.

Aggregate dynamics

Here we consider the aggregate stochastic log-deviations from a steady state in the model. In equilibrium, these must satisfy the following system of first order conditions

$$\begin{bmatrix} w \\ r \\ l \\ \lambda \\ y \\ p \end{bmatrix} = \begin{bmatrix} 0 & 0 & \alpha-1 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{\psi} & 0 & 0 \\ 1 & 0 & -\gamma & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ r \\ l \\ \lambda \\ y \\ p \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{\psi} \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \mathtt{E}(p) \\ \varepsilon \end{bmatrix}$$

obtained from (42) where notice log-constants cancel out each other from both sides of the equalities. The solution to the system is

$$\begin{bmatrix} w \\ r \\ l \\ \lambda \\ y \\ p \end{bmatrix} = \begin{pmatrix} \frac{\gamma}{1+\gamma-\alpha} & \frac{1-\alpha}{1+\gamma-\alpha} \\ \frac{1+\gamma}{1+\gamma-\alpha} & \frac{-\alpha}{1+\gamma-\alpha} \\ \frac{1}{1+\gamma-\alpha} & \frac{-1}{1+\gamma-\alpha} \\ \frac{-\alpha\psi}{1+\gamma-\alpha} & \frac{1+\gamma-\alpha+\alpha\psi}{1+\gamma-\alpha} \\ \frac{\alpha}{1+\gamma-\alpha} & \frac{-\alpha}{1+\gamma-\alpha} \\ \frac{-\alpha\psi}{1+\gamma-\alpha} & \frac{1+\gamma-\alpha+\alpha\psi}{1+\gamma-\alpha} \end{pmatrix} \begin{bmatrix} \mathrm{E}\left(p\right) \\ \varepsilon \end{bmatrix}, \tag{56}$$

where in equilibrium $\mathrm{E}\left(p\right) = \mathtt{e}_{\varepsilon}\varepsilon$. The system gives the relations (26) and (27) in the model.

Reaction to island-specific shocks

From respectively (46)-(47), (48b)-(48c), (49b)-(49c) and (51)-(52) we have

$$\mathtt{k}_{\phi} = \frac{1}{\gamma}\left(\frac{1+\gamma}{\alpha}\mathtt{e}_{\phi} - 1\right),\ \mathtt{k}_{\eta} = \frac{1}{\gamma}\left(\frac{1+\gamma}{\alpha}\mathtt{e}_{\eta} - \frac{1+\gamma-\alpha}{\alpha}\right), \tag{57}$$

$$\mathtt{l}_{\phi} = \frac{1}{\gamma}\left(\frac{1}{\alpha}\mathtt{e}_{\phi} - 1\right),\ \mathtt{l}_{\eta} = \frac{1}{\gamma}\left(\frac{1}{\alpha}\mathtt{e}_{\eta} - \frac{1}{\alpha} + 1\right), \tag{58}$$

$$\mathtt{w}_{\phi} = \frac{1}{\alpha}\mathtt{e}_{\phi},\ \mathtt{w}_{\eta} = 1 + \frac{1}{\alpha}\left(\mathtt{e}_{\eta} - 1\right), \tag{59}$$

and

$$\mathtt{y}_{\phi} = -\frac{1}{\alpha\gamma}\left(\alpha - \left(1 + (1-\alpha)\gamma\right)\mathtt{e}_{\phi}\right),$$
$$\mathtt{y}_{\eta} = -\frac{1}{\alpha\gamma}\left(\left(1-\alpha\right)\left(1+\gamma\right) - \left(1 + \left(1-\alpha\right)\gamma\right)\mathtt{e}_{\eta}\right).$$

The values at the perfect-information and dispersed information limit equilibria obtain plugging respectively $\mathtt{e}_{\phi,\circ} = 0$ and $\mathtt{e}_{\eta,\circ} = 1$, and

$$\mathtt{e}_{\phi,\pm} = \frac{1+\gamma-\alpha+\alpha\psi}{1+\gamma}\sigma_{\phi}^{-1},\ \mathtt{e}_{\eta,\pm} = \frac{1+\gamma-\alpha}{1+\gamma}\mathbf{b}_{\pm},$$

where remember that $\lim_{\mathbf{b}\to\mathbf{b}_{\pm}} \mathbf{b}^2\sigma_i$ is a finite value. Notice from (54) and (55) that the cross sectional volatility of consumption across islands remains the same.

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