Inflation, default and the denomination of sovereign debt

Laura Sunder-Plassmann
University of Minnesota

PRELIMINARY

November 2013

Abstract

A government that faces an unsustainably high debt burden can either inflate the value of its debt away or default on it outright. In this paper I study this decision and how it interacts with the denomination of sovereign debt. I build a model in which the government lacks commitment to its borrowing, inflation, and default policy. Default is endogenous and costly; inflation is distortionary because of a cash in advance constraint on consumption. Issuing real instead of nominal debt has two effects in the model. On the one hand, real debt reduces the incentive to create costly inflation today because the value of the debt is fixed in real terms. It thus helps mitigate the commitment problem. On the other hand, precisely because the commitment problem is smaller, inducing future inflation is less costly when debt is real, and real debt facilitates more debt accumulation over time, causing the government to resort to the printing press after all to finance the debt burden. I quantify these effects in a calibrated version of the model and show that the second effect tends to dominate: Inflation, and default rates are higher in a real debt economy compared with an otherwise identical nominal debt economy. This is consistent with empirical regularities: across emerging markets over the last two decades, high nominal debt shares have been associated with low inflation and default rates.

1 Introduction

Emerging market governments actively manage the denomination of their sovereign debt. Brazil has a declared target of 30-35% inflation indexed debt, South Africa aims for 70% of its domestic debt to be nominal with a fixed coupon rate, citing the composition of debt as "one of the major risk concerns".[1] Being able to evaluate such policies is important because of their implications for sovereign borrowing and the possibility of debt crises: A government can expropriate nominal bondholders through either inflating or outright default, whereas for real bonds default is the only option. One common perception for example is that real debt is the better option for a government that lacks commitment because it removes the temptation to inflate. Nonetheless, there are few formal studies of sovereign debt and default crises that take into account debt denomination.

This paper contributes to filling this gap by analyzing the implications of debt denomination for sovereign borrowing, inflation and default in emerging market economies. I document that, empirically, real and foreign currency emerging market government debt is the exception rather than the rule. The top issuers in emerging sovereign bond markets have relied largely on nominal local currency bonds over the last two decades. In addition, I show that both inflation and default rates vary systematically with the denomination of the debt: Emerging market countries with high shares of nominal debt tend to experience lower than average inflation and default rates and vice versa.

I then build a monetary model of sovereign debt with endogenous default that can rationalize this pattern. I show in a stylized version of the model that debt denomination has two countervailing effects on default and inflation. On the one hand, when debt is real, inflation is less attractive than default because it does not erode the real value of the debt and relax the government's budget constraint. The only benefit to inflating for the government is seigniorage revenue. On the other hand, real bond returns carry no inflation premium, such that borrowing in real terms is cheaper. Investors do not need to be compensated for increased inflation risk since real returns are indexed to the price level. The government takes this into account when setting policy today. As a result, it is less reluctant to induce higher inflation for the same level of real debt, and is able to accumulate more debt in equilibrium which puts upward pressure on both inflation and default.

The model is a monetary version of quantitative sovereign default models as in Arellano

[1] Sources: "Optimal Federal Public Debt Composition: Definition of a Long-Term Benchmark" by the Brazilian Treasury, 2011. "Debt Management Report 2011/2012" by the South African Treasury.

(2008). It differs from existing papers in two key dimensions. First, it focuses on the difference between expropriation through inflation versus outright default as qualitatively different phenomena. Other studies restrict attention to inflation when analyzing the role of debt denomination (for example **Diaz-Gimenez et al. (2008)**), while the sovereign default literature predominantly assumes real, foreign currency external debt. Second, the paper distinguishes between the cost of inflation and debt denomination. In particular, even when bonds are real there is still an incentive to inflate in my framework. This corresponds to an economy where the government issues indexed debt but still has control over its own currency. Issuing real debt is not equated to dollarization or joining a monetary union.

Borrowing and the level of debt in the model are driven by the government's goal to smooth distortions combined with lack of commitment. Bonds provide a lump sum means of raising revenue to service debt and pay for public consumption, unlike inflation or default which are distortionary. Ex-post, however, the government is tempted to devalue its outstanding debt through default or, if nominal, inflation. This is reflected in bond prices in equilibrium and thus restricts its ability to borrow and smooth distortions over time. I introduce costs of inflation and default in standard ways. Inflation is costly because of a cash in advance constraint on consumption as in Lucas and Stokey (1987) and Svensson (1985). Households need to enter each period with sufficient money balances to cover consumption expenditures. Default is costly because it incurs a cost in terms of resources, akin to output costs used in many sovereign default studies including Arellano (2008).

I evaluate the quantitative effect of debt denomination on inflation and default in a stochastic version of the model that includes cash and credit consumption goods as well as labor income taxes. I compare the implications of debt denomination across two economies, one where the government issues nominal bonds, and one where its bonds are real. I find that in an otherwise identical real debt model economy, inflation is on average 2.7 times as high as in a nominal debt economy, and default rates are 5.7 times as high. This captures around 60% and 75% of the difference in the data. The government is able to borrow more in the real debt model economy, which contributes to upward pressure on equilibrium inflation and default rates, but the difference across economies is relatively small.[2] In terms of dynamics, for the same sequence of government expenditure shocks, the government delays inflating in the real debt economy for longer, but inflates more aggressively when it does. In my framework, issuing nominal debt is welfare improving, with small but positive

[2]In the data, nominal debt issuers tend to have higher debt to GDP ratios.

lifetime consumption equivalent welfare gains of around 0.12%.

The results shed light on the importance of the connection between lack of commitment to monetary and fiscal policy. In a setting where the government cannot commit to either policy, addressing the commitment problem on the fiscal front by issuing real debt may exacerbate the inflationary commitment problem.

Related Literature

This paper builds on the literature on sovereign debt and default based on papers like Eaton and Gersovitz (1981) and more recently Arellano (2008). I share with these papers that default is modeled as endogenous and dependent on fundamentals, and that governments lack commitment. One key difference is that papers in this literature predominantly focus on foreign currency external debt and default. I model costs of inflation and the presence of money as in the cash-credit good economy in Lucas and Stokey (1987). The computation of Markov equilibria in macroeconomic dynamic models was first developed by Klein et al. (2008). There is a large literature that explores optimal taxation, including through inflation and default, under full commitment, including in Chari and Kehoe (1999).

Two papers that are closely related to mine are Martin (2009) and Diaz-Gimenez et al. (2008). The former studies the determination of nominal public debt levels in a setting without commitment. His application focuses on war finance in advanced economies. He does not consider real debt or default. The latter analyzes monetary policy under different debt denominations in an economy as in Nicolini (1998). The authors find that the welfare effect of nominal versus indexed debt are in general ambiguous and show how they depend on parameters, specifically the intertemporal elasticity of substitution. Both papers model money demand as arising from a cash in advance constraint on consumption, and both address lack of commitment on the part of the government, as does this paper. Neither considers the interaction of monetary policy with default which is a key focus here.

Domestic or nominal debt and self-fulfilling sovereign debt crises are the topic of a number of recent papers, including Aguiar et al. (2013), Lorenzoni and Werning (2013), Da-Rocha et al. (2013) and Araujo et al. (2013). They focus on self-fulfilling, expectations driven debt crises as in Calvo (1988) and Cole and Kehoe (2000) whereas I consider default driven by weak fundamentals. Another important difference is that these papers compare economies without any role for domestic monetary policy - a currency union of dollarization - with economies with nominal debt and monetary policy. I focus on an environment where money always plays a role and the country has control over its monetary policy; the issue of debt denomination is distinct from the choices of whether to relinquish control of the printing press. Other related papers that study sovereign default and foreign currency debt are Arellano and Heathcote (2010) in a model of dollarization and limited enforcement, and Gumus (2013) in a two-sector model and bonds that are either denominated in terms of tradables or nontradables.

Less closely related in terms of modeling approach, but related in terms of topic are numerous papers that address the benefits and costs of indexed versus nominal debt, including Missale (1997), Bohn (1990)who discusses the benefits of nominal debt in terms of making returns state contingent, Barro (1997) and Alfaro and Kanczuk (2010) who argue in favor of indexed debt (without considering explicit default as an option for the government.

On the empirical side, there are a number of papers that study the currency composition of sovereign debt, as well as the connection between domestic default and inflation. Reinhart and Rogoff (2011) focus on domestic debt and default over a long period of time. They document that inflation and default episodes tend to occur together, that domestic default, even though less prevalent than external default, does occur with some frequency. Claessens et al. (2007) explore empirically the determinants of local currency debt as well as debt shares using non-publicly available data from the BIS. Cowan et al. (2006)construct a detailed debt database with a focus on Latin American countries, Guscina and Jeanne (2006) analyze debt composition for a subset of the countries I consider.

2 Nominal government debt in the data

In this section I will first document that the majority of emerging market government bond debt is denominated in nominal, local currencies, and second show that in these economies high nominal debt shares tend to be associated with low inflation and default rates.

2.1 Bond data

I construct local and foreign currency government bond debt estimates for a range of emerging market countries using Bloomberg bond-level data. The dataset contains all sovereign bond issues that were outstanding at some point between January 1, 1990 and December 31, 2012. For each issue, it includes information on the face value, the currency, the coupon structure, the maturity and issue date.

Table 1: Emerging bond market characteristics in 2012

	All	LatAm	Asia	Europe	Africa
Local currency	85.44	72.45	96.28	77.11	91.55
Domestic market	84.60	72.42	96.04	72.45	94.46
Zero coupon	18.05	28.05	11.83	12.38	35.63
Fixed rate	64.36	47.40	71.93	71.67	64.27
Pay at maturity	93.78	82.22	99.72	94.91	100.00
Inflation indexed	4.93	10.51	0.47	7.10	5.27
Defaulted/ Restructured	1.31	4.54	0.01	0.02	0.00

Percent. Bonds are valued using discounted US dollar face values. Percentages for regions are out of region totals.

The countries I consider are a broad set of 27 emerging market countries, as classified by the IMF in 2012. They account for just over 80% of total emerging market GDP on average over the last two decades. Seven of them are in Eastern Europe, eight in Latin America, nine in Asia, and three in Africa. I include Korea which is classified as advanced according to the IMF. The set contains both both defaulters and non-defaulters.[3]

The set includes the top 20 largest emerging market bond issuers. The top 5 as of 2012 are Brazil, China, India Korea and Mexico.

To calculate debt stocks for each of these countries, I restrict attention to bonds with a simple payout structure. I only include bonds whose face value is paid back at maturity ("bullet" bonds rather than callable, sinkable or hybrid bonds with equity-like structure), and that have deterministic coupon payments (zero, fixed rate or step rate coupons). I also exclude defaulted bonds.

This is not restrictive as the vast majority of bonds do in fact such a simple payout structure. Table (1) shows a breakdown of the dataset by various bond characteristics at the end of 2012.[4] It shows that across regions the majority of debt is in local currencies,

[3]See a full list in the appendix. For some countries the time series starts later than 1990 because there are no bonds in the dataset early in the sample. See precise start dates in the appendix.

[4]These statistics are based on bonds valued as the discounted sum of their face value. I use this simple measure because it is easy to construct for all bonds, including the ones that I do not include in the calculation of the debt stock later on. It abstracts from coupon payments and assume that bonds that can be redeemed before maturity are never expected to be. Note that many measures of government debt stocks do in fact not include any coupon payments, including the external debt estimates by the World

issued on domestic markets, with either fixed or zero coupons and payable at maturity. Less than 5% of al debt is indexed, with the highest share in Brazil. Defaulted (and not restructured) bonds are a small fraction of the total.

2.2 Debt stocks

I construct debt stock estimates as the sum of discounted future payments over all outstanding bonds. I first calculate the value $b_{n,t}$ of each bond issue n at time t, as

$$b_{n,t} = \sum_{s=0}^{S} \frac{C_{n,s} + P_{n,s}}{(1+r)^s}$$

where $C_{n,s}$ and $P_{n,s}$ are coupon and principal payments, respectively, at time $t+s$, and S is the number of periods to maturity. I discount future payments at the constant annual interest rate $r = 4\%$.

If the bond is denominated in a currency other than US dollars, I convert the future payment stream using the exchange rate at time t. This means that the value of a bond can change over its lifetime not just because coupon payments are made, but also because of revaluation due to exchange rate movements. The principal and coupon values of indexed bonds are adjusted by multiplying by the so-called index factor - the change in the CPI between the issuance date and time t. The CPI is available monthly for most countries so I interpolate between dates.

The stock of bond debt B_t is then calculated as the sum over all bonds outstanding at time t:

$$\begin{aligned} B_t &= \sum_{n=1}^{N} b_{n,t} \\ &= \sum_{n=1}^{N} \sum_{s=0}^{\bar{S}} \frac{C_{n,s} + P_{n,s}}{(1+r)^s} \end{aligned}$$

where N are the number of bond issues outstanding at time t, and $\bar{S}$ is the maximum number of periods left to maturity for all bonds. The frequency I choose is daily.

Figure (2.1) plots the aggregate debt stock for all countries in my sample over time as a fraction of aggregate GDP. This amounted to around 30% in 2012. In the Appendix,

Bank WDI.

Figure 2.1: Aggregate bond debt as a fraction of aggregate GDP

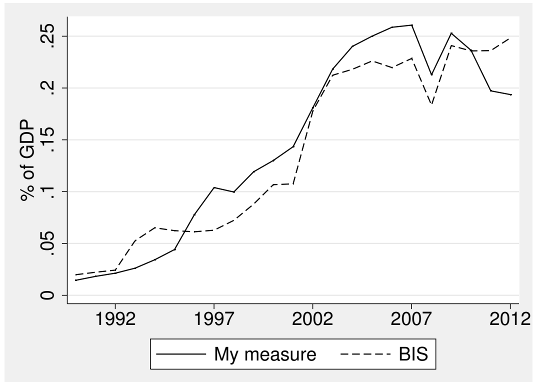


Figure (2.1) shows that total bond debt issued in my sample of emerging market countries was worth around 4 trillion current US$ in 2012 - much smaller than in advanced economies but growing over time.

In order to assess the accuracy of my estimates I compare them to aggregate data available from the BIS debt securities database. This database records quarterly general government debt securities outstanding for both domestic and international bonds from 1993Q3. The series for domestic securities is less complete - there are data for 16 of my countries and the series start late for some of them.

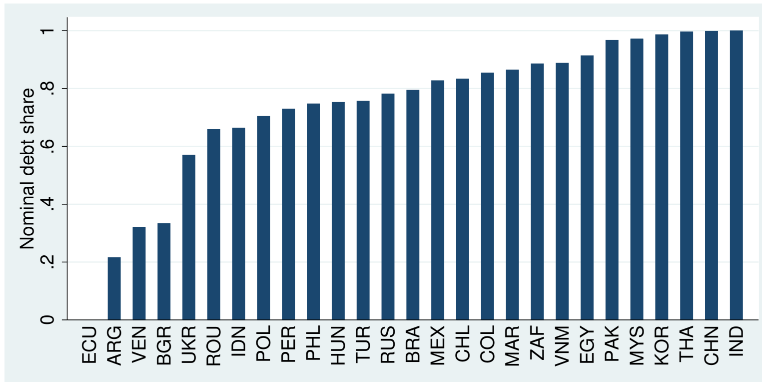
The sum of domestic and international debt from the BIS is plotted in the Figures alongside my estimates. The two series comove closely and the levels match up well. The uptick and subsequent drop around 1998/1999 in my series reflects the Russian default and increases in mainly local currency debt - one reason why this is not captured by the BIS is that the series for domestic debt does not start until 2005 for Russia.

The hump-shaped path of debt-to-GDP ratios in the early 2000s is driven by Latin American countries. All of them except Ecuador increased their debt stocks rapidly at the beginning of the millennium from very low levels, and experienced a sharp drop at the onset of the financial crisis, relative to GDP. Most countries saw their bond debt-to-GDP ratios fall around the time of the financial crisis in 2009.[5]

It is worth noting that the clear upward trend in bond debt levels is not exclusively

[5]The gap towards the end of the sample between my and the BIS measure is driven by my estimates of Brazil's debt stock being too low especially for the last five years of the sample. I am working on improving this.

Figure 2.2: Nominal debt shares in 2012



driven by the move towards bond markets and away from bank debt finance. Gross general government debt according to the IMF WEO shows a similar pattern as the bond debt measure shown here. On average over time, bond debt according to both my and the BIS measure constitutes around half of gross general government debt (which includes in addition liabilities such as social security and pensions), and this fraction actually falls slightly over time.

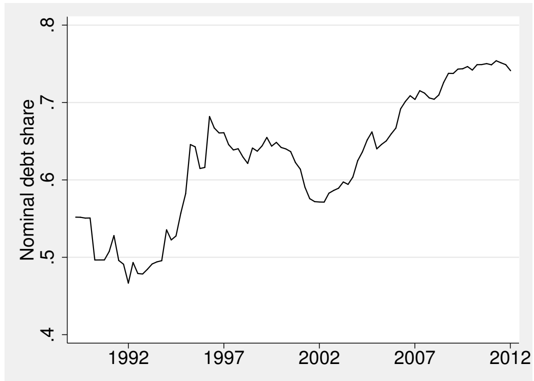
2.3 Nominal debt shares

I now turn to the question of how much emerging markets rely on foreign currency compared to local currency debt. I construct measures of the debt stock as above, separately for local currency and foreign currency denominated bonds. Figure (2.2) plots the resulting local currency debt share for each country at the end of 2012.

The Figure shows that the majority of emerging market debt is in local currencies. The unweighted average across countries in 2012 is 75%, a GDP-weighted average is slightly higher. All countries except four have local currency shares above 40%. Ecuador has a share of zero since it is dollarized, Argentina, Venezuela and Bulgaria are the other three countries that rely mostly on foreign currency denominated debt. At the other end of the spectrum, India has not issued a single foreign currency bond (in 2012 or the entire sample). Thailand, Korea, Malaysia, China and Pakistan all have local currency shares in excess of 95%.

Over time the share of debt issued in local currencies has increased substantially. This

Figure 2.3: Nominal debt share over time, simple average across countries



is shown in Figure (2.3) where we can see that the average local currency share stood at around 50% in 1990. The GDP-weighted average is lower at around 20% as the two largest countries in the sample in the early 1990s (in terms of constant US$ GDP), Mexico and Brazil, issued exclusively foreign currency debt.

Latin American countries generally have seen the largest shifts over time from foreign to local currency. In Asia local currency was more prevalent even early in the sample. Some countries have consistently issued mostly local currency debt: India, Korea, Malaysia, Pakistan, South Africa and Morocco.

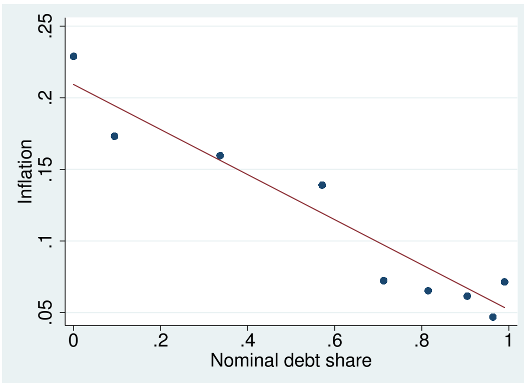
I cross-checked my results against data from the IMF's Public Sector Debt (PSD) database as far as possible. Data on the currency composition of public debt is available in the PSD for only 7 of my countries and a short sample. For Brazil, Indonesia, Peru, the Philippines and Poland the series start in 2010, for Mexico in 2005 and only for Hungary do they go back to 1999. Where available, the local currency shares from this data source match mine well.

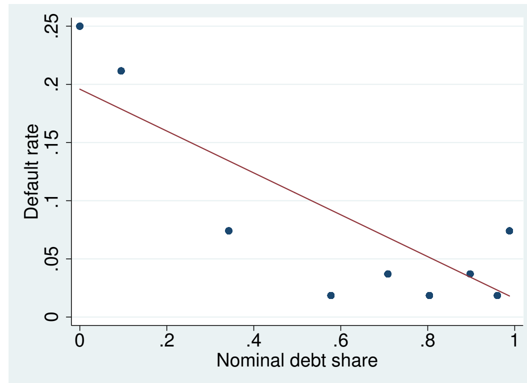
2.4 Nominal debt, inflation and default rates

Next I document the correlation between high nominal debt shares and low inflation and default rates.

High inflation and default were widespread in the countries I am considering for at least part of the sample. Only 9 countries never defaulted between 1990 and 2012, with default defined by Standard & Poors sovereign ratings: China, Colombia, Egypt, Hungary, India,

Figure 2.4: Inflation rates and default against deciles of nominal debt shares





Pooled data. Y-axis: Nominal debt share deciles, 642 obs per decile.

X-axis left panel: Average inflation per decile. X-axis right panel: #default obs / #obs per decile.

Korea, Malaysia and Thailand and Turkey. Turkey's sovereign crisis in 2001 crisis is not classified as a default by S&P.

Inflation was very high in many of the countries in the sample: 11 of them, all in Latin America or Europe, recorded annual CPI inflation of at least 100% at some point.[6] Turkey, Venezuela, Ecuador and Romania had the highest median inflation over the whole period, all over 20%. Median inflation was the lowest in Morocco, Malaysia, Thailand and Korea with under 4%. Peru's median is actually also just below 4%, mainly because its hyperinflation ended just after the beginning of my sample.

I pool the data across countries and time, and split the observations into deciles by nominal debt share. I then compute average inflation and default rates for each decile. For inflation I exclude episodes of inflation that exceed 100% annually. Including these would make the results only stronger since the hyperinflation episodes are concentrated in the lower deciles of local currency shares.

The left panel of Figure (2.4) shows the resulting graph for inflation. There is a clear negative relationship between nominal debt shares and inflation. For the observations with the highest 10% of nominal debt shares, inflation was on average 8% annually. The lowest 10% in terms of nominal debt shares saw prices increase at a rate of 25%. The right panel

[6] Argentina, Brazil, Bulgaria, Ecuador, Peru, Poland, Romania, Russia, Turkey, Venezuela and the Ukraine.

of Figure (2.4) shows the analogous graph for default rates.[7] As in the case of inflation, nominal debt debt shares and default rates are negatively correlated, with default rates for the highest nominal debt decile of around 7% compared with 27% for the set with the lowest local share of nominal debt.[8]

The correlations from both previous graphs are significant and robust to several modifications, including using median instead of mean inflation, period averages or period end inflation rates, and computing the deciles using local currency shares weighted by GDP. In addition, the Appendix contains the results from pooled and panel regressions of nominal debt shares on inflation and multinomial logistic regressions on default that control for a variety of factors. These controls include GDP, GDP per capita, debt levels, reserves, an index of democratic institutions, exchange rate regimes, independence of monetary authorities and inflation targeting dummies. Even after controlling for these factors, nominal debt shares are found to have a significant effect on inflation and the probability of default.

3 Model

In this section I will present a dynamic monetary model of sovereign borrowing without commitment, and use it to study the interaction between inflation, default and the denomination of debt. I will first discuss an economy where the government issues only nominal bonds, and then show what changes when instead it sells claims that are indexed to the price level.

Environment Time is discrete and infinite. The economy is populated by a representative agent with preferences over consumption and leisure, and a benevolent government that faces stochastic exogenous public consumption expenditures. Asset markets are incomplete with only money and one-period noncontingent bonds. The government has the monopoly over printing money and issuing bonds, and it lacks commitment to inflation, default and borrowing policies. When bonds are nominal it can devalue its debt through inflation, whereas real bonds are indexed to the price level. It can default on bonds of either denomination at any point. Inflation is costly because agents are subject to a cash-in-advance

[7] Default rates are defined as the total number of default states relative to the total number of states per decile. A default state is a country/year pair in which the country was in default - say Argentina in 2003. All states are all country/year pairs that fall in the given decile.

[8] See the Appendix for unpooled, raw scatter plots of inflation against nominal debt shares. Raw scatter plot of default are not informative obviously since it is a binary variable.

constraint on a subset of their consumption purchases, and default incurs a resource cost and temporary exclusion from credit markets.[9]

Households The representative agent maximizes the expected discounted lifetime utility from consumption of cash and credit goods, and leisure. He enters every period with nominal money balances $\bar{m}_t$ and, if the government is not in default, sovereign bonds $\bar{B}_t$. Households split into a shopper who makes consumption purchases and a producer who transforms labor into output with a linear technology. He receives labor income net of taxes. I assume that the shopper must purchase the cash good c_{1t} using money balances that they hold at the start of the period

$$\bar{p}_t c_{1t} \leq \bar{m}_t \tag{3.1}$$

The can use bond holdings and labor income to finance credit good consumption. My timing assumption follows Svensson (1985) and Lucas and Stokey (1987) and implies that unexpected inflation is costly since agents are unable to adjust their balances after uncertainty is resolved. If the cash in advance constraint binds, agents spend all their money on cash consumption goods ((3.1) holds with equality). Higher than expected inflation then reduces the real purchasing power of the money that agents hold to buy c_{1t}.

Securities markets opens and households decide how to allocate receipts from cash god sales and invoices from credit good sales between money $\bar{m}_{t+1}$ and bonds $\bar{B}_{t+1}$ to carry into the next period. In equilibrium prices will adjust such that money and bond markets clear and households hold exactly as much money and bonds as the government issues.

The household therefore faces the budget constraint

$$\bar{p}_t c_{1t} + \bar{p}_t c_{2t} + \bar{m}_{t+1} + q_{nt} \bar{B}_{t+1} = (1 - \tau)\bar{p}_t n_t + \bar{m}_t + \bar{B}_t$$

if the government is not in default and

$$\bar{p}_t c_{1t} + \bar{p}_t c_{2t} + \bar{m}_{t+1} = (1 - \tau)\bar{p}_t n_t + \bar{m}_t$$

if it has defaulted. The labor tax rate τ is fixed exogenously. I introduce it since labor income taxes are an important source of government revenue empirically, but their determination is not the focus of this paper.

[9] I abstract from endogenous renegotiation and partial default in this paper.

Following Cooley and Hansen (1991), in order to make the problem stationary I divide all nominal variables by the aggreagte money supply, that is $x \equiv \frac{\bar{x}}{\bar{M}}$, and define the money growth rate as $\mu_t \equiv \frac{\bar{M}_{t+1}}{\bar{M}_t} - 1$. Note that $\frac{\bar{x}_{t+1}}{\bar{M}_t} = \frac{\bar{x}_{t+1}}{\bar{M}_{t+1}} \frac{\bar{M}_{t+1}}{\bar{M}_t} = \frac{\bar{x}_{t+1}}{\bar{M}_{t+1}}(1+\mu_t)$. With this normalization, the households solves the problem

$$\max_{\{c_{1t}, c_{2t}, n_t, m_{t+1}, B_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, 1-n_t)$$

subject to

$$p_t c_{1t} + p_t c_{2t} + (1+\mu_t)(m_{t+1} + q_{nt} B_{t+1}) = (1-\tau) p_t n_t + m_t + B_t$$

if the government is not in default,

$$p_t c_{1t} + p_t c_{2t} + (1+\mu_t) m_{t+1} = (1-\tau) p_t n_t + m_t$$

if it is in default, and the cash in advance constraint

$$p_t c_{1t} \leq m_t$$

as well as a nonnegativity constraint on money balances. I do not impose the constraint that the agent cannot be in debt ($\bar{B}_t < 0$) but in a distortionary equilibrium that features positive inflation and default rates they will lend to the government rather than the other way around. The utility function satisfies the standard properties.

Competitive Equilibrium and Asset Prices We can use the first order conditions of the household problem to characterize the competitive equilibrium in this economy. The cash-in-advance constraint implies

$$u_{1t} - u_{2t} \geq 0 \tag{3.2}$$

and as already mentioned in a competitive equilibrium where the cash in advance constraint binds such that $u_{1t} - u_{2t} > 0$, we have

$$c_{1t} = \frac{m_t}{p_t}$$

Labor is pinned down by

$$u_{lt} = (1-\tau)u_{2t}$$

We can derive expressions for asset prices from the household's problem. The first order conditions to his problem give rise to an equation for the money growth rate

$$\mu_t = \beta \mathbb{E}_t \left[\frac{u_{1,t+1}}{u_{2,t}} \frac{p_t}{p_{t+1}} \right] - 1 \tag{3.3}$$

Define consumer price inflation as $\pi_{t+1} \equiv \frac{p_{t+1}}{p_t} - 1$ and note that consumer price inflation and the money growth rate are related through

$$1 + \bar{\pi}_{t+1} = (1 + \pi_{t+1})(1 + \mu_t)$$

so we can alternatively express equation (3.3) as an equation for the price of money

$$1 = \beta \mathbb{E}_t \left[\frac{u_{1,t+1}}{u_{2,t}} \frac{1}{1 + \bar{\pi}_{t+1}} \right] \tag{3.4}$$

From equations (3.3), (3.4) and (3.2) we see that in a nonstochastic steady state monetary policy follows the Friedman rule - negative money growth and inflation at the rate of time preference - only if the cash in advance constraint does not bind. Outside the steady state, higher expected future inflation implies higher expected marginal utility of the cash good: If inflation is high, agents are cash-strapped and value cash good consumption relatively more.

Denote the states of the world in which the government chooses to default tomorrow by δ_{t+1}. The household problem implies the following epxression for the bond price:

$$q_{nt} = \beta \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{2,t}} \frac{1}{1 + \bar{\pi}_{t+1}} (1 - \delta_{t+1}) \right] \tag{3.5}$$

The price of nominal bonds reflects default, inflation and risk premia. The higher the risk of default δ_{t+1} and future inflation $\bar{\pi}_{t+1}$, the lower the price at which the bond sells today - agents demand a higher return to be compensated for the risk. The ratio of marginal utilities $\frac{u_{2,t+1}}{u_{2t}}$ reflects movements in the risk free rate. If this ratio is high, agents want to shift consumption from today to tomorrow by saving in bonds, driving up their price.

Government The government is benevolent and wants to finance exogenous public expenditures in the least distortionary way. It can print money, issue one-period noncontingent bonds, raise taxes, and default. It makes its decisions before the goods and securities markets for households open but after the shock has been realized. The government budget constraint if it repays is given by

$$\bar{M}_t + \bar{B}_t + \bar{p}_t g_t = \tau \bar{p}_t n_t + \bar{M}_{t+1} + q_{nt} \bar{B}_{t+1}$$

whereas in default they need to finance all of their expenditures with labor tax revenues and by printing money

$$\bar{M}_t + \bar{p}_t g_t^d = \tau \bar{p}_t n_t + \bar{M}_{t+1}$$

with $g^d \geq g$.[10] As in the household case, I normalize to get

$$1 + B_t + p_t g_t = \tau p_t n_t + (1 + \mu_t)(1 + q_{nt} B_{t+1})$$

and

$$1 + p_t g_t^d = \tau p_t n_t + (1 + \mu_t)$$

in repayment and default respectively.

I assume that the government is unable to commit to its policies and analyze Markov perfect equilibria of the economy throguhout the paper. Thus I assume that the government, when making its decisions, can only condition them on current fundamentals - debt B_t and the shock z_t - and take as given the policies implemented by future governments as well as the competitive equilibrium. In particular, it is unable to take into account that its policies today affected yesterday's outcomes. To make this concrete, the government would like to be able to promise to only inflate today and never again, but it cannot credibly commit to doing that. The government tomorrow only considers current and future tradeoffs and ignores promises made yesterday. The government todaty therefore knows and must take into account that inflating today will make it more expensive for the government tomorrow to borrow. Mechanically this can be seen from expressions (3.4) and (3.5), which enter the government budget constraint today but are functions of future inflation and default policies.

[10]Whether it is households or the government paying the cost is not crucial.

Real Debt In an economy where bonds are real instead of nominal, their value is fixed in units of the consumption good. Denote a claim to one unit of consumption by $b = \frac{\bar{B}}{\bar{p}}$ and its price by q_r. Then the household faces the following budget constraint if the government does not default

$$\bar{p}_t c_{1t} + \bar{p}_t c_{2t} + \bar{m}_{t+1} + \bar{p}_t q_{rt} b_{t+1} = (1-\tau)\bar{p}_t n_t + \bar{m}_t + \bar{p}_t b_t$$

and the government budget constraint is

$$\bar{M}_t + \bar{p}_t \bar{b}_t + \bar{p}_t g_t = \tau \bar{p}_t n_t + \bar{M}_{t+1} + \bar{p} q_{rt} b_{t+1}$$

There are two main difference between issuing real compared to nominal debt. The first is that the value of oustanding liabilities is fixed in terms of consumption units for real debt. This can be seen by comparing the budget constraints of households and government in the two economies. An increase in the price level does not devalue the debt - the government cannot inflate it away.

The other difference arises from bond prices. We can derive bond prices in the same way as for the nominal debt economy to obtain

$$q_{rt} = \beta \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{2,t}} (1 - \delta_{t+1}) \right] \tag{3.6}$$

Compare this with (3.5): Because real debt cannot be inflated away, it does not carry an inflation premium and future inflation does not lower revenue from selling bonds today.

It is useful to note that bond prices can alternatively be written as follows, using the expression for the money growth rate (3.3) and inflation (3.4) which are the same in either economy:

$$q_{nt} = \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{1,t+1}} (1 - \delta_{t+1}) \right] \tag{3.7}$$

$$q_{rt} = \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{1,t+1}} (1 + \bar{\pi}_{t+1})(1 - \delta_{t+1}) \right] \tag{3.8}$$

This way of writing bond prices highlights the link between money and bonds as alternative assets in the model. For both real and nominal debt we see that the more the cash in advance constraint is expected to bind, that is the lower $\frac{u_{2,t+1}}{u_{1,t+1}}$, the lower bond prices are today. The reason is that money is in higher demand. Households would prefer to hold more money to avoid being short of cash in the next period.

In addition we can see that real bond prices are *higher* the higher inflation is tomorrow. This may seem counterintuitive, but is in fact also related to money and bonds being alternative assets. When inflation is expected to be high, real bonds are a more appealing investment than money since households know it cannot be devalued. This drives up bond prices relative to the price of money.

It is important to remember then that even if debt is real, the bond price is not independent of inflation because households are the holders of both money and bonds in the economy. The net effect of higher inflation on real bond prices is ambiguous: On the one hand, agents want to hold more money to avoid the cash in advance constraint binding which drives bond prices down, on the other, inflation makes real bonds the more attractive investment which drives their price up. In a nominal debt economy, bond prices unambiguously fall with higher inflation.

I will discuss the implications of this in the next sections after defining equilibria of both economies.

3.1 Recursive Equilibrium

I will state the problem recursively in order to define an equilibrium. The state of the nominal debt economy is B, the bond to money ratio, and the shock to governemt expenditure z. Assume $B \in \mathbb{B} \subset \mathbb{R}_+$ and $z \in \mathbb{Z} \subset \mathbb{R}$. In an equilibrium the government maximizes the representative households's utility subject to the government's budget constraint and the competitive equilibrium conditions. I am going to let B', p and d be the government's choice variables. The cash in advance constraint links prices to cash consumption, and the first order conditions pin down equilibrium money growth residually.[11] The commitment problem, as discussed above, means that borrowing today affects future price and default policies and that the government recognizes this, that is $p' = P(B', z')$ and $d' = D(B', z')$. Denote the exogenous probability of re-entering capital markets after a default by η.

[11] Alternatively and equivalently, think about the economy starting the period with a fixed money stock. Then the price level is pinned down by the cash in advance constraint during the goods market, and the difference between start and end of period money stocks pin down the money growth rate. The government decides on the money stock next period which shapes expectations for the price level tomorrow (see equation 3.4), and similarly for borrowing levels which affect default expectations.

Nominal Debt The government's option value of default is then given by

$$V(B, z) = \max \left\{ V^r(B, z), dV^d(z) \right\} \tag{3.9}$$

where the value of repayment is

$$V^r(B, z) \quad = \quad \max_{p, B'} u\left(c_1, c_2, 1 - n\right) + \beta \int_{z'} V(B', z') dFz' \tag{3.10}$$

subject to

$$\begin{aligned}
1 + B + pg(z) &= \tau pn + \left[1 + \mu(p, B', z)\right]\left[1 + q_n(p, B', z)B'\right] && (3.11)\\
c_1 + c_2 + g(z) &= n && (3.12)\\
c_1 &= \frac{1}{p} && (3.13)\\
u_c - u_l &\geq 0 && (3.14)\\
u_l &= (1 - \tau)u_2 && (3.15)\\
\mu(p, B', z) &= \beta \int_{z'} \left[\frac{u_1'(P^r(B', z'), z')}{u_2(p, z)} \frac{p}{P^r(B', z')}\right] dF(z', z) - 1 && (3.16)\\
q_n(p, B', z) &= \beta \int_{z'} \left[\frac{u_2'(P^r(B', z'), z')}{u_2(p, z)} \frac{p}{P^r(B', z')}(1 - D(B', z'))\right] dF(z', z) && (3.17)
\end{aligned}$$

that is, the budget constraint, the resource constraint, the cash in advance constraint, the intratemporal competitive equilibrium condition pinning down labor, the cash in advance inequality condition from the household problem, and expressions for the money growth rate and bond prices.

In default the value is

$$V^d(z) = \max_p u\left(c_1, c_2, 1 - n\right) + \beta \int_{z'} \left[\eta V(0, z') + (1 - \eta)V^d(z')\right] dFz' \tag{3.18}$$

subject to the analogous conditions

$$\begin{aligned}
1 + pg^d(z) &= \tau pn + [1 + \mu(p, 0, z)] && (3.19)\\
c_1 + c_2 + g^d(z) &= n && (3.20)\\
c_1 &= \frac{1}{p} && (3.21)\\
u_l &= (1-\tau)u_2 && (3.22)\\
u_c - u_l &\geq 0 && (3.23)\\
\mu(p, B', z) &= \beta \int_{z'} \left[\frac{u_1'(P^d(z'), z')}{u_2(p, z)} \frac{p}{P^d(z')} \right] dF(z', z) - 1 && (3.24)
\end{aligned}$$

with $g^d = g + h(g), h(g) \geq 0$. Note that both in repayment and default money growth rates depend on future prices, but the relevant price functions are different - in repayment, future price levels depend on the level of borrowing. In default - as I will show below - they reflect the probability of re-entering capital markets with zero debt. We can define an equilibrium price function $P(B, z)$ as

$$P(B, z) = \mathbb{1}\left(V^r(B, z) \geq V^d(z)\right) P^r(B, z) + \mathbb{1}\left(V^r(B, z) < V^d(z)\right) P^d(z) \tag{3.25}$$

and for default

$$D(B, z) = \mathbb{1}\left(V^r(B, z) < V^d(z)\right) \tag{3.26}$$

Definition 1. *Let* $V : \mathbb{B} \times \mathbb{Z} \to \mathbb{R}, P : \mathbb{B} \times \mathbb{Z} \to \mathbb{R}_{++}, H : \mathbb{B} \times \mathbb{Z} \to \mathbb{B}$ *and* $D : \mathbb{B} \times \mathbb{Z} \to \{0, 1\}$*. A Markov perfect equilibrium of the nominal debt economy are functions* V, P, D, H *as well as* $c_1(B, z), c_2(B, z), n(B, z)$ *and prices* $\mu(P, H, z)$ *and* $q_n(P, H, z)$ *that solve the government's problem (3.9) through (3.26) and where* $B' = H(B)$*.*

Real Debt Analogously to the nominal debt economy, the recursive problem of the government in a real debt economy is the following: Its option value to default is given by

$$V(b, z) = \max\left\{V^r(b, z), dV^d(z)\right\} \tag{3.27}$$

where the value of repayment is

$$V^r(b,z) = \max_{p,b'} u\left(c_1, c_2, 1-n\right) + \beta \int_{z'} V(b', z') dFz' \tag{3.28}$$

subject to

$$\begin{aligned}
1 + pb + pg(z) &= \tau pn + \left[1 + \mu(p, b', z)\right] + q_r(p, B', z)pb' && (3.29)\\
c_1 + c_2 + g(z) &= n && (3.30)\\
c_1 &= \frac{1}{p} && (3.31)\\
u_l &= (1-\tau)u_2 && (3.32)\\
u_1 - u_2 &\geq 0 && (3.33)\\
\mu(p, b', z) &= \beta \int_{z'} \left[\frac{u_1'(P^r(b', z'), z')}{u_2(p, z)} \frac{p}{P^r(b', z')}\right] dF(z', z) - 1 && (3.34)\\
q_r(p, b', z) &= \beta \int_{z'} \left[\frac{u_2'(P^r(b', z'), z')}{u_2(p, z)} (1 - D(b', z'))\right] dF(z', z) && (3.35)
\end{aligned}$$

and the value of default is

$$V^d(z) = \max_{p} u\left(c_1, c_2, 1-n\right) + \beta \int_{z'} \left[\eta V(0, z') + (1-\eta) V^d(z')\right] dFz' \tag{3.36}$$

subject to

$$\begin{aligned}
1 + pg^d(z) &= \tau pn + \left[1 + \mu(p, B', z)\right] && (3.37)\\
c_1 + c_2 + g^d(z) &= n && (3.38)\\
c_1 &= \frac{1}{p} && (3.39)\\
u_l &= (1-\tau)u_2 && (3.40)\\
u_1 - u_2 &\geq 0 && (3.41)\\
\mu(p, B', z) &= \beta \int_{z'} \left[\frac{u_1'(P^d(z'), z')}{u_2(p, z)} \frac{p}{P^d(, z')}\right] dF(z', z) - 1 && (3.42)
\end{aligned}$$

with $g^d = g + h(g), h(g) \geq 0$, and P and D satisfy

$$P(B, z) = \mathbb{1}\left(V^r(B, z) \geq V^d(z)\right) P^r(B, z) + \mathbb{1}\left(V^r(B, z) < V^d(z)\right) P^d(z) \tag{3.43}$$

$$D(B,z) = \mathbb{1}\left(V^r(B,z) < V^d(z)\right) \tag{3.44}$$

Definition 2. *Let $V : \mathbb{B}\times\mathbb{Z} \to \mathbb{R}$, $P : \mathbb{B}\times\mathbb{Z} \to \mathbb{R}_{++}$, $H : \mathbb{B}\times\mathbb{Z} \to \mathbb{B}$ and $D : \mathbb{B}\times\mathbb{Z} \to \{0,1\}$. A Markov perfect equilibrium of the real debt economy are functions V, P, D, H as well as $c_1(B,z), c_2(B,z), n(B,z)$ and prices $\mu(P,H,z)$ and $q_n(P,H,z)$ that solve the government's problem (3.27) through (3.44) and where $b' = H(b)$.*

Note that we can use the competitive equilibrium conditions for prices as well as the resource constraint to express the government's problem purely in terms of the state, choice for borrowing as well as current and future prices and default decisions (see the Appendix).

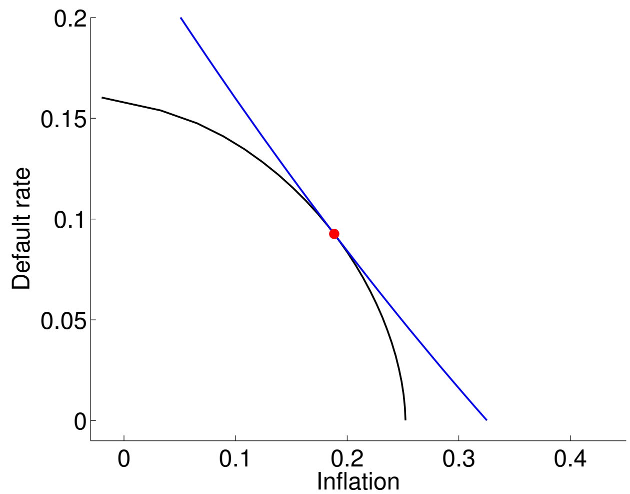
4 Inflation, Default and Borrowing in Equilibrium

In this section I analyze the channels through which the denomination of bonds affects the equilibrium of this model. We will see that the government in general prefers to spread the distortionary costs of inflation and default and thus uses both. Real debt on the one hand makes inflation less appealing because the debt burden is fixed in real terms and cannot be devalued. But on the other hand the government is less worried about causing inflation in the future because expected inflation does not depress bond prices when debt is real. In addition, the equilibrium level of borrowing affects the degree to which the government chooses to repudiate and monetize its liabilities. If the government borrows more then this adds to upward pressure on prices and default risk.

In order to draw out the main forces at work in the model I use a simplified version in this section. The key simplification is that I assume default is a continuous variable with the government choosing a default rate $d \in [0,1]$ each period. Defaulting incurs costs $t(d), t_d > 0, t_{dd} \geq 0$. There is no exclusion after default. With this assumption I can use first order conditions at an interior solution to the problem and characterize tradeoffs more precisely. I also abstract from uncertainty, labor income taxes and credit consumption goods. See the Appendix for a full definition of this simplified model.

Consider a nominal debt economy first. We can rewrite the government budget constraint purely in terms of allocation by using the competitive equilibrium conditions to substitute out prices. Then the constraints on the government's problem can be written

Figure 4.1: Static tradeoff between inflation and default



For a given B and income (borrowing B' with associated seigniorage and bond revenue)

as[12]

$$G^n(B,p,d,B',P(B'),D(B')) \equiv -u_2\left(\frac{1+(1-d)B}{p}+g+t(d)\right)+\beta\left(\frac{u_1'+u_2'(1-D(B'))B'}{P(B')}\right)=0 \tag{4.1}$$

and

$$F(p,d) \equiv u_1(p,d)-u_2(p,d) \geq 0$$

The first order conditions are

$$\begin{aligned} -\frac{u_c-u_l}{p^2}+\lambda G_p+\gamma F_p &= 0 \\ -u_l T_d+\lambda G_d+\gamma F_d &= 0 \\ \lambda\left(G_{b'}+G_P\frac{\partial P(B')}{\partial B'}+G_D\frac{\partial D(B')}{\partial B'}\right)+\beta\lambda' G_b' &= 0 \end{aligned} \tag{4.2}$$

At an interior solution where $F(p,d)>0$ such that the cash in advance constraint is binding, the ratio of the first two conditions characterize the intratemporal optimal tradeoff between inflation and default

$$\frac{u_1-u_2}{p^2(u_2 t_d)}=\frac{G_p^n}{G_d^n} \tag{4.3}$$

[12]See the Appendix for a full derivation

where

$$G_p^n = \frac{\partial G^n}{\partial p} \quad = \quad u_2 \frac{(1+(1-d)B)}{p^2} + (u_{12} - u_{22}) \frac{1}{p^2} \left[\frac{(1+(1-d)B)}{p} + g + t(d) \right] \quad (4.4)$$

$$G_d^n = \frac{\partial G^n}{\partial d} \quad = \quad u_2 \frac{B}{p} - u_2 t_d + u_{22} t_d \left[\frac{(1+(1-d)B)}{p} + g + t(d) \right] \quad (4.5)$$

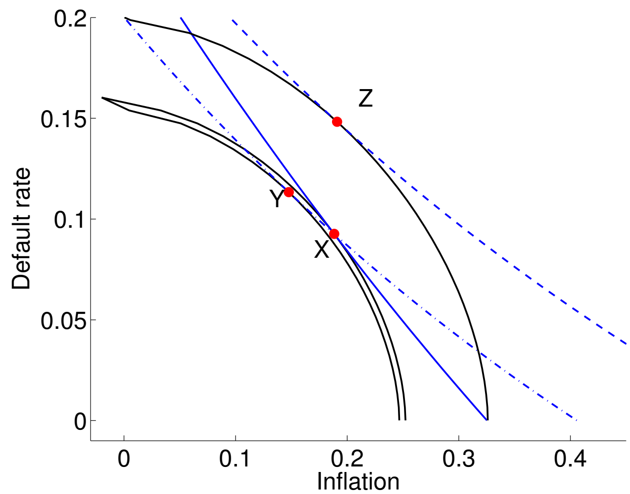
The left hand side of (4.3) is the relative cost of inflating compared to defaulting. The government is willing to trade off a fall in prices that reduces distortions by $\frac{u_1 - u_2}{p^2}$ for an increase in default that creates distortions $u_2 t_d$. Figure (4.1) illustrates this. It shows an "indifference curve" for default and inflation rates.[13] Along the curve, utility is held constant. The marginal rate of substitution between default and inflation, in other words the slope of this indifference curve, is given by the left hand side of equation (4.3).

The right hand side of equation (4.3) represents the relative benefit to the government of devaluing through inflation compared to default in terms of relaxing its budget constraint. For a given level of income (the right hand side of (4.1)) and debt B, the picture plots the pairs of default and inflation rates that satisfy the government budget constraint. The slope of this line is equal to the negative of the right hand side of (4.3). At an optimum the government picks the lowest default and inflation pair that is feasible, at the point of tangency, the red dot in the picture.

The previous discussion focused purely on the intratemporal decision, holding borrowing and revenue fixed, but there are of course dynamic effects. It is not possible to solve analytically for the policy function for debt or the steady state, but we can use the intertemporal first order condition of the government's problem to gain insight into what drives equilibrium borrowing and how it affects the tradeoff between inflation and default. The intertemporal Euler condition for the government's problem, equation(4.2), describes the tradeoffs involved. It states that the government at an optimum equates the marginal benefit of borrowing today with the cost of repaying tomorrow. Borrowing more today lowers bond revenues but increases seigniorage revenue, which is reflected in the terms $G_P \frac{\partial P(B')}{\partial B'}$ and $G_D \frac{\partial D(B')}{\partial B'}$. The faster the sum of these terms falls with the level of debt, the less debt the government will accumulate in equilibrium. Note that we can derive analytical expressions for G_P and G_D but not for the partials with respect to the equilibrium policy

[13] The figure plots the tradeoffs in terms of inflation rather than price levels for expositional clarity. Analogous figures in terms of the price level look the same. See the Appendix for functional forms and parameters used in this example.

Figure 4.2: Nominal and real debt: Substitution and income effects on inflation and default



Fixed $\frac{B}{p} = b$. X: Nominal bonds. Z: Real bonds. $X \to Y$: Substitution effect. $Y \to Z$: Income effect.

functions, $\frac{\partial P(B')}{\partial B'}$ and $\frac{\partial D(B')}{\partial B'}$.[14] Numerically both prices and default rates are increasing in debt levels, such that higher equilibrium debt levels translate to upward pressure on inflation and default.

In terms of Figure (4.1), higher borrowing and debt tend to shift the budget constraint out. The government partly finances a higher debt burden through expropriation since additional bond sales and pure seigniorage do not generate sufficient revenue.

4.1 Debt Denomination

When debt is real, incentives to inflate and default change. The relative cost of inflation compared to default is unaffected by the denomination of the debt since we are in a closed economy and debt does not affect utility directly via a resource constraint. What does change is the relative benefit from inflating, both today and tomorrow.

[14] An alternative way of expressing the Euler equation without substituting out prices is $\lambda \left(\frac{1+\mu}{p} q_n + \frac{\partial \mu}{\partial B'} \left(\frac{1}{p} + \frac{q_n B'}{p} \right) + \frac{\partial q_n}{\partial B'} \frac{(1+\mu) B'}{p} \right) = \beta \lambda' \frac{1-d'}{p'}$. The first term in brackets represents the direct additional revenue from selling a marginal unit of debt, q_n, in real terms. The second term captures additional seigniorage revenue. An increase in borrowing increases money growth and thus seigniorage revenue. The third term is the effect of increased borrowing on bond prices. Future expected inflation and default drive down bond prices and thus revenue today. The right hand side is the marginal cost of repaying the debt, in real terms.

In terms of the *intra*temporal trade off that the government faces, inflation becomes less appealing when debt is real. We can see this by looking at the budget constraint for real debt

$$G(b,p,d,b',P(b'),D(b')) \equiv -u_2\left(\frac{1}{p}+(1-d)b+g+t(d)\right)+\beta\left(\frac{u_1'}{P(b')}+u_2'(1-D(b'))b'\right)$$

and comparing

$$\begin{aligned} G_p &= u_2\frac{1}{p^2}+(u_{12}-u_{22})\frac{1}{p^2}\left[\frac{1}{p}+(1-d)b+g+t(d)\right] \\ G_d &= u_2 b-u_2 t_d+u_{22}t_d\left[\frac{1}{p}++(1-d)b+g+t(d)\right] \end{aligned}$$

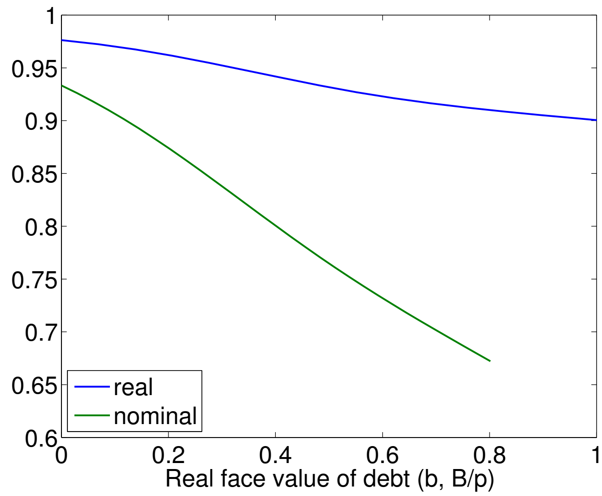
with the corresponding expressions for nominal debt, equations (4.4) and (4.5). G_d is unchanged, but the second term in (4.4), $u_2\frac{(1-d)B}{p^2}>0$ is missing in the analogous expression for real debt. Intuitively, the same drop in inflation now requires a smaller increase in default rates to satisfy the budget constraint, since the lower inflation did not increase the real value of outstanding bonds.

Figure (4.2) plots this along with the previous tradeoff from the nominal debt economy. It shows that the dashed budget constraint for the real debt economy has a flatter slope. Everything else equal, if the government in the nominal debt economy chooses a point like X, the government in the real debt economy with the same real debt burden and revenue would choose point Y - higher default and lower inflation.

How is revenue affected when debt is real instead of nominal? Recall that nominal bond prices in this model incorporate risk, default and inflation premia. In particular, the higher expected default and inflation tomorrow, the lower the bond price today since investors need to be compensated for the risk. This provides incentives for the government today to avoid inducing inflation tomorrow by borrowing more. The payout of real bonds on the other hand is not affected by inflation - there are no inflation premia in these bonds. As a result, the government today has less of a disincentive to cause inflation tomorrow by borrowing. Figure (4.3) shows this by plotting bond prices for both real and nominal bonds as a function of the real value of debt. Real bond prices fall more slowly with the level of debt.

An alternative way of seeing this is to consider a steady state in which the real value of bond debt is the same across the two economies. Suppose the economy has been a

Figure 4.3: Nominal and real bond prices: Incentives to issue bonds are higher, and fall more slowly, when debt is real



nominal debt economy at a point like X in Figure (4.2), and suppose one morning all bond debt is suddenly indexed. As discssed above, holding revenue constant, the static effect is to increase default rates and lower inflation rates (moving to Y). We held revenue fixed, but the Markov government takes into account future policies which change with the denomination of the debt. In particular, with debt being real, a decrease in inflation rates will not be reflected in bond prices and thus give no boost to revenue, while an increase in default rates still hurts revenues. In terms of the budget constraint, as a result, without adjusting borrowing, revenues are too low. The government must either increase borrowing, or increase distortions today to make up the difference, and move to a point like Z in Figure (4.2). In the Appendix I prove under simplifying assumptions on utility and default costs, inflation and default are higher in a steady state where real debt levels are identical across a real and a nominal debt economy.

In this section we discussed how inflation, default rates and the level of borrowing are determined in the model. The government in general uses both default and inflation to smooth the tax burden on households both intratemporally and intertemporally. When debt is real instead of nominal, there are broadly two countervailing effects on equilibrium default and inflation: On the one hand, the government is less likely to use inflation because it is less effective at relaxing its budget constraint - real debt cannot be devalued through inflation. On the other hand, inducing future inflation is less costly for the government when debt is real since real bond prices carry no inflation premium. It can create inflation without hurting bond revenues too much. This tends to increase inflation and default. Which effect dominates is a quantitative question which I will explore in the next section.

5 Quantitative Exercise

In this section I use the quantitative version of the model to evaluate which of the forces identified in the previous section dominate, and whether the model can capture the data. Recall that empirically real debt issuers experience higher inflation and default rates on average.

5.1 Parameters and Functional Forms

Utility is assumed to exhibit constant elasticity of substitution between cash and credit good consumption, and is separable in leisure:

$$u(c_1, c_2, 1-n) = \frac{\left((\alpha c_1^{\rho} + (1-\alpha)c_2^{\rho})^{\frac{1}{\rho}}\right)^{1-\sigma}}{1-\sigma} + \nu\frac{(1-n)^{1-\theta}}{1-\theta}$$

Government spending in the model is given by $g = Ae^z$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}, \epsilon \sim N(0, \sigma_\epsilon)$$

I calibrate the economy to match inflation and default rates in the average nominal debt issuing country in my sample. Average public consumption to GDP ratio in the data is 15%. Given a calibrate value for average hours worked, this pins down the value for A. Instead of estimating the shock process I pick standard values for its persistence and volatility that are observed in a range of emerging market countries. I do not calibrate it since my question and calibration target is a cross-sectional observation. My chosen values for ρ and σ_ϵ imply a persistent, relatively volatile process. The average income tax rate is 15%. The long run growth rate of GDP is around 2% annually, so I choose $\beta = 0.98$ which implies an annual real risk free rate in the model economy of 2%.

Government expenditures in default are parameterized as

Table 2: Parameters

Parameter	Value	Description/ Target	
β	0.98	Discount factor	Long run growth 2%
τ	0.15	Tax rate	Average income tax rate
A	0.15	$\mathbb{E}[g]$	Average $\frac{g}{y}$
ρ	0.95	Persistence of AR(1) of g	
σ_ϵ	0.01	Volatility of AR(1) of of g	
ν, θ	1.8,6.0	Leisure level/ curvature	Fraction working 30%
η	0.25	Probability of exclusion	Average duration of default episode 4 years
α	0.01	Cash good weight	
ρ	-2.8	Cash - credit elasticity	Inflation 6%
χ_1	-0.59	Default cost	Default rate 2.5%
χ_2	0.66	Default cost	Cash-credit ratio 30%
σ	3	Risk aversion	

$$
\begin{aligned}
g^d &= g + h(g) \\
&= g + \max\left\{0, \chi_1 \frac{\mathbb{E}[g]}{g} + \chi_2 \left(\frac{\mathbb{E}[g]}{g}\right)^2\right\}
\end{aligned}
$$

This is analogous to papers in the sovereign default literature that assume the source of uncertainty are shocks to output and that the cost of default is higher in good times when output is high. My specification implies that default is more expensive in good times, when government expenditures are low. I choose the probability of re-entry such that the average default episode takes 4 years.

The remaining parameters are not clearly tied to one particular statistic. I choose them to replicate observed inflation and default rates in the average nominal debt issuing country in my sample - 5.5% and 2.5%, respectively - as well to match average hours worked of 30%

Table 3: Model statistics

	Model (nominal)	Model (real)
Inflation	5.1	13.7
Default probability	2.6	14.7
Cash-credit good ratio	29.6	29.5
Debt-to-GDP ratio	24.0	25.5
Nominal interest rate	9.6	
Real interest rate		17.1
Seigniorage revenue/GDP	1.0	3.5
Net bond revenue/GDP	-1.1	-7.5

All statistics are in percent, averages over periods of good credit standing, excluding the first 10 periods after the end of a default episode.

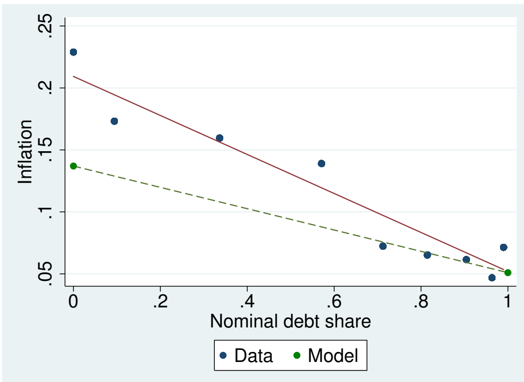
and a ratio of cash to credit goods of 30%. Table (2)summarizes the chosen parameter values.

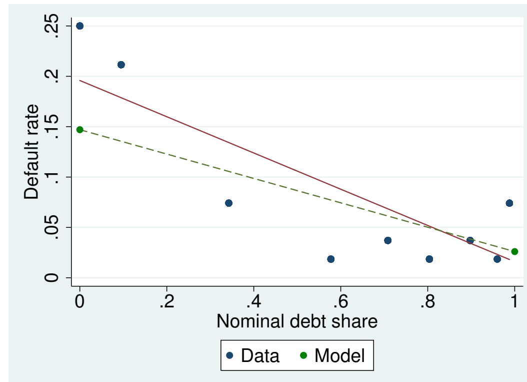
5.2 Simulation results

Table (3) presents key statistics from the simulated model data. The first column shows statistics for the nominal debt economy, the second column for the real debt economy. Inflation, default rates and cash-credit ratios in the nominal debt economy match the calibrated values as can be seen in the first three rows of the table.

Inflation and default rates in the real debt economy are higher by a factor of 2.7 and 5.7, respectively. The incentive to inflate less in the real debt economy is more than offset by the lower future cost of creating inflation. This is mostly driven by the lower cost of inflating due to the absence of inflation premia in bond prices rather than by an increased incentive to accumulate debt in equilibrium: Debt to GDP ratios are around 25% in both models and only slightly higher in the real debt economy. Interest rates on bonds reflect the default and inflation risk: Real bond yields are on average around 17%. Hypothetical real rates in the nominal economy, computed as nominal rates less inflation, are much lower at 4.5%. The same is true for nominal rates. Since inflation is higher in the real debt economy, seigniorage revenue as a percentage of GDP is higher. Net bond revenue is lower as a result of both higher premia and borrowing levels.

Figure 5.1: Model vs data





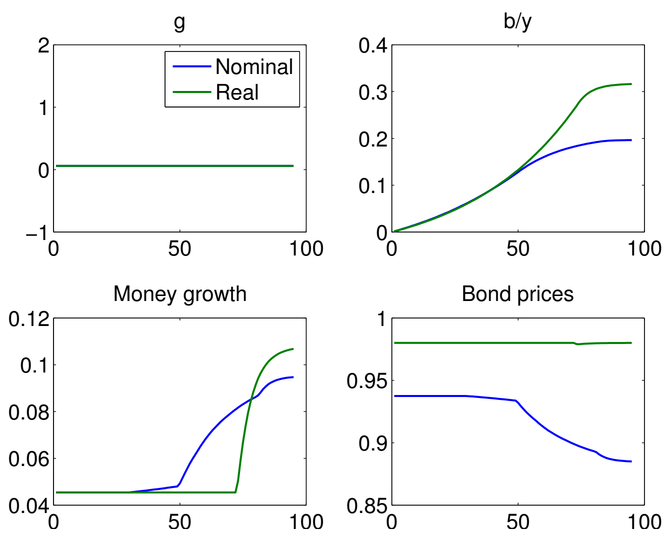
Figures (5.1) visualizes the fit of the model to the data along the two key dimensions, inflation and default. The model reproduces just over 60% of the increase in inflation among real debt issuers compared to nominal debt issuers in the data, and around 75% of the increase in default rates.

Figure (5.2) illustrates the dynamics of the model. Starting from zero debt and when faced with constant expenditures at their mean level, the government initially starts accumulating debt to finance the outlays and maintains low inflation, before eventually switching to high inflation. In the real debt economy, this switch occurs later, at higher debt levels. This captures the sense in which real debt reduces inflation: It is less useful for relaxing the budget constraint and hence the government is only willing to incur the costs for sufficiently high debt levels. Once it does inflate, however, it does so more aggressively in the real debt economy.

To assess the welfare consequences of debt denomination policies I compute the consumption equivalent welfare gain of living in nominal debt economy. Specifically, I compute the fraction of aggregate lifetime consumption ω that agents born into the in real debt economy with no assets give up to live in nominal debt economy:

$$\mathbb{E}_0 \sum_t \beta^t u(\hat{c}_{n,t}, 1-\hat{n}_{nt}) \quad = \quad \mathbb{E}_0 \sum_t \beta^t u(\hat{c}_{r,t}(1+\omega), 1-\hat{n}_{rt})$$

Figure 5.2: Dynamics in the model economies



which with power utility is the ω that solves

$$\omega \quad = \quad \left(\frac{\int V_r(0,z)dF(z)}{\int V_n(0,z)dF(z)} \right)^{\frac{1}{1-\sigma}} - 1$$

In my benchmark calibration the welfare gain from living in the nominal debt economy is 0.12% of lifetime consumption - small but positive. Lower distortions contribute to this welfare result.

6 Conclusion

In this paper I have studied sovereign inflation and default policies, and how these depend on debt denomination. In the data, countries that issue nominal debt tend to achieve lower inflation and default rates. A monetary model of sovereign borrowing and lack of commitment can account for this observation. Real debt lower incentives to inflate today since it cannot be monetized, but it increases incentives to inflate tomorrow since real bond prices carry no inflation premia and inducing future inflation is less costly for the government. The second of these two forces dominates in a quantitative version of the model, leading to higher inflation and default rates, and lower welfare, in a real debt economy compared with an otherwise identical nominal debt economy.

The model highlights that even if bonds are indexed to the price level, money still plays a role and affects borrowing decisions. In terms of policy, the model suggests that issuing real debt in an attempt to address weak commitment by the policy maker may be short sighted and in fact exacerbate inflationary commitment problems. It would be interesting to contrast this framework and its implications for the incidence and consequences of sovereign debt crises with other debt structures, including a monetary union or dollarization.

Bibliography

Aguiar, M., Amador, M., Farhi, E. and Gopinath, G. (2013), Crisis and commitment: Inflation credibility and the vulnerability to sovereign debt crises, mimeo.

Alfaro, L. and Kanczuk, F. (2010), 'Nominal versus indexed debt: A quantitative horse race', *Journal of International Money and Finance* **29**(8), 1706–1726.

Araujo, A., Leon, M. and Santos, R. (2013), 'Welfare analysis of currency regimes with defaultable debts', *Journal of International Economics* **89**(1), 143–153.

Arellano, C. (2008), 'Default risk and income fluctuations in emerging economies', *American Economic Review* **98**(3), 690–712.

Arellano, C. and Heathcote, J. (2010), 'Dollarization and financial integration', *Journal of Economic Theory* **145**(3), 944–973.

Barro, R. J. (1997), Optimal management of indexed and nominal debt, NBER Working Papers 6197, National Bureau of Economic Research, Inc.

Bohn, H. (1990), 'Tax smoothing with financial instruments', *American Economic Review* **80**(5), 1217–30.

Calvo, G. A. (1988), 'Servicing the public debt: The role of expectations', *American Economic Review* **78**(4), 647–61.

Chari, V. and Kehoe, P. J. (1999), Optimal fiscal and monetary policy, *in* J. B. Taylor and M. Woodford, eds, 'Handbook of Macroeconomics', Vol. 1 of *Handbook of Macroeconomics*, Elsevier, chapter 26, pp. 1671–1745.

Claessens, S., Klingebiel, D. and Schmukler, S. L. (2007), 'Government bonds in domestic and foreign currency: the role of institutional and macroeconomic factors', *Review of International Economics* **15**(2), 370–413.

Cole, H. L. and Kehoe, T. J. (2000), 'Self-fulfilling debt crises', *Review of Economic Studies* **67**(1), 91–116.

Cooley, T. F. and Hansen, G. D. (1991), 'The welfare costs of moderate inflations', *Journal of Money, Credit and Banking* **23**(3), 483–503.

Cowan, K., Yeyati, E. L., Panizza, U. and Sturzenegger, F. (2006), Sovereign debt in the americas: New data and stylized facts, Research Department Publications 4480, Inter-American Development Bank, Research Department.

Da-Rocha, J.-M., Giménez, E.-L. and Lores, F.-X. (2013), 'Self-fulfilling crises with default and devaluation', *Economic Theory* **53**(3), 499–535.

Diaz-Gimenez, J., Giovannetti, G., Marimon, R. and Teles, P. (2008), 'Nominal debt as a burden on monetary policy', *Review of Economic Dynamics* **11**(3), 493–514.

Eaton, J. and Gersovitz, M. (1981), 'Debt with potential repudiation: Theoretical and empirical analysis', *Review of Economic Studies* **48**(2), 289–309.

Gumus, I. (2013), 'Debt denomination and default risk in emerging markets', *Macroeconomic Dynamics* **17**(05), 1070–1095.

Guscina, A. and Jeanne, O. (2006), Government debt in emerging market countries: A new data set, IMF Working Papers 06/98, International Monetary Fund.

Klein, P., Krusell, P. and Ros-Rull, J.-V. (2008), 'Time-consistent public policy', *Review of Economic Studies* **75**(3), 789–808.

Lorenzoni, G. and Werning, I. (2013), Slow moving debt crises, NBER Working Papers 19228, National Bureau of Economic Research, Inc.

Lucas, Robert E, J. and Stokey, N. L. (1987), 'Money and interest in a cash-in-advance economy', *Econometrica* **55**(3), 491–513.

Martin, F. (2009), 'A positive theory of government debt', *Review of Economic Dynamics* **12**(4), 608–631.

Missale, A. (1997), 'Managing the public debt: The optimal taxation approach', *Journal of Economic Surveys* **11**(3), 235–65.

Nicolini, J. P. (1998), 'More on the time consistency of monetary policy', *Journal of Monetary Economics* **41**(2), 333–350.

Reinhart, C. M. and Rogoff, K. S. (2011), 'The forgotten history of domestic debt', *Economic Journal* **121**(552), 319–350.

Svensson, L. E. O. (1985), 'Money and asset prices in a cash-in-advance economy', *Journal of Political Economy* **93**(5), 919–44.

Appendix A: Equilibrium - Simplified Model

Assume households have preferences over cash and credit good consumption and supply labor inelastically: $U = \sum_t \beta^t u(c_1, c_2)$. Assume the government can choose to default partially every period, where default incurs a resource cost $t(d)$ with $t(0) = 0$ and $t_d > 0$.

Nominal bonds

The household faces the following constraints

$$pc + (1+\mu)(m' + q_n B') = pn + m + (1-d)B$$

$$pc \leq m$$

The competitive equilibrium first order conditions imply

$$\begin{aligned} u_c - u_l &\geq 0 \\ \mu &= \beta \frac{u_1' p}{u_2 p'} - 1 \\ q_n &= \beta \frac{u_2'}{u_2} \frac{p}{p'(1+\mu)}(1-d') \\ &= \frac{u_2'}{u_1'}(1-d') \end{aligned}$$

Note that $c_1 + c_2 + g + t(d) = n = 1$.

The government faces the budget constraint

$$1 + (1-d)B + pg - pt(d) = (1+\mu)(1 + q_n B')$$

which can be rewritten in terms of allocations as

$$G(B, B', p, P(B'), d, D(B)) \equiv -u_2 \left[\frac{(1+(1-d)B)}{p} + g + t(d)\right] + \beta \frac{(u_1' + u_2'(1-D(B'))B')}{P(B')}$$

Definition 3. *Let* $V : \mathbb{B} \to \mathbb{R}, P : \mathbb{B} \to \mathbb{R}_{++}, H : \mathbb{B} \to \mathbb{B}$ *and* $D : \mathbb{B} \to [0,1]$*. A Markov perfect equilibrium of the simplified economy with nominal debt are functions* V, P, D, H

such that

$$V(B) = \max_{p,d,B'} u\left(\frac{1}{p}, 1 - \left(\frac{1}{p} + g + t(d)\right)\right) + \beta V(B')$$

subject to

$$\begin{aligned} G(B, B', p, P(B'), d, D(B)) &= 0 \\ F(p, d) \equiv u_1(p, d) - u_2(p, d) &\geq 0 \\ B' &= H(B) \end{aligned}$$

Real bonds

The household faces the following constraints

$$pc + (1 + \mu)m' + q_r b' = pn + m + (1 - d)b$$

$$pc \leq m$$

The competitive equilibrium first order conditions imply

$$\begin{aligned} u_c - u_l &\geq 0 \\ \mu &= \beta \frac{u_1' p}{u_2 p'} - 1 \\ q_r &= \beta \frac{u_2'}{u_2}(1 - d') \\ &= \frac{u_2'}{u_1'}(1 - d') \frac{p'(1 + \mu)}{p} \end{aligned}$$

Note that $c_1 + c_2 + g + t(d) = n = 1$.

The government faces the budget constraint

$$1 + (1 - d)b + pg - pt(d) = (1 + \mu) + q_r b'$$

which can be rewritten in terms of allocations as

$$G(b, b', p, P(b'), d, D(b)) \equiv -u_2 \left[\frac{1}{p} + (1-d)b + g + t(d) \right] + \beta \left(\frac{u_1'}{P(b')} + u_2'(1 - D(b'))b' \right)$$

Definition 4. *Let $V : \mathbb{B} \to \mathbb{R}, P : \mathbb{B} \to \mathbb{R}_{++}, H : \mathbb{B} \to \mathbb{B}$ and $D : \mathbb{B} \to [0,1]$. A Markov perfect equilibrium of the simplified economy with real debt are functions V, P, D, H such that*

$$V(b) \quad = \quad \max_{p,d,b'} u\left(\frac{1}{p}, 1 - \left(\frac{1}{p} + g + t(d)\right)\right) + \beta V(b')$$

subject to

$$\begin{aligned} G(b, b', p, P(b'), d, D(b)) &= 0 \\ F(p,d) \equiv u_1(p,d) - u_2(p,d) &\geq 0 \\ b' &= H(b) \end{aligned}$$

First Order Conditions

$$\begin{aligned} -\frac{u_c - u_l}{p^2} + \lambda G_p + \gamma F_p &= 0 \\ -u_l T_d + \lambda G_d + \gamma F_d &= 0 \\ \lambda \left(G_{B'} + G_P \frac{\partial P(B')}{\partial B'} + G_D \frac{\partial D(B')}{\partial B'} \right) + \beta \lambda' G_B' &= 0 \end{aligned}$$

where for nominal debt

$$\begin{aligned} G_p^N &= u_l \frac{(1 + (1-d)B)}{p^2} + (u_{lc} - u_{ll}) \frac{1}{p^2} \left[\frac{(1 + (1-d)B)}{p} + g + T(d) \right] \\ G_d^N &= u_l \frac{B}{p} - u_l T_d + u_{ll} T_d \left[\frac{(1 + (1-d)B)}{p} + g + T(d) \right] \end{aligned}$$

and

$$
\begin{aligned}
G_P &= -\frac{\beta}{P(B')}\left(\frac{u'_c}{P(B')} + \frac{u'_l(1-D(B'))B'}{P(B')} + \frac{u'_{cc}-u'_{cl}}{P(B')^2} + (1-D(B'))\frac{B'}{P(B')}\frac{(u'_{cl}-u'_{ll})}{P(B')}\right) \\
&= -\frac{\beta}{P(B')}\left(\frac{u'_c}{P(B')} + \frac{u'_l(1-D(B'))B'}{P(B')}\right. \\
&\quad + \frac{1}{P(B')}\left[\frac{u'_{cc}}{P(B')} - u'_{ll}(1-D(B'))\frac{B'}{P(B')}\right] \\
&\quad \left. + \frac{u'_{cl}}{P(B')}\left[-\frac{1}{P(B')} + (1-D(B'))\frac{B'}{P(B')}\right]\right) \\
G_D &= -\beta\frac{B'}{P(B')}(u'_l + u'_{ll}t_D(1-D(B')))
\end{aligned}
$$

while for real debt

$$
\begin{aligned}
G_p^R &= \frac{u_l}{p^2} + (u_{lc}-u_{ll})\frac{1}{p^2}\left[\frac{1}{p} + (1-d)b + g + T(d)\right] \\
G_d^R &= u_l b - u_l T_d + u_{ll}T_d\left[\frac{1}{p} + (1-d)b + g + T(d)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
G_P &= -\frac{\beta}{P(b')}\left(\frac{u'_c}{P(b')} + \frac{u'_{cc}-u'_{cl}}{P(b')^2} + (1-D(b'))b'\frac{(u'_{cl}-u'_{ll})}{P(b')}\right) \\
&= -\frac{\beta}{P(b')}\left(\frac{u'_c}{P(b')}\right. \\
&\quad + \frac{1}{P(b')}\left[\frac{u'_{cc}}{P(b')} - u'_{ll}(1-D(b'))b'\right] \\
&\quad \left. + \frac{u'_{cl}}{P(b')}\left[-\frac{1}{P(b')} + (1-D(b'))b'\right]\right) \\
G_D &= -\beta b'(u'_l + u'_{ll}t_D(1-D(b')))
\end{aligned}
$$

Appendix B: Equilibrium - Full Model

The government's problem can be expressed purely in terms of allocations by substituting out competitive equilibrium conditions. An alternative equilibirum definition therefore is:

Definition 5. *Let* $V : \mathbb{B}\times\mathbb{Z} \to \mathbb{R}$, $P : \mathbb{B}\times\mathbb{Z} \to \mathbb{R}_{++}$, $H : \mathbb{B}\times\mathbb{Z} \to \mathbb{B}$ *and* $D : \mathbb{B}\times\mathbb{Z} \to \{0,1\}$*. Define also* $\bar{z}(H(B))$ *as the highest* $z \in \mathbb{Z}$ *for which the government chooses not to default.*

Then a Markov perfect equilibrium of the economy with nominal debt are funtions V, P, D, H and a corresponding $\bar{z}(H(B))$ such that

$$V(B,z) = \max\left\{V^r(B,z), V^d(z)\right\}$$

where the value of repayment is

$$V^r(B,z) \quad = \quad \max_{p,B'} u\left(\frac{1}{p}, n(p) - \frac{1}{p} - g, 1 - n(p)\right) + \beta \int_{z'} V(B', z') dF(z', z)$$

subject to

$$\begin{aligned}
u_2\left[\frac{(1+B)}{p} + g - n(p)\right] + u_l n(p) &= \beta\left(\mathbb{E}\left[\zeta_1(B',z')\right] + \mathbb{E}\left[\zeta_{2n}(B',z')\right]B'\right) \\
u_c - u_l &\geq 0 \\
u_l(n(p)) &= (1-\tau)u_2(p) \\
B' &= H(B)
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{E}\left[\zeta_1(B',z')\right] &= \int_{z'\leq\bar{z}'} \frac{(u_1^r)'}{P^r(B',z')} f(z',z)dz' + \int_{z'>\bar{z}'} \frac{\left(u_1^d\right)'}{P^d(z')} f(z',z)dz' \qquad (6.1) \\
\mathbb{E}\left[\zeta_{2n}(B',z')\right] &= \int_{z'\leq\bar{z}'} \frac{(u_2^r)'}{P^r(B',z')} f(z',z)dz' + \int_{z'>\bar{z}'} \frac{\left(u_2^d\right)'}{P^d(z')} f(z',z)dz'
\end{aligned}$$

and the value of default is

$$V^d(z) \quad = \quad \max_{p} u\left(\frac{1}{p}, n(p) - \frac{1}{p} - g^d, 1 - n(p)\right) + \beta \int_{z'} \left[\eta V(0,z') + (1-\eta)V^d(z')\right] dF(z',z)$$

subject to

$$\begin{aligned}
u_2\left[\frac{1}{p} + g^d - n(p)\right] + u_l n(p) &= \beta\left(\eta\mathbb{E}\left[\zeta_1(0,z')\right] + (1-\eta)\mathbb{E}\left[\zeta_1^d(z')\right]\right) \\
u_c - u_l &\geq 0 \\
u_l(n(p)) &= (1-\tau)u_2(p) \\
B' &= H(B)
\end{aligned}$$

where ζ_1 is defined as in 6.1, and if the country stays in default

$$\mathbb{E}\left[\zeta_1^d(z')\right] = \int_{z'} \frac{(u_1^d)'}{P^d(z')} f(z', z) dz'$$

and P and D satisfy

$$P(B, z) = \mathbb{1}\left(V^r(B, z) \geq V^d(z)\right) P^r(B, z) + \mathbb{1}\left(V^r(B, z) < V^d(z)\right) P^d(z)$$

$$D(B, z) = \mathbb{1}\left(V^r(B, z) < V^d(z)\right)$$

Bond prices and money growth rates are given by, respectively

$$\begin{aligned}
q_n(B', z) &= \frac{\mathbb{E}[\zeta_{2n}(B', z')]}{\mathbb{E}[\zeta_1(B', z')]} \\
q_r(b', z) &= \frac{\mathbb{E}[\zeta_{2r}(b', z')]}{\mathbb{E}[\zeta_1(b', z')]} \frac{(1 + \mu(b', z))}{p} \\
&= \beta \frac{\mathbb{E}[\zeta_{2r}(b', z')]}{u_2} \\
\mu(B', z) &= \beta \frac{\mathbb{E}[\zeta_1(B', z')]}{\frac{u_2}{p}} - 1
\end{aligned}$$

Appendix D: Proofs

Claim. Consider the simple model (defined in Appendix A). Suppose utility is logarithmic in consumption and linear in leisure, and suppose default costs are linear $t(d) = \chi_1 d$. Consider a given steady state level of debt that is identical in the real and nominal debt economies: $b_r = b_n \equiv \frac{B}{p_n}$. Then inflation is higher in the nominal debt economy if and only if default rates are higher.

Proof. Suppose utility is log in consumption and linear in leisure: $u = \log c + \alpha l$ where $l = 1 - n$. Bond prices and money growth rates are determined in the competitive equilibrium and given by

$$\begin{aligned}
\mu &= \beta \frac{u_1' p}{u_2 p'} - 1 \\
&= \beta \frac{p}{\alpha}
\end{aligned}$$

and

$$
\begin{aligned}
q_n &= \beta \frac{u_2'(1-d')p}{u_2 p'(1+\mu)} \\
&= \frac{u_2'(1-d')}{u_1'} \\
&= \frac{\alpha(1-d')}{p'}
\end{aligned}
$$

The budget constraint for the government then is

$$
\frac{1}{p} + \frac{(1-d)B}{p} + g + t(d) = \frac{1+\mu}{p}\left(1 + q_n B'\right) \tag{6.2}
$$

$$
\frac{1}{p} + \frac{(1-d)B}{p} + g + t(d) = \frac{\beta}{\alpha}\left(1 + \frac{\alpha(1-d')B'}{p'}\right) \tag{6.3}
$$

The other constraint is the intratemporal condition relating default and inflation:

$$
\frac{u_1 - u_2}{p^2 u_2 t_d} = \frac{G_p}{G_d}
$$

which under our assumptions reduces to

$$
\frac{p-\alpha}{\alpha t_d} = \frac{1+(1-d)B}{-\frac{B}{p} + t_d} \tag{6.4}
$$

The analogous equations for a real debt economy are given by

$$
\begin{aligned}
q_r &= \beta \frac{u_2'(1-d')}{u_2} \\
&= \frac{u_2'(1-d')}{u_1'} \frac{(1+\mu)p'}{p} \\
&= \alpha(1-d')\frac{1+\mu}{p}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{p} + (1-d)b + g + t(d) &= \frac{1+\mu}{p} + q_r b' \\
\frac{1}{p} + (1-d)b + g + t(d) &= \frac{\beta}{\alpha}\left(1 + \alpha(1-d')b'\right)
\end{aligned}
$$

and

$$\frac{p-\alpha}{\alpha t_d} = \frac{1}{-b+t_d}$$

Use subscripts to denote objects from the two different types of economy (n for nominal and r for real). Now consider the steady state of these economies with the same real value of debt taken as given, $b_r = b_n \equiv \frac{B}{p_n}$. Then we can solve two equations in two unknowns for equilibrium prices and default rates: For nominal debt

$$\begin{aligned}\frac{1}{p_n} + (1-\beta)(1-d_n)b_n + t(d_n) &= \frac{\beta}{\alpha} - g \\ (p_n - \alpha)(t_{dn} - b_n) &= \alpha t_{dn}(1 + (1-d_n)p_n b_n)\end{aligned}$$

and for real debt

$$\begin{aligned}\frac{1}{p_r} + (1-\beta)(1-d_r)b_r + t(d_r) &= \frac{\beta}{\alpha} - g \\ (p_r - \alpha)(t_{dr} - b_r) &= \alpha t_{dr}\end{aligned}$$

Suppose $p_r < p_n$. Then we must have

$$-d_n b + t(d_n) \quad < \quad -d_r b + t(d_r)$$

from the budget constraint. The net cost from inflating has to be lower in the nominal case. Under the assumption of linear default costs, this implies $d_r < d_n$ provided debt is not too low and the default cost is not too high, that is provided $b - \chi > 0$. This will hold in any equilibrium by the intratemporal FOC. Note that if $b - \chi < 0$, then the left hand side of the intratemporal condition is negative. But the right hand side is strictly positive for $d > 0$. $\square$

More generally, for different functional forms, provided the cost of defaulting is not disproportionately larger than the debt burden, we have that $d_r < d_n$. This is in general true the higher debt, and the lower the default cost. In practice I have not encountered equilibria where the equilibrium level of debt is such that this fails to hold.

Appendix E: Data

Sources and Coverage

All data run from 1990 through 2012 unless otherwise noted. Annual GDP data in current and constant US$ are from the WDI. The constant price series for Argentina ends in 2006. Monthly CPI inflation is from the IFS (except for the index for China which is from Global Financial Data). The series start later than January 1990 for Romania (Oct 1991), Ukraine (Jan 1993), Russia (Jan 1993) and Vietnam (Jan 96). Exchange rate data are from Global Financial Data. The series are daily except for Russia (weekly until Jan 1992) and Romania (monthly until Feb 1990). Bond data are from Bloomberg. The default data are based on Standard & Poors from 1990 to 2006, and extended thereafter through various other sources (news articles etc.). Quarterly data on aggregate government debt securities are from the BIS debt securities database. They are the sum of domestic and international debt securities; the international series is only available from 1993Q3. Annual general government debt is from the IMF World Economic Outlook.

Table 4: Countries, regions and first bond data observations in my sample

Latin America		Asia	
Argentina	15 Oct 1992	China	2 Jan 1990
Brazil	2 Jan 1990	Indonesia	30 Jul 1996
Chile	6 Dec 1994	India	2 Jan 1990
Colombia	8 May 1992	Korea	2 Jan 1990
Ecuador	8 May 1997	Malaysia	2 Jan 1990
Mexico	13 Mar 1991	Pakistan	6 Mar 1991
Peru	9 Mar 1998	Philippines	2 Jan 1990
Venezuela	15 Jan 1991	Thailand	2 jan 1990
		Vietnam	28 Jul 2000

Europe		Africa	
Bulgaria	29 Jan 1998	Egypt	22 May 1997
Hungary	2 Jan 1990	Morocco	18 Mar 1993
Poland	12 Mar 1994	South Africa	2 Jan 1990
Romania	28 May 1996		
Russia	14 May 1993		
Turkey	2 Jan 1990		
Ukraine	21 Mar 1995		

Regression evidence

Regressions results documenting the negative relationship between nominal debt shares and inflation and default, respectively, are shown in Table (5). I estimate a linear model with country fixed effects for inflation rates

$$\log \text{inflation}_{it} \quad = \quad \alpha_i + \beta x_{it} + \epsilon_{it}$$

where x_{it} is the nominal debt share.

For default probabilities I use a simpe logit model

$$\text{logit} P(d_{it} = 1 | x_{it}, \zeta_i) = \alpha + \beta x_{it}$$

and also estimate two other versions where I allow for either random or fixed effects.

The tables show a significant negative relationship between nominal debt shares and both inflation and default. In terms of magnitude, the left panel shows that issuing nominal instead of real debt increases inflation by 1.5 log points. This corresponds roughly to a change from 5% annual inflation when all debt is nominal to around 20% when it is real. With year fixed effects the effect is somewhat smaller but still significant at 10%. The right panel of the Table shows that the probability of default is 90% lower for a country that issues all nominal instead of all real debt (I report the odds ratio of 0.118 in square brackets). Allowing for country random effects the magnitude of this effect shrinks to 50% but remains significant.

Table 5: Inflation and default probabilities

	Log inflation			Default		
Nominal debt share	-1.435**	-0.896*	Nominal debt share	-2.138***	-0.675***	-0.649***
	(0.580)	(0.473)		(0.621)	(0.205)	(0.205)
				[0.118]	[0.509]	[0.523]
N	6220	6220	N	6418	6418	2663
adj. R^2	0.434	0.570				
Time FE	No	Yes				
Country	FE	FE	Country	-	RE	FE

Standard errors in parentheses * $p<0.10$ ** $p<0.05$ *** $p<0.01$ Odds ratios in square brackets

Additional Figures

Figure A.1: Aggregate bond debt in my sample of countries

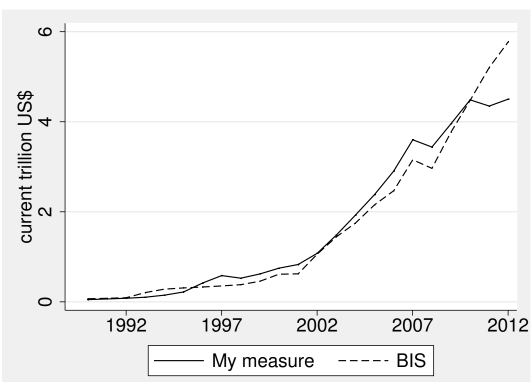


Figure A.2: Raw scatter plot of inflation against nominal debt share