

# Financial Shocks and the Cyclical Behavior of Skilled and Unskilled Unemployment\*

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## Abstract

In this paper we study the effect of a tightening in the credit conditions to the productive sector on labor markets for both skilled and unskilled workers. We build a model with two types of labor, two types of capital and both search and financial frictions, in the form of borrowing constraints for firms. The wage bill is included in the latter and only one type of capital is pledgeable. We find that a financial shock (an exogenous shock to the borrowing constraint) is capable of generating a big drop in employment in this economy through three channels: the increase in the shadow cost of labor in terms of financing, the existence of two capitals that allows the pledgeable one to increase while overall capital goes down (and with it the marginal product of labor) and the increase in consumption, which results in a temporary increase in the reservation wage of workers. Moreover, the model is able to reproduce a fairly volatile and countercyclical ratio of skilled to unskilled employment, as in the data. Finally, when conducting model simulations by feeding in productivity and financial shocks estimated from the data and computing the response of output, employment and relative employment, we find that financial shocks are capable of replicating the behavior of these variables during the whole sample period (1976.II-2012.II).

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# 1 Introduction

The recent financial and economic crisis in the US has brought renewed interest in models of financial frictions and their impact on economic activity.<sup>1</sup> However, and despite a growing literature on the topic, the importance of financial shocks, that is, shocks that originate directly in the financial sector, has only recently begun to be explored<sup>2</sup>. Moreover, works that study the interaction between financial frictions and labor markets in particular remain limited<sup>3</sup>. In this paper we address this issue by constructing a model with search labor frictions and financial frictions, in the form of a borrowing constraints, and ask how a disruption to credit changes both the firms' incentives to hire labor and the optimal mix between skilled and unskilled workers.

We find that a financial shock, understood as a tightening of the borrowing constraint, generates almost two-thirds of the volatility of aggregate employment as observed in the data, as well as a fairly volatile and counter-cyclical ratio of skilled to unskilled workers. In addition, we find that productivity shocks fail to produce enough movements in labor, as in the standard search model, and that both financial and productivity shocks generate more employment volatility when the model is calibrated to have higher leverage in steady state.

When conducting model simulations, by feeding in productivity and financial shocks estimated from the data, and computing the response of employment and relative employment, we find that financial shocks are capable of replicating the behavior of employment during the whole sample period and in particular, generate sharp drops in employment during all four recessions: 1980s, 1990-1991, 2001 and 2008-2009. The drop in employment generated by financial shocks in the last recession is almost of the same magnitude as that observed in the data.

Something similar holds for relative employment of skilled to unskilled workers, the model with financial shocks captures the dynamics of the variable well, and in particular is able to explain all of the increase in relative employment during the last crisis.

Our model economy is populated by relative impatient entrepreneurs and a representative household with two types of workers: skilled and unskilled. Households supply labor in exchange for labor income and provide funds to finance the entrepreneurs' activity in the real sector. Entrepreneurs use the skilled and unskilled labor input to run a constant returns to scale production function with two types of capital: structures and equipment. Because of a limited enforceability of contracts, the entrepreneur can only borrow up to a fraction of the value of its stock of capital structures. Entrepreneurs cannot use capital equipment as a collateral and they have to finance their wage bill

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<sup>1</sup>Quadrini (2011) has a recent survey on the literature.

<sup>2</sup>See, for example, Jermann and Quadrini (2011), Christiano, Motto and Rostagno (2008), Del Negro, Eggertson, Ferrero and Kiyotaki (2010) and Kiyotaki and Moore (2008). Gertler and Karadi (2009) as well as Gertler, Kiyotaki and Queralto (2011) also consider financial shocks but they explicitly model the frictions in the financial intermediation sector.

<sup>3</sup>Some examples are: Wasmer and Weil (2004), Petrosky-Nadeau and Wasmer (2012), Hristov(2009), Dromel et al (2010), Arellano, Bai and Kehoe (2010), Guerrieri and Lorenzoni (2010), and Monacelli, Quadrini and Trigaru (2010)

using external funds.

Facing a tighter borrowing constraint, entrepreneurs are forced to reduce their debt and scale down their operation. Given that capital in structures serve as a collateral, firms accumulate this type of capital in order to offset the tighter borrowing constraint, at the expense of their consumption (the firm's profits), investment in equipment and labor input.

At the center of the success of the model regarding aggregate dynamics of employment lies the assumption of the need of external funding to cover payroll costs. The combination of financial shocks and the presence of the wage bill in the borrowing constraint allow the model to overcome the usual subdued volatility of employment (the so-called Shimer Puzzle), common to search models with a conservative calibration regarding the flow utility of unemployment. In fact, the model not only delivers high volatility of employment but also a counter-cyclical and volatile "labor wedge", thanks to the distortion introduced by the shadow cost of labor in terms of the financial constraint.

The introduction of financial shocks, instead of the standard TFP shocks as a source of business fluctuations, is also key for this result. Under our benchmark parameterization, TFP shocks cannot on their own produce enough volatility of employment as wage dynamics tend to offset any changes in labor productivity. On the contrary, after a financial shock, wages tend to fall less than labor productivity, given the short-lived increase in consumption, resulting in an increase in the outside option of workers.

Another element of the model that is critical for our results is the fact that we have one type of capital that is unpledgeable. If both capital of structures and equipment were pledgeable, or equivalently there was only one type of capital, a tightening of the borrowing constraint would lead to a higher stock of capital in the economy, and with it an increase in the marginal product of labor. In that setup a financial shock not only would be unable to explain a drop in labor, but it could result in an expansion of output rather than in a contraction, depending on the strength of the accumulation of the collateral asset<sup>4</sup>.

In our benchmark model the fall in employment caused by the scale down of the firm, tends to be more acute for unskilled workers, as entrepreneurs internalize that future re-hiring is easier on the unskilled market given the larger pool of unemployed. This asymmetry is result of our calibration of the search parameters consistent with higher steady state unemployment of unskilled workers and does not rely on the financial constraint present in our environment. Nonetheless, changes in the relative ratio of skilled-to-unskilled employment are quantitatively important only when the drop in total employment is relative big.

This paper is organized as follows. In the next section we present the model and the basic equilibrium conditions. In section 3 we discuss the effect of leverage in the steady state employment

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<sup>4</sup>The challenge of generating significant drops in output is shared by most of the literature on financial frictions. The usual solution to overcome this problem relies on the assumption of a fixed-supply collateral, such as land. Olivella and Roldan (2012) show that the standard amplification mechanism fails when the financial constraint is defined in terms of a reproducible asset.

of the model. Section 4 is devoted to discuss the quantitative results by analyzing the impulse-responses of the model, the standard business cycle statistics and the dynamics of the model induced by estimated shocks in the data. In section 5 we do some sensitivity analysis regarding the most important elements of the model. Finally, we conclude.

## 2 Model Description

### 2.1 Households

There is a representative household composed by a unit measure of infinitely lived individuals with the same preferences over consumption and disutility of working. There are two types of family members, skilled and unskilled, with measure  $s$  and  $u$  respectively, where  $u = 1 - s$ . In every period, each member of the household can be either employed or unemployed. The disutility of working of the two types,  $\hat{\gamma}_S$  and  $\hat{\gamma}_U$ , is potentially different and each type searches for employment in a separate labor market, but there is only one level of consumption within the household for both types and for employed and unemployed members. Hence, there is consumption risk sharing as if markets were complete. Wages of the two types of workers are determined in separate markets and in equilibrium there is a fraction  $n_s$  and  $n_u$  of employed family members of the skilled and unskilled type, respectively.

The representative household supplies funds to entrepreneurs in the form of non-contingent one-period bonds,  $D_t$ . There is a household-level budget constraint, which states that the consumption of all family members and the amount of funds supplied to the firm should be equal to the total labor income of the household plus the return on the previous period bond holdings.

The optimization problem of the representative household can be written as:

$$\max_{c_t, D_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t) - \hat{\gamma}_S n_{s,t} - \hat{\gamma}_U n_{u,t}]$$

subject to the budget constraint

$$c_t + D_{t+1} = n_{s,t} W_{S,t} + n_{u,t} W_{U,t} + (1 + r_t) D_t$$

and laws of motion of the fraction of skilled and unskilled employed workers given by:

$$n_{S,t+1} = (1 - x_S) n_{S,t} + f_{S,t}(\theta_{S,t})(s - n_{S,t})$$

$$n_{U,t+1} = (1 - x_U) n_{U,t} + f_{U,t}(\theta_{U,t})(u - n_{U,t})$$

The job finding rate  $f_t$  of each type depends on the market tightness of each labor market and the laws of motion indicate that skilled and unskilled employment in the next period will be determined by the surviving matches today, after an exogenous death shock  $x$ , specific to each

type, and the new matches formed in the current period. The first-order optimal conditions for the household problem are summarized in the following Euler equation:

$$\frac{1}{c_t} = \beta E_t \left[ \frac{1}{c_{t+1}} (1 + r_{t+1}) \right]$$

where  $r_{t+1}$  is the interest rate determined in the bond market.

## 2.2 Entrepreneurs

Entrepreneurs own capital and produce final goods using capital structures  $K_{S,t}$ , capital equipment  $K_{E,t}$ , skilled labor  $L_{S,t}$  and unskilled labor  $L_{U,t}$ . The production technology exhibits constant returns to scale and is given by:

$$F(K_{S,t}, K_{E,t}, L_{S,t}, L_{U,t}) = Y_t = Z_t K_{S,t}^{\alpha_{K_S}} K_{E,t}^{\alpha_{K_E}} L_{S,t}^{\alpha_{L_S}} L_{U,t}^{1-\alpha_{K_S}-\alpha_{K_E}-\alpha_{L_S}}$$

where  $\alpha_{K_S}$ ,  $\alpha_{K_E}$ ,  $\alpha_{L_S}$  are the income shares of structures, equipment and skilled labor and  $Z_t$  is a technology shock.

Firms use some of their labor input to recruit new workers<sup>5</sup>. For this purpose the firm divides workers of each type between two tasks: production ( $L_{S,t}$ ,  $L_{U,t}$ ) and recruiting ( $V_{S,t}$ ,  $V_{U,t}$ )<sup>6</sup>. Total labor demand for each type can be written as:

$$n_{S,t} = L_{S,t} + V_{S,t}$$

$$n_{U,t} = L_{U,t} + V_{U,t}$$

and their laws of motion are given by:

$$n_{S,t+1} = (1 - x_S) n_{S,t} + \omega(\theta_{S,t}) V_{S,t}$$

$$n_{U,t+1} = (1 - x_U) n_{U,t} + \omega(\theta_{U,t}) V_{U,t}$$

From the entrepreneurs's point of view, next-period employment will be a function of the surviving current matches and the amount of new workers that current recruiters are able to hire. The function  $\omega(\theta_{\{s,u\}t})$ , which is a function of the market tightness in the labor market for each type of worker, indicates the number of workers that one recruiter can hire. Hence,  $\omega(\theta_{S,t}) V_{S,t}$  equals the total number of new skilled workers hired.

<sup>5</sup>This specification is the same as in Shimer (2010) and it is consistent with a balance growth model in which aggregate productivity growth doesn't drive recruiting costs to zero in terms of wages, as it would be the case when the cost of posting a vacancy is determined in units of the final good. In addition, as pointed out by Shimer (2010), this assumption seems reasonable as recruiting is a time-intensive activity.

<sup>6</sup>We then modify this assumption by making recruiting an activity exclusive of the skilled type. Our main conclusions remain robust to this specification but for expositional purposes its easier to start with a model in which recruiting is done by the two types of workers.

Entrepreneurs maximize their expected discounted flow of consumption  $C_t^E$ , by choosing how much to invest in physical capital of structures and equipment, how two split workers of each type between production and recruiting activities and how much to borrow from households. The optimization problem of the entrepreneur is thus summarized by:

$$\max_{C_t^E, K_{S,t+1}, K_{E,t+1}, V_{S,t}, V_{U,t}, B_{t+1}} E_0 \sum_{t=0}^{\infty} \gamma^t \frac{(C_t^E)^{1-\sigma_E}}{1-\sigma_E}$$

subject to the following budget constraint:

$$C_t^E + (K_{S,t+1} - (1 - \delta_S)K_{S,t}) + (K_{E,t+1} - (1 - \delta_E)K_{E,t}) + W_{U,t}n_{U,t} + W_{S,t}n_{S,t} = F(K_t, L_{S,t}, L_{U,t}) + B_{t+1} - (1 + r_t)B_t$$

where  $\delta_S$  and  $\delta_E$  are depreciation rates of capital structures and equipment respectively. The entrepreneurs are assumed to be less patient and less risk adverse than households, thus, their discount factor satisfies  $\gamma < \beta$  and their coefficient of risk aversion  $\sigma_E < 1$ . In fact, in order to insure that entrepreneurs are financially constrained, we set their risk aversion coefficient such that they are very close to being risk neutral<sup>7</sup>. Finally, entrepreneurs cannot commit to repaying their loans, and thus face the following collateral constraint:

$$B_{t+1}(1 + r_{t+1}) \leq \chi_t K_{S,t+1} - (W_{U,t}n_{U,t} + W_{S,t}n_{S,t})$$

This constraint implies that entrepreneurs are able to borrow up to the point where the repayment equals a fraction  $\chi_t$  of the total value of their capital structures minus the wage bill. This means that even in the event of default in which the entrepreneur appropriates  $1 - \chi_t$  of the collateral and liquidates the firm, there is enough pledgable asset to pay the workers. The hair-cut of the pledgable asset  $\chi_t$  evolves stochastically in this economy and reflects shocks to the terms of loans or current financial conditions. For example, a negative shock to  $\chi_t$  implies that creditors are willing to lend to the entrepreneur less funds relative to the same level of collateral. The timing of the constraint implies the following sequence of events: In every period the entrepreneur repays its debt,  $B_t(1 + r_t)$ , and asks for a new loan,  $B_{t+1}$  to fund production. Then it produces, pay workers, invests in physical capital and consumes. At the end of the period the entrepreneur decides whether to liquidate the firm or not.

Let  $\mu_t(c_t^E)^{-\sigma}$  be the multiplier associated with the borrowing constraint. The Euler equation for capital structures is described by:

$$1 - \mu_t \chi_t = \gamma \left( \frac{c_{t+1}^E}{c_t^E} \right)^{-\sigma} E_t [r_{S,t+1} + (1 - \delta_S)]$$

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<sup>7</sup>The risk aversion coefficient is set to be very small but not zero to insure that there consumption of the entrepreneurs don't turn negative in any of the simulations after the economy is hit with shock.

where  $r_{S,t+1}$  is the return on physical capital in terms of the final good defined by the standard capital-output ratio:  $r_{S,t+1} = \alpha_{K_S} \frac{Y_{t+1}}{K_{S,t+1}}$ . Note that the stochastic fraction of accepted collateral  $\chi_t$  affects this Euler equation directly whenever the borrowing constraint is binding. Given its role as collateral, capital of structures serves provides a way to relax the borrowing constraint. The Euler equation of capital equipment, as defined as follows, is completely standard.

$$1 = \gamma \left( \frac{c_{t+1}^E}{c_t^E} \right)^{-\sigma} E_t [r_{E,t+1} + (1 - \delta_E)]$$

where the return of equipment is defined as  $r_{E,t+1} = \alpha_{K_E} \frac{Y_{t+1}}{K_{E,t+1}}$ . The Euler equation for debt can be expressed as:

$$\mu_t = \frac{1}{(1+r_{t+1})} - \gamma E_t \left( \frac{c_{t+1}^E}{c_t^E} \right)^{-\sigma}$$

Given the differences in discount factors between the representative household and the entrepreneur, it is easy to see that the multiplier  $\mu_t$  will be positive in steady-state, which implies that in steady-state the borrowing constraint is satisfied with equality. In addition, a low value of the risk-aversion coefficient for the entrepreneur ensures that the borrowing constraint remains binding in the stochastic approximation around the steady-state.

Regarding the decision of splitting workers between productive labor and recruiting, the next two Euler equations describe the trade-off faced by the entrepreneur:

$$\begin{aligned} \frac{MPL_{S,t}}{\omega(\theta_{S,t})} &= \gamma \left( \frac{c_{t+1}^E}{c_t^E} \right)^{-\sigma} E_t \left[ MPL_{S,t+1} \left( 1 + \frac{1-x_S}{\omega(\theta_{S,t+1})} \right) - W_{S,t+1} (1 + \mu_{t+1}) \right] \\ \frac{MPL_{U,t}}{\omega(\theta_{U,t})} &= \gamma \left( \frac{c_{t+1}^E}{c_t^E} \right)^{-\sigma} E_t \left[ MPL_{U,t+1} \left( 1 + \frac{1-x_U}{\omega(\theta_{U,t+1})} \right) - W_{U,t+1} (1 + \mu_{t+1}) \right] \end{aligned}$$

In each of these two equations the left-hand side indicates the opportunity cost of hiring one extra worker, which implies increasing the number of recruiters today, at the expense of productive labor. As we indicated before, the function  $\omega(\theta_{\{s,u\}t})$  indicates the number of workers that a recruiter can hire of each type, so its inverse,  $1/\omega(\theta_{\{s,u\}t})$ , amounts to the number of recruiters needed to hire one worker. Thus, the left-hand side of each euler equation is equal to the value of foregone production associated with increasing the number of recruiters to increase the number of productive workers by one. The right-hand side of both equations is the expected discounted value of increasing the size of the firm by one worker. The first term in this expression is given by the benefit in terms of marginal product of hiring a new worker plus the saving (also in terms of marginal product) associated to the need of less recruiters in t+1 to keep the number of workers constant. The second term is the cost of increasing the firm's payroll which includes the wage of the newly hired worker plus the shadow cost in terms of financial funds. In all these expressions  $MPL_{S,t}$  and  $MPL_{U,t}$  stands for the marginal product of skilled and unskilled labor in terms of the final good:  $MPL_{S,t} = \alpha_{L_S} \frac{Y_t}{L_{S,t}}$  and  $MPL_{U,t} = \alpha_{L_U} \frac{Y_t}{L_{U,t}}$ .

### 2.3 Wage Determination

Following most of the literature, we assume Nash Bargaining in the wage determination in both the skilled and unskilled markets. Each type of worker bargains with the entrepreneur, separately, over the surplus created in the match. We present here the characterization of the bargaining problem for the skilled worker, but there is an analogous condition for the unskilled. The Nash Bargaining solution to this negotiation protocol is given by the wage that maximizes the following Nash product:

$$W_S = \operatorname{argmax} \left( \tilde{V}_{n_S} \right)^\phi \left( \tilde{J}_{n_S} \right)^{1-\phi}$$

where  $\tilde{V}_{n_S}$  is the marginal value for a household of having a skilled worker employed at wage  $W_s$  and  $\tilde{J}_{n_S}$  is the marginal value of this worker for the firm. The bargaining power of workers is represented by  $\phi$  (same for skilled and unskilled workers).

Using a recursive representation for the household's preferences we obtain the following expression:

$$\frac{\partial \tilde{V}_{n_S}}{\partial W_S} = \tilde{V}'_{n_S} = \frac{1}{c}$$

Taking logs to the Nash product and deriving with respect to the equilibrium wage, we can express the first term as:

$$\phi \frac{\tilde{V}'_{n_S}(W_S)}{\tilde{V}_{n_S}(W_S)} = \phi \frac{(1/c)}{V_{n_S}} = \phi \frac{[1-\beta(1-x_S-f(\theta_S))]}{W_S - c \cdot \hat{\gamma}_S}$$

For the firm side, we can compute the marginal value of a worker employed at a wage  $W_S$  for one period, instead of the equilibrium wage  $\bar{W}_S$  :

$$\tilde{J}_{n_S}(K_E, K_S, B, n_S, n_U, W_S) = (W_S - \bar{W}_S)(1 + \mu) + J_{n_S}$$

Taking logs and the derivative with respect to the wage, we have:

$$(1 - \phi) \frac{\tilde{J}'_{n_S}}{J_{n_S}} = \frac{-(1-\phi)(1+\mu)}{J_{n_S}} = \frac{(1+\mu)(1-\phi)\gamma\omega(\theta_S)}{MPL_S}$$

Putting together the expressions from the household and firm side:

$$\phi \frac{\tilde{V}'_{n_S}(W_S)}{\tilde{V}_{n_S}(W_S)} + (1 - \phi) \frac{\tilde{J}'_{n_S}}{J_{n_S}} = \phi \frac{(1-\beta(1-x_S-f(\theta_S)))}{W_S - c \cdot \hat{\gamma}_S} = \frac{(1-\phi)\gamma\omega(\theta_S)(1+\mu)}{MPL_S}$$

and simplifying and solving for the equilibrium wage, we get:

$$W_S = \frac{\beta\phi}{\phi\beta+(1-\phi)\gamma} \frac{MPL_S}{(1+\mu)} \left( 1 + \theta_S + \frac{\gamma-\beta}{\gamma\beta} \frac{1}{\omega(\theta_S)} \right) + \frac{\gamma(1-\phi)}{\phi\beta+(1-\phi)\gamma} \hat{\gamma}_S \cdot c$$



This equilibrium condition for the wage is similar to the one obtained in standard search models but it has some differences introduced by the wedge in the discount factor between household and firms and the shadow cost of hiring from the borrowing constraint. As can be seen from the equation, the wage is a weighted average of two terms: 1) the marginal product of labor, adjusted by the financing cost of labor in terms of the borrowing constraint and 2) the marginal value of the time of the worker, that is, the marginal rate of substitution between leisure and consumption. The wedge between  $\beta$  and  $\gamma$  changes the weights of these two terms. If these two parameters were equal, as in the standard model, these two weights would only be function of the bargaining power of workers ( $\phi$ ). In addition, the difference in the impatience factor between households and entrepreneurs introduces a new term, which modifies the value of the surplus of a match relative to the standard model. Increasing the size of the firm by one worker, frees up resources by an amount equivalent to the cost of recruiting in terms of units of the final good:  $\frac{MPL_S}{(1+\mu)} \frac{1}{\omega(\theta_S)}$ . The present value associated to these savings is valued differently by households and entrepreneurs, as the two agents discount the future at different rates. If there was no wedge between  $\beta$  and  $\gamma$ , this additional term would disappear and our expression for the wage would converge to that of the standard model.

## 2.4 Equilibrium

An equilibrium in this economy consists of a consumption and bond-holdings allocation for the representative household:  $\{c_t, D_{t+1}\}_{t=0}^{\infty}$ , and one for the entrepreneur's optimization problem regarding its consumption, capital structures, capital equipment, recruiting and debt:

$$\{C_t^E, K_{S,t+1}, K_{E,t+1}, V_{S,t}, V_{U,t}, B_{t+1}\}_{t=0}^{\infty}$$

such that, given prices  $\{r_t, W_{S,t}, W_{U,t}\}_{t=0}^{\infty}$ , initial conditions, the borrowing constraint of the entrepreneur and the stochastic processes, these allocations are optimal, as defined by the first-order conditions before mentioned and markets clear:

$$f_{S,t}(\theta_{S,t})(s - n_{S,t}) = \omega(\theta_{S,t})V_{S,t}$$

$$f_{U,t}(\theta_{U,t})(u - n_{U,t}) = \omega(\theta_{U,t})V_{U,t}$$

for labor and  $D_{t+1} = B_{t+1}$  for debt. A final assumption needed to complete the model refers to the matching function between workers and recruiters. We will follow the literature here and assume a constant returns to scale matching function in which the elasticity with respect to the market tightness is the same as the bargaining power of workers, meeting the Hosios (1990) condition. Under these assumptions the recruiting cost function can be expressed as:

$$\omega(\theta_S) = (1 - \phi)\bar{\omega}_S\theta_S^{-\phi}$$

$$\omega(\theta_U) = (1 - \phi)\bar{\omega}_U\theta_U^{-\phi}$$

for the skilled and unskilled market respectively. Finally, the market-tightness for each market is defined by:

$$\theta_U = \frac{V_U}{u-n_U} \text{ and } \theta_s = \frac{V_s}{s-n_s}$$

### 3 Leverage, Financial Shocks and Elasticity of Employment

Before solving the model numerically we explore the response of employment to financial shocks around the steady state. Working with the equations that characterize the steady state of the model, we find that aggregate productivity carries no effect on labor markets, product of the assumed balanced-growth preferences. On the contrary, the fraction  $\chi$  has a first-order effect on market tightness as well as all other endogenous variables.

To simplify the analysis, we focus on a parameterization of the model in which there are no differences between skilled and unskilled workers. We begin by solving for the wage from the steady state Euler equation for employment as a function of the marginal product of labor:

$$W = \frac{MPL}{(1+\mu)} \left( 1 + \frac{[1-x-1/\gamma]}{\omega(\theta)} \right)$$

Also, from the Nash Bargaining equation we get:

$$W = \lambda \frac{MPL}{(1+\mu)} \left( 1 + \theta + \frac{\gamma-\beta}{\gamma\beta} \frac{1}{\omega(\theta)} \right) + (1-\lambda) \hat{\gamma} \cdot c$$

In the steady state allocation, in which bond asset holdings are constant across periods, the budget constraint of the household, combined with the borrowing constraint of entrepreneurs allows us to write consumption as follows:

$$c = \beta W n + (1-\beta)\chi K_s$$

Combining the steady state Euler equation with the Nash-Bargaining equation and substituting for consumption we get the following labor market clearing expression:

$$\frac{MPL}{(1+\mu)} \left( 1 + \frac{[1-x-1/\gamma]}{\omega(\Theta)} \right) = \lambda \frac{MPL}{(1+\mu)} \left( 1 + \theta + \frac{\gamma-\beta}{\gamma\beta} \frac{1}{\omega(\theta)} \right) + (1-\lambda) \hat{\gamma} (\beta W n + (1-\beta)\chi K_s)$$

where  $\lambda = \frac{\beta\phi}{\phi\beta+(1-\phi)\gamma}$ .

From the Euler-equation we get the capital structures-output ratio:

$$\frac{K_s}{Y} = \frac{\gamma\alpha K_s}{1-\mu\chi-\gamma(1-\delta_s)}$$

Finally, using the fact that  $MPL = \alpha_L \frac{Y}{L}$ ,  $L = n - V = \frac{f-\theta x}{f+x}$  and  $\omega(\theta) = \frac{f(\theta)}{\theta}$ , we can rewrite the labor market clearing expression (after diving both sides by Y and simplifying) as:

$$(1 - \lambda) \hat{\gamma} \beta \left( \frac{f(\theta) + x}{f(\theta) - \theta x} \right) \left[ 1 + \frac{\theta}{f(\theta)} (1 - x - 1/\gamma) - \lambda \left( 1 + \theta + \frac{\gamma - \beta}{\gamma \beta} \frac{\theta}{f(\theta)} \right) \right] =$$

$$\left[ 1 + \frac{\theta}{f(\theta)} (1 - x - 1/\gamma) \right] + (1 - \lambda) \hat{\gamma} (1 - \beta) \frac{(1 + \mu) \gamma (\alpha_{K_S} / \alpha_L)}{1 - \mu \chi - \gamma (1 - \delta_S)} \chi$$

Solving for the labor market equilibrium implies solving for the market tightness  $\theta$  and this last equation defines market tightness as a function only of parameters and  $\chi$ . Productivity shocks do not appear in this equation, which means that in the steady state the ratio of recruiters to unemployed workers is independent of productivity<sup>8</sup>. This neutrality result only works for the non-stochastic equilibrium allocation, however, as long as the dynamics of the model do not move the equilibrium allocation too far from steady state, and given our specification of vacancy costs in terms of labor units, we can ascertain that TFP shocks will have small-order effects on labor variables, even in the presence of borrowing constraints.

On the other hand, the level of  $\chi$  does appear in the equation, implying that it has a direct effect on the labor market. The high non-linearity of this equation prevents us from getting a closed form solution for  $\theta$  as a function of  $\chi$ , but using total derivatives we can show that there is a negative relationship between the two. The numerical simulation allows us to verify this relationship, where we change the steady state level of  $\chi$  and compare the results.

The intuition behind this is the following. A higher  $\chi$  implies that higher debt can be sustained in equilibrium. This in turn implies that a higher fraction of household income is unrelated to the employment status of its members. Hence, as more borrowing and lending can be sustained in the economy, the outside option of workers increase, and consumption falls less when workers become unemployed. Moreover, as entrepreneurs can borrow more and become less constrained, they produce more, thus using more labor and reducing its marginal product. Overall, the outside option of workers increase and the marginal product of labor falls, thus reducing the total surplus of a match in the steady state.

Our calibration with high steady state leverage can be compared to that in Hangerdorn and Manovski (2008), where they show that large unemployment benefits and a high value for the worker's bargaining power parameter also lead to a low value of a match surplus in steady state. They also show that this implies higher employment volatility. In our model we are not able to show this analytically, so for this we concentrate on our quantitative results, which we present below.

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<sup>8</sup>Shimer (2010) shows that this neutrality result for the steady state also carries through to the stochastic simulation of a model with no capital and one type of labor

## 4 Quantitative Analysis

### 4.1 Calibration

As in most labor search models, we calibrate the model on a monthly basis. Table 1 presents the complete list of parameter values used in our numerical simulation. Using BLS data for civilian labor force and employment by educational attainment we calculate the average labor force participation rate and the unemployment rate for skilled and unskilled workers<sup>9</sup>. Based on these numbers we set up the fraction of skilled workers in the labor force to 0.31. We use the disutility parameters  $\gamma_s$  and  $\gamma_u$  to match the long-run average unemployment rate for skilled and unskilled workers of 2.5% and 5.2%, respectively. The discount factor of the representative household,  $\beta$ , is set to 0.996, implying a real annual interest rate of 5%. The income share of capital structures and equipment,  $\alpha_{K_S}$  and  $\alpha_{K_E}$  are set equal to 0.13 and 0.17, following estimates by Greenwood, Hercowitz, and Krusell, (1997), used also in Krusell et al (2000) and Lindquist (2004). The depreciation rates of capital structures and of capital equipment,  $\delta_S$  and  $\delta_E$ , are set equal to 0.01 and 0.05 respectively, to match the ratio of investment in capital structures and capital equipment to output, as observed in the National Income and Product Accounts data. The income share of skilled workers is consistent with a wage premium of 60%, as suggested by the estimations of Acemoglu (1998) and Krusell et al (2000).

The entrepreneur's discount factor,  $\gamma$ , is set equal to 0.85, so as to match, together with the mean value of the share of collateralized capital,  $\bar{\chi}$ , a steady state ratio of debt to quarterly GDP of 0.63 (1.89 in our monthly calibration). This is the average ratio over the period 1976.II-2012.III for the nonfinancial business sector based on data from the Flow of Funds (for debt) and National Income and Product Accounts (for GDP). The mean value of the share of collateralized capital is computed by assuming that the borrowing constraint is always binding. We first compute  $\chi_t$  as a residual using empirical series for end of period debt (Flow of Funds), capital structures (NIPA and own calculations) and wage bill (also from NIPA), all relative to GDP and transformed into monthly frequency. Then,  $\bar{\chi}$  is determined as the average of this residual over the sample period.

The intertemporal elasticity of substitution of the entrepreneur,  $\sigma_E$ , which is also the risk aversion parameter, is set to 0.08, so that the entrepreneur is close to being a risk neutral agent but always exhibits non-negative consumption. Regarding the workers' bargaining power we follow the literature, in particular the parameterization of Shimer (2010), and set  $\phi$  equal to 0.5, assuming that the Hosios (1990) condition holds. The separation rates for both skilled and unskilled are set to 0.0113 and 0.0268, respectively, to match their empirical counterpart as recently reported by Elsby et al (2010). The parameters governing the matching efficiency for both skilled and unskilled

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<sup>9</sup>Series: LNS11027659, LNS12027659, LNS11027660 for less than High School Diploma, LNS11027660, LNS12027660 for High School Graduates, LNS11027689, LNS12027689 For less than Bachelor's Degree and LNS11027662, LNS12027662 for College Graduates. Monthly data from January 1992 to July 2012. We also complete this data we time series for job finding and destruction rates from Elsby et al (2010).

are chosen to match the recruitment costs in hours of each type as reported by the Employment Opportunity Pilot Project (EOPP) 1982 survey and reported by Cajner and Cairo (2012). In our robustness section we change the configuration of the recruiting technology so that only skilled workers can be used to recruit new workers.

**Table 1. Parameter Values**

Proportion of skilled workers in the labor force	$s$	0.31
Household's discount factor	$\beta$	0.996
Skilled workers' disutility of work	$\hat{\gamma}_s$	1.328
Unskilled workers' disutility of work	$\hat{\gamma}_u$	0.842
Entrepreneur's discount factor	$\gamma$	0.85
Entrepreneur's risk aversion	$\sigma_E$	0.08
Unconditional mean of financial shock	$\bar{\chi}$	0.85
Depreciation of capital structures	$\delta_S$	0.01
Depreciation of capital equipment	$\delta_E$	0.05
Income share of capital structures	$\alpha_{K_S}$	0.13
Income share of capital equipment	$\alpha_{K_E}$	0.17
Income share of the skilled (Wage Premium)	$\alpha_{L_S}$	0.3
Workers' bargaining power	$\phi$	0.5
Exogenous separation rate for skilled	$x_s$	0.0113
Exogenous separation rate for unskilled	$x_u$	0.0268
Matching function efficiency for skilled	$\bar{\omega}_s$	1.91
Matching function efficiency for unskilled	$\bar{\omega}_u$	2.75

## 4.2 Stochastic Processes

The economy is subject to both aggregate productivity shocks  $z_t$  and financial shocks, that is, shocks to the share of collateralized capital  $\chi_t$ . To obtain productivity we use the standard solow residual approach. From the production function we get:

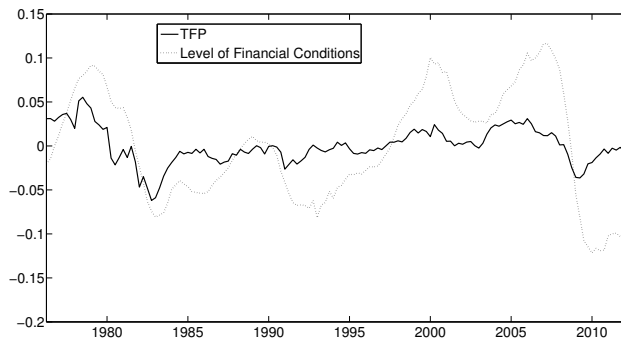
$$\hat{z}_t = \hat{y}_t - \alpha_{K_S} \hat{k}_{S,t} - \alpha_{K_E} \hat{k}_{E,t} - \alpha_{L_S} \hat{l}_{S,t} - (1 - \alpha_{K_S} - \alpha_{K_E} - \alpha_{L_S}) \hat{l}_{U,t}$$

where hat variables are log or percentage deviations from the deterministic trend. Given the values for the parameters  $\theta_{K_S}, \theta_{K_E}$  and  $\theta_{L_S}$  and empirical series for  $\hat{y}_t, \hat{k}_{S,t}, \hat{k}_{E,t}, \hat{l}_{S,t}$  and  $\hat{l}_{U,t}$  we can construct the  $\hat{z}_t$  series.

To construct the series for the financial shock, we express all the variables relative to output and assume that the borrowing constraint holds with equality:

$$\frac{B_{t+1}(1+r_{t+1})}{Y_t} + \frac{W_{U,t}n_{U,t} + W_{S,t}n_{S,t}}{Y_t} = \chi_t \frac{K_{S,t+1}}{Y_t}$$

Each term in this equation has a clear counterpart in the data. The first term is the leverage of the private sector, which can be computed using the Flow of Funds data. The second term is the labor-share of the economy which can be measured using the NIPA accounts. The right-hand side term is the capital structures-output ratio which can be also computed using data from the NIPA accounts. We can thus compute the series for  $\chi_t$  as a residual.<sup>10</sup> Figure 1 plots the series for both shocks over the period 1976.II-2012.II.



Finally, we estimate the autoregressive system:

$$\begin{pmatrix} \hat{z}_{t+1} \\ \hat{\chi}_{t+1} \end{pmatrix} = \begin{pmatrix} \rho_{zz} & \rho_{z\chi} \\ \rho_{\chi z} & \rho_{\chi\chi} \end{pmatrix} \begin{pmatrix} \hat{z}_t \\ \hat{\chi}_t \end{pmatrix} + \begin{pmatrix} \epsilon_{z,t+1} \\ \epsilon_{\chi,t+1} \end{pmatrix}$$

where  $\epsilon_{z,t+1}$  and  $\epsilon_{\chi,t+1}$  are iid shocks with standard deviations  $\sigma_z$  and  $\sigma_\chi$  respectively<sup>11</sup>. The results are presented in Table 2.

**Table 2. Stochastic Properties of Shocks**

Standard deviation productivity shock	$\sigma_z$	0.0044
Standard deviation financial shock	$\sigma_\chi$	0.0057
Covariance parameter innovations	$\sigma_{z\chi}$	0.0008
	$\sigma_{\chi z}$	0.0008
Autoregressive parameter for productivity shock	$\rho_{zz}$	0.9824
Autoregressive parameter for financial shock	$\rho_{\chi\chi}$	0.9738
Spill-over from financial shock to productivity	$\rho_{z\chi}$	-0.0032
Spill-over from productivity to financial shocks	$\rho_{\chi z}$	0.1072

<sup>10</sup>As in Jermann and Quadrini (2002), the validity of this procedure depends on the validity of the assumption that the borrowing constraint is always binding. We check this condition ex-post by feeding the constructed series into the model and checking whether the constraint is always binding.

<sup>11</sup>Please see appendix 8.1 for details on how we obtain our monthly estimates from our quarterly results.

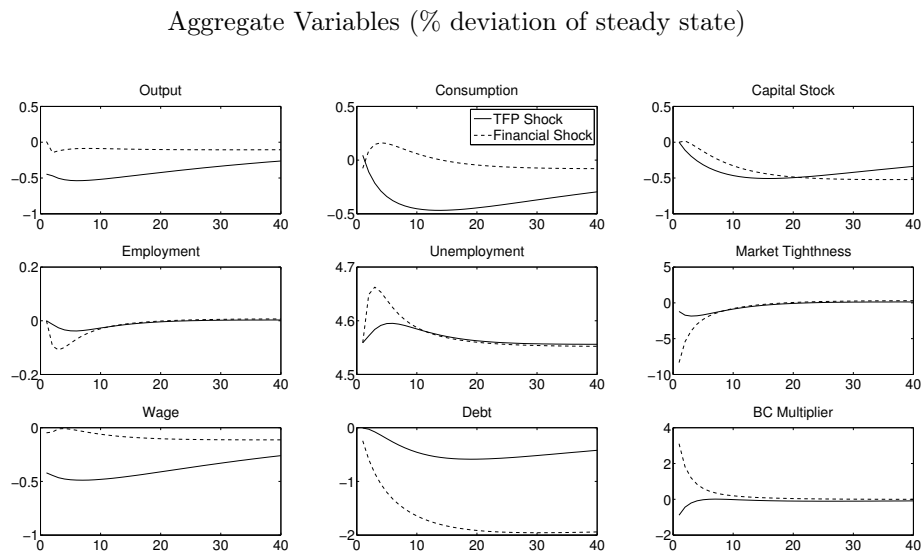
### 4.3 Impulse-Responses

In this section, we analyze the behavior of our benchmark model when hit by a one standard deviation negative shock to aggregate productivity and to the parameter governing the rate of acceptable collateral. Figures 2 and 3 show the impulse response functions for each case.

We begin by looking at the effects of a TFP shock. This shock impacts output directly, and is capable of generating a larger fall in output and consumption relative to a financial shock, but exhibits a dampened response of labor market variables and the aggregate capital stock over a long horizon. This small decline in employment and other labor market variables is not surprising in light of two results. First, Shimer's (2010) neutrality result that, as discussed earlier, predicts low volatility of employment when changes in productivity affect in a similar way both consumption and the marginal product of labor, leaving the surplus of a match unchanged. As seen from the impulse response functions, TFP shocks produce a fall in wages of a similar magnitude as the drop in consumption. Second, the fact that in models with reproducible capital, general equilibrium effects dampen the response of real variables to productivity shocks in the presence of financial frictions. See, for example, Olivella and Roldan (2012).

Regarding relative variables, a TFP shock has little effect on relative employment of skilled to unskilled workers and also a negligible effect on the composition of capital<sup>12</sup>. Changes in TFP affect all production inputs uniformly and have little reallocation effects between the two types of workers and the two types of capital.

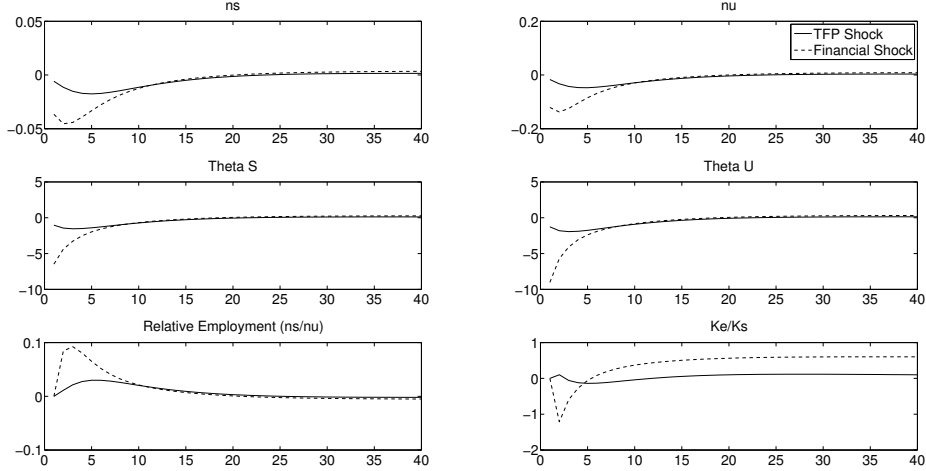
Figure 2. Impulse-Responses Full Model: TFP and Financial Shock



<sup>12</sup>Note that the ratio  $K_E/K_S$  remains relatively constant after a TFP shock.

Figure 3. Impulse-Responses Full Model: TFP and Financial Shock

Selected Variables (% deviation of steady state)



Financial shocks have altogether a quite different impact on all variables. They produce a larger drop in employment and in the stock of debt, while generating a much higher increase in the multiplier of the borrowing constraint, where shocks of this nature have a direct impact.

Three effects explain the large decline in employment after a financial shock. The first one is related to the fact that there are two types of capital, but only one,  $K_S$ , is pledgeable for collateral. When a financial shock hits the economy, firms find themselves more financially constrained and the pledgeable asset  $K_S$  becomes more valuable as it because of its role as collateral. This provides incentives for entrepreneurs to accumulate more of the pledgeable asset at the expense of the unpledgeable one, capital equipment, so that  $K_E/K_S$  declines sharply. The decrease in  $K_E$  is such that aggregate capital stock also falls, bringing down the marginal product of labor and thus, incentives to hire labor.

Second, the temporary increase in reservation wages associated with a lower marginal utility of consumption in the periods following the shock, generates additional incentives to reduce employment. The intuition behind this is the following. In the period in which the financial shock hits the economy, output remains constant as capital and employment are fixed. Given that firms are not demanding as much loans, consumption by households goes up, and this serves as a cushion for the fall in wages. As a result, wages do not fall as much as the marginal product of labor, making the value of a match surplus to fall and with this, employment.

Finally, as a result of the tightening of credit the shadow cost of labor increases proportionally to the multiplier of the borrowing constraint. This provides extra incentives for firms to reduce



hiring and appears in the model as labor wedge.

With respect to the responses of skilled and unskilled employment to a financial shock, we observe that what holds for the aggregate also holds for each type. In particular, a financial shock generates larger movements in both skilled and unskilled employment relative to those generated by a TFP shock. The same is true of the market tightness for both types of labor.

A financial shock also causes the ratio of skilled to unskilled employment ( $n_s/n_u$ ) to move in favor of skilled workers, much more than what a TFP shock does. This arises as a by-product of the large volatility in employment that the financial shock generates and the interaction of the shock with the search friction. Note that although a TFP shock also generates a counter-cyclical ratio of skilled to unskilled workers, it is unable to reproduce almost any of the volatility of this ratio, as observed in the data. In general, as we discuss in the next section, various specifications of the model can generate a counter-cyclical employment ratio. This counter-cyclicality is a result of the parameterization of the model, which incorporates the fact that the pool of unskilled unemployed workers is much larger than the pool of skilled unemployed. Facing the decision to scale down production, firms find it optimal to hire proportionally less unskilled workers than skilled, as attracting unskilled workers in the future is much easier than re-hiring skilled ones. This is the case for both TFP and financial shocks, but it is quantitatively significant only for the latter, as employment falls much more.

In the next subsection we present the basic RBC statistics of the model with both shocks as well as with one shock at the time.

#### 4.4 Business Cycle Statistics

Table 3 presents the standard Real Business Cycle statistics for the data, for our benchmark model with the two shocks and for model specifications that only include one shock at the time. The volatility of the data is computed as the standard deviation of the cyclical component of the detrended quarterly series<sup>13</sup>. To compute volatilities implied by the model we follow the same procedure but adjust the HP filter to control for the fact that the model is calibrated at a monthly frequency<sup>14</sup>.

The statistics for the benchmark model and the two alternative specifications with only one shock at a time confirm what we discussed in the previous subsection when discussing the IRF of the model. The statistics for the TFP shock show that the introduction of the borrowing constraint has no effect on the typical successes and failures of the standard search model. In this way, the model with the TFP shock delivers a volatility of output and consumption similar to the data but

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<sup>13</sup>The sample period is the same used for estimating the stochastic processes: 1976:II-2012:II. We use data from NIPA accounts and report statistics using the Hodrick-Prescott (HP) filter with the standard smoothing parameter  $\lambda = 1,600$  for quarterly data. The volatility for  $\theta$  is taken from Shimer (2010).

<sup>14</sup>The statistics for the model are the theoretical moments using an smoothing parameter of  $10^5$  as in Shimer (2011)

fails to reproduce the volatility of employment variables. The numbers reported for this exercise are very similar to the ones in Shimer (2010).

**Table 3. Second Moments of the Simulated Model**

	Data	Full Model		
		Both Shocks	Z Shock	$\chi$ Shock
Volatilities				
$\sigma_Y$	1.46	1.74	1.30	0.60
Volatility Relative to Output				
$\sigma_C$	0.77	0.75	0.82	2.09
$\sigma_N$	0.66	0.38	0.10	0.81
$\sigma_{K/Y}$	1.38	1.99	0.68	4.17
$\sigma_{n_U}$	0.77	0.47	0.12	0.99
$\sigma_{n_S}$	0.62	0.20	0.05	0.40
$\sigma_\theta$	15.30	17.41	4.63	39.25
$\sigma_{\theta_U}$	-	18.07	4.81	41.04
$\sigma_{\theta_S}$	-	15.21	4.02	33.49
$\sigma_{n_S/n_U}$	0.84	0.28	0.07	0.61
Correlations				
$corr(C, Y)$	0.87	0.38	0.87	-0.35
$corr(N, Y)$	0.79	0.79	0.81	0.55
$corr(K, Y)$	0.38	0.49	0.74	0.72
$corr(n_S/n_U, Y)$	-0.57	-0.77	-0.81	-0.52

The financial shock however, produces statistics that are significantly different. Output has a lower standard deviation, but consumption, aggregate capital stock relative to output, employment (of both skilled and unskilled workers) and the market tightness for both labor markets are more volatile than their counterpart with TFP shocks. The volatilities of most labor market variables in this specification are higher than what is observed in the data. The volatility of employment relative to output, for example, is 0.81, higher than the 0.66 reported in the data. The same is true for the volatility of unskilled employment (0.99 in the model vs 0.77 in the data) and the aggregate market tightness (39 in the model vs 15 in the data). As we discussed earlier, the financial shock has a first order effect on employment as it affects the shadow cost of hiring as well as the reservation wages through the movement in consumption. In fact, the financial shock predicts a counterfactual negative correlation of consumption with output, given that, as shown before, during the first few months after the shock, consumption is above steady-state levels while output is falling.

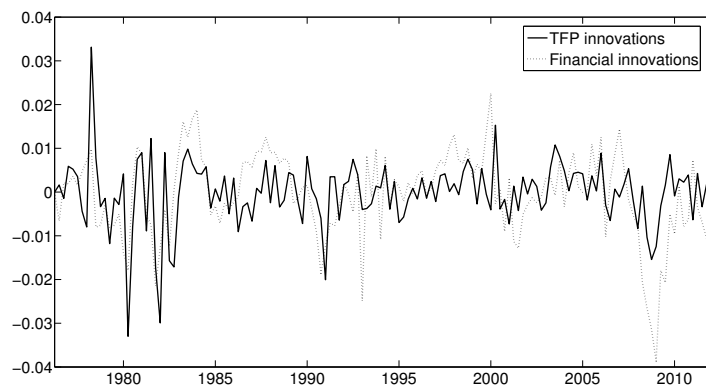
The model that includes both shocks (with properties as estimated in section 4.2) is able to deliver statistics closer to the data. The model explains almost 58% of the volatility in employment relative to output (0.38 in the model vs 0.66 in the data), and 69% of the absolute volatility of employment (0.66 in the model and 0.96 in the data). The volatility of theta relative to output is 17.41 in the model with both shocks, close to the 15.3 of the data and almost 4 times bigger than the volatility delivered by the standard search model with TFP shocks. The benchmark model also delivers the correct correlation among aggregate variables, although the correlation of consumption and output is lower than in the data and than in the standard search model with TFP shocks.

Finally, in the three specifications the ratio of the employment rate of skilled to unskilled is countercyclical, although for the TFP shock this ratio has a very low volatility. The countercyclicity of relative employment results from the presence of search frictions and the calibration of the model that replicates destruction rates of both skilled and unskilled workers as observed in the data. However, only when the model exhibits high volatility of employment is that this ratio moves enough to emulate the volatility of the data.

#### 4.5 Dynamics Induced by Shocks

To study the dynamics of the model induced by the constructed series of shocks, we conduct the following simulation. Starting with initial values of  $\hat{z}_{1976.II}$  and  $\hat{\chi}_{1976.II}$  we compute the quarterly innovations (recall that we use quarterly data to construct the series of shocks) for the period going up to 2012.II. These are shown in Figure 4. It is important to point out how the decline in  $\hat{\chi}$  during the last crisis has been the largest in all of our sample period, and it is in this sense that the recent crisis is characterized by the most severe financial conditions experienced by the US economy for decades.

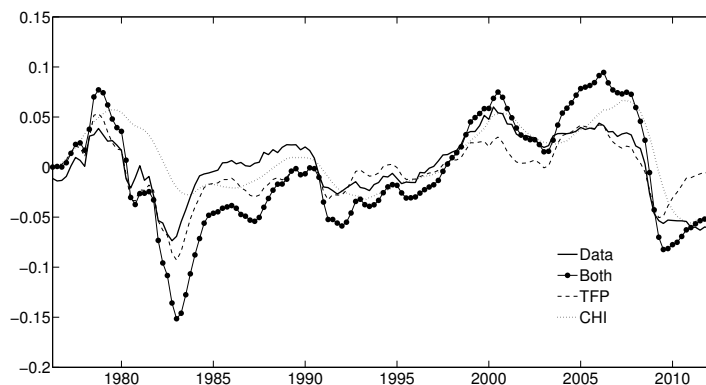
Figure 4. TFP and Financial Innovations



Once we have the quarterly innovations, we transform them into monthly (see appendix for details on how we do this) which we feed into the model and compute the responses for output, employment and relative employment. We do this for three different specifications of the model, one in which we only have productivity shocks (financial variable  $\hat{\chi}$  is kept constant at its unconditional mean  $\bar{\chi}$ ), one with financial shocks only and one with both. Note that although we use the actual sequence of shocks, the agents do not perfectly anticipate them, they forecast their future values using the autoregressive system described earlier. Finally, it is important to mention that when computing the response of the Lagrange multiplier for the borrowing constraint, the negative deviations from its steady state never exceed 100 percent, implying that the multiplier is always positive during the whole simulation period and thus the constraint is always binding.

Figure 5 to 7 plot the response of output, employment and relative employment respectively under the three different model specifications together with the corresponding data (GDP and employment are in logs and linearly detrended over the period 1976.II-2012.II, relative employment is also linearly detrended during the same period but not logged).

Figure 5. Output Simulation With Estimated Shocks  
(% deviations from trend)

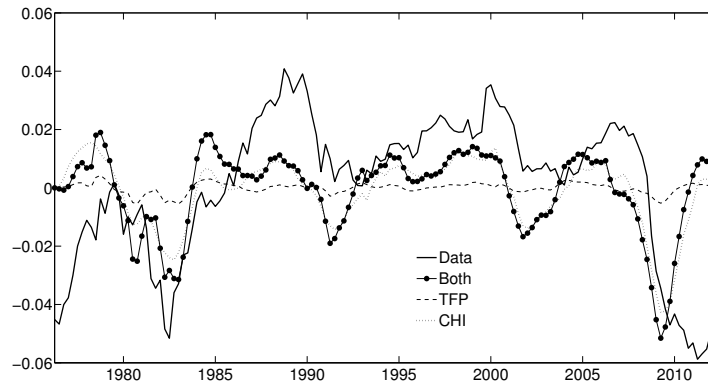


As we can see for the case of output, all of our model specifications fare well in explaining the behavior of the data. This is not surprising for the series generated by the model in response to TFP shocks only, and for the model with financial shocks only, it can be explained by the magnitude of the financial shock during the last crisis. Even more, financial shocks seem to do pretty well in explaining the behavior of output during the last couple of years, while TFP shocks appear to predict a much faster recovery than the one observed.

With respect to employment, the dynamics induced by the model in response to TFP shocks only fail to capture any of the behavior displayed by the data, that is, the model does not generate enough volatility in employment. Not only this, but the drop in employment captured by TFP shocks during

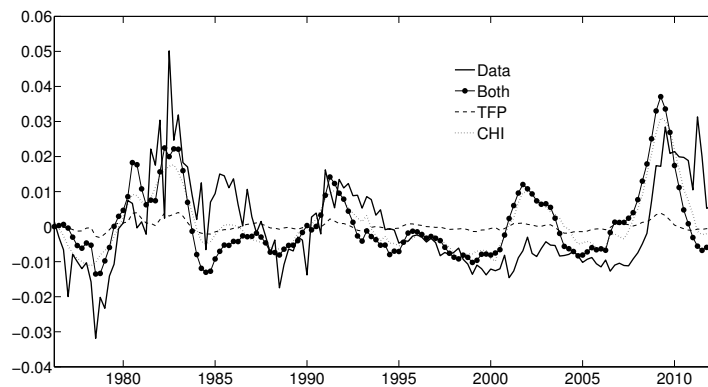
the last recession is barely noticeable relative to the fall observed in the data. On the other hand, the model with financial shocks only is capable of replicating the behavior of employment during the whole sample period quite accurately. In particular, financial shocks generate sharp drops in employment during in all four recessions: 1980s, 1990-1991, 2001 and 2008-2009. The drop in employment generated by financial shocks in the last recession is almost of the same magnitude as that observed in the data.

Figure 6. Employment Simulation With Estimated Shocks  
(% deviations from trend)



Something similar holds for relative employment of skilled to unskilled workers. While the model with TFP shocks can account for almost none of the volatility in relative employment, the model with financial shocks captures the dynamics of the variable quite well, and in particular is able to explain all of the increase in relative employment during the last crisis.

Figure 7. Relative Employment  $n_S/n_U$  Simulation With Estimated Shocks  
(% deviations from trend)



Finally, for employment and relative employment the performance of the model with both shocks is very similar to that of the model with financial shocks only.

## 5 Sensitivity Analysis

In this section we discuss how results change as we modify some of the assumptions of the model. Given that our main interest is on the effects of financial shocks on the elasticity of employment, we focus on the response of the volatility of key employment variables to various modifications of the benchmark model.

Table 4 reports selected statistics from alternative specifications of the model subject to a financial shock. We change the model along 3 dimensions: 1) we change some of the features of the model on the production side, 2) we test how the model changes as we depart from balance growth preferences introducing more or less curvature in the utility function regarding consumption and 3) we recalibrate the model to have lower or higher leverage in equilibrium.

**Table 4. Financial Shocks and Employment Volatility Across Specifications**

	Benchmark	Firms			Households		Leverage	
		1 Capital	No WB	1 Recruiter	Neutral	Averse	Low	High
$\sigma_n/\sigma_Y$	0.81	0.37	0.53	0.65	0.34	1.23	0.43	1.00
$\sigma_{n_U}/\sigma_Y$	0.99	0.44	0.66	0.75	0.42	1.52	0.52	1.24
$\sigma_{n_S}/\sigma_Y$	0.40	0.20	0.26	0.44	0.18	0.62	0.22	0.50
$\sigma_{n_S/n_U}/\sigma_Y$	0.61	0.25	0.40	0.32	0.24	0.92	0.34	0.76
$corr(n_S/n_U, Y)$	-0.52	-0.05	-0.35	-0.40	-0.96	-0.59	-0.23	-0.73

Along the first dimension, our first exercise is to check how results change when the two types of capital can be used as a collateral (which is equivalent to have only one type of capital in the economy). At it can be seen from the second column of Table 4, the model with only one capital generates smaller changes in employment. The reason for this is that when the entrepreneur has only capital, the substitution effect between the two capitals in the benchmark model is no longer present, hence the only way entrepreneurs can escape from the tightening of the borrowing constraint is by accumulating aggregate capital, which in turn increases the marginal product of labor. This partially offsets the negative effect of the financial shock in the labor markets coming from the higher cost of employment in terms of financing.

The second exercise is related to the presence of the wage bill in the borrowing constraint. If labor costs are not part of the borrowing constraint, hiring decisions are not affected by the financing situation of the firm directly. In this case, the model exhibits around 65% of the volatility of employment relative to the benchmark model. This exercise allows us to infer that a large fraction

of the volatility of employment in the benchmark model is coming from the general equilibrium effect of the financial shock not associated to the direct effect it has on the cost of labor.

Column 4 reports the results when the model has only one recruiter of the skilled type. In general, the model exhibits lower volatility but sufficiently high to show that this particular feature is not key for the results of the benchmark model. One important difference between this specification and the benchmark model is the volatility of unskilled workers, which falls to 0.75 from 0.99. When only skilled workers can recruit, the opportunity cost of hiring unskilled is determined by skilled wages, and hence the differences between the two types of labor narrow.

A second dimension of alternative specifications we explore is the one regarding the parameterization of preferences on the household side. Column 5 and 6 present the results of a model recalibrated to have the same steady state but in which the risk aversion coefficient (the intertemporal substitution parameter), is different from one. In the first of these two columns we report the results when households are very close to having linear utility in consumption and in the second we present the case when the risk aversion coefficient is 2. Given that the two specifications have the same steady state as in the benchmark model, this allows to isolate the effect of the dynamics on consumption on wages and hence in the overall elasticity of employment. When agents are risk-neutral, the relative volatility of employment to output is 42% of the one in the benchmark, falling to 0.34 from 0.81. This drop in volatility is common also to both skilled and unskilled employment, and to the relative employment ratio. Altogether this result suggests that the volatility of employment due to financial shocks in the benchmark model is explained roughly 3/5 by the general equilibrium effect of consumption on wages and employment, and 2/5 by the direct effect of the tightening of the borrowing constraint on the shadow cost of labor in terms of financing. Note that Column 6 shows that increasing the curvature on utility leads to an even higher employment volatility.

Finally, we show that the elasticity of employment is directly related to the level of leverage in the economy. For the benchmark calibration we chose parameter values of  $\chi$  and  $\gamma$  consistent with a debt-to-gdp ratio of 0.63 at quarterly basis. In the last two columns we present a calibration of the model that favors low and high leverage respectively. For the first case we re-calibrate  $\gamma$  to be consistent with a steady state debt-to-gdp ratio of 0.2 at quarterly basis. For the high leverage case we target a ratio of 1. The economy with higher leverage shows higher volatilities across of labor market variables, as suggested in our earlier discussion on leverage and employment.

## 6 Conclusions

TO BE COMPLETED

## 7 References

## 8 Appendix

### 8.1 Estimation Stochastic Processes

We use quarterly data to construct our series of TFP and the credit shock, but in our model the stochastic processes have a monthly frequency. Hence, we need to transform our quarterly estimates into monthly numbers. In order to do so, we solve the non-linear system of equations associated to the problem of temporal aggregation as follows.<sup>15</sup>

The monthly auto-regressive process can be represented as:  $\hat{X}t^M = A\hat{X}_{t-1}^M + B\xi_t^M$ , where  $\hat{X}_t^M = \begin{bmatrix} \hat{z}_t^M \\ \hat{\chi}_t^M \end{bmatrix}$ ,  $A = \begin{bmatrix} \rho_{\hat{z}\hat{z}}^M & \rho_{\hat{z}\hat{\chi}}^M \\ \rho_{\hat{\chi}\hat{z}}^M & \rho_{\hat{\chi}\hat{\chi}}^M \end{bmatrix}$ ,  $B = \begin{bmatrix} \sigma_{\hat{z}\hat{z}}^M & \sigma_{\hat{z}\hat{\chi}}^M \\ \sigma_{\hat{\chi}\hat{z}}^M & \sigma_{\hat{\chi}\hat{\chi}}^M \end{bmatrix}$  and  $\xi_t^M \sim N(0, \sum_{z,\chi^M} = I_2)$ . We can re-write it as  $\hat{X}t^M = A^3\hat{X}_{t-3}^M + B\xi_t^M + AB\xi_{t-1}^M + A^2B\xi_{t-2}^M$ , and then, re-stated in quarterly terms  $\hat{X}t^Q = C\hat{X}_{t-1}^Q + D\epsilon_t$ , where  $\hat{X}_t^Q = \begin{bmatrix} \hat{z}_t^Q \\ \hat{\chi}_t^Q \end{bmatrix}$ ,  $C = \begin{bmatrix} \rho_{\hat{z}\hat{z}}^Q & \rho_{\hat{z}\hat{\chi}}^Q \\ \rho_{\hat{\chi}\hat{z}}^Q & \rho_{\hat{\chi}\hat{\chi}}^Q \end{bmatrix}$ ,  $B = \begin{bmatrix} \sigma_{\hat{z}\hat{z}}^Q & \sigma_{\hat{z}\hat{\chi}}^Q \\ \sigma_{\hat{\chi}\hat{z}}^Q & \sigma_{\hat{\chi}\hat{\chi}}^Q \end{bmatrix}$ ,  $A^3\hat{X}_{t-3}^M = C\hat{X}_{t-1}^Q$  and  $D\epsilon_t = B\xi_t^M + AB\xi_{t-1}^M + A^2B\xi_{t-2}^M$ , with  $\epsilon_t \sim N(0, \sum_{z,\chi^Q} = I_2)$ . From our quarterly estimates we get matrices  $C$  and  $D$ . The matrix  $A$ , governing the persistence of the monthly process can be calculated using  $A = C^{\frac{1}{3}}$ . We can recover the  $B$  matrix, by solving the following non-linear system:  $DD' = BB' + ABB'A' + A^2BB'A'^2$ .

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<sup>15</sup>For a discussion on this topic see Marcellino (1999)