How buybacks increase retail competition

Abstract

Many manufacturers offer retailers the possibility to return unsold stocks for a fraction of the wholesale price. We show that, when competing retailers do not observe each others' stocks before choosing prices, returns intensify retail competition by squeezing retailers' margins. This can increase significantly the manufacturer's profit—by up to 25% if demand is linear. There is however a trade-off: returns intensify retail competition by giving retailers the incentive to order excess stocks, the production cost of which is paid up-front by the manufacturer. Therefore the optimal return price, as a percentage of the wholesale price, decreases as the manufacturing marginal cost increases.

1 Introduction

Manufacturers often let retailers return unsold goods for a fraction of the wholesale price. Such policies have been widely documented in a variety of contexts and for a multitude of products, and previous formal explanations rely on uncertainty about market demand.

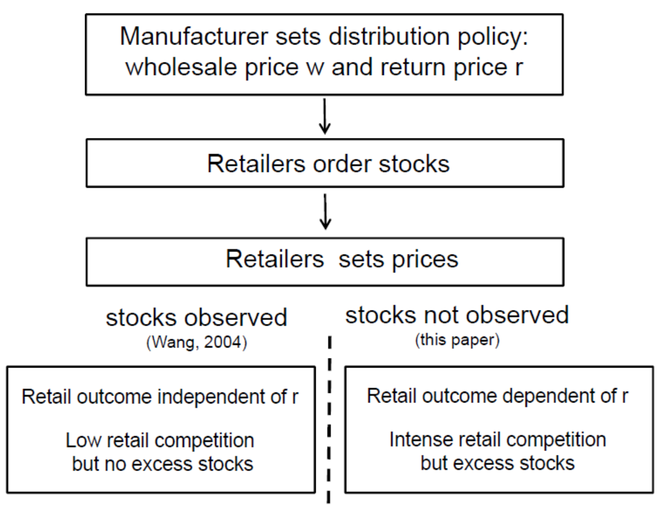
In this paper we abstract from market uncertainty to focus on the strategic use of returns by a monopolist manufacturer to intensify retail competition. As initially suggested by Padmanabhan and Png (1997)—PP hereafter—, it seems natural that returns reduce retailers' marginal cost of doing business, and they should therefore affect the way retailers compete with each other. Manufacturers may then use this marketing tool to manage retail competition to their advantage. However Wang (2004) showed that in a model where *retailers observe each other's stocks* before choosing their prices, which PP assumed, returns have no effect on the equilibrium margins or profits. Thus Wang concluded that "returns policies do not intensify retail competition in the model proposed by PP."

Here we show that if instead retailers do not observe each other's stocks before choosing their prices then returns will have a significant effect on both retail margin and manufacturer's profit, thus formally showing that in the absence of market uncertainty a return policy can also have a significant effect on profitability.

So far the relevance of information on stocks seems to have been overlooked. Models where competing firms observe each others' production capacity before choosing prices are a staple of industrial economics. When first studied by Kreps and Scheinkman's (1983), this representation intended to describe industries where firms make long-term capacity choices, such as the size of a plant, that are either observed or can be accurately inferred by competitors over time.

Yet, that assumption seems hard to justify in a retail setting. Indeed, retailers are unlikely to know the exact stocks the other retailers have in store of a particular good when they choose their prices. Unlike the size of a plant, retail stocks have a transient nature and are hard to observe—reliable information on rivals' current stocks would require access to current and private warehouse information, as historical data is insufficient to provide a good estimate. The view that stocks are not observed is more common in the operations literature as, for example, the newsvendor problem with multiple retailers also assumes that retailers choose prices and stocks without knowing the choices of rival retailers.

Here extend the analysis to a situation where retailers *do not observe each others' stocks* or quantities before choosing their prices. This arguably more realistic situation changes retail competition in a fundamental way and we find that in this case the original intuition of PP is vindicated: higher returns reduce retail margins and a return policy can have a significant effect on manufacturer's profit even in the absence of demand uncertainty—increasing this profit by up to 25% when demand is linear.



The original idea proposed by PP was that, in the absence of returns, if the retailers buy from single manufacturer at a linear wholesale price and *observe the stocks of each other* before choosing their prices then retail competition is of the Cournot type—following the argument by Kreps and Scheinkman (1983). Offering retailers the possibility to return unsold stock at the wholesale price, i.e. *full returns*, eliminates retailers' cost of holding excessive stocks. If for this reason retailers were to order large stocks and choose not to be capacity constrained, then this would create intense Bertrand price competition among retailers. PP concluded that returns would then have the effect of squeezing retail margins and increase the manufacturer's profit.

Wang (2004) observed that if two retailers are identical then PP's logic with full returns results in a perfectly competitive retail outcome with both retailers making zero profits. He then noted that retailers would make positive profits if either ordered less stock. "Hence", he concluded, "ordering 'insufficient' stock makes each retailer better off" and provides a profitable unilateral deviation. Therefore Bertrand style competition cannot be an equilibrium outcome of that game.

By formally studying PP's game, Wang could also show that retailers would never order excess stocks but they would instead use their stock choice as a precommitment to soften price competition. In that case returns do not change the equilibrium strategies or profits of the game. As Kreps and Scheinkman (1983) had also noted, even "situations that 'look' very Bertrand-like", i.e. situations where most of the cost remains variable at the pricing stage, "will still give the Cournot outcome."[1] We find that this result relies on the assumption that retailers not only choose stocks but also that those stocks are perfectly observed by their rivals.

If retailers do not observe each others' stocks or quantities before choosing their prices, then they cannot use stocks as a precommitment to soften price competition. In that case, for any linear wholesale and return price, in the unique equilibrium the competing retailers use mixed strategies: Each retailer sometimes orders a small quantity and charges a high price, while other times he orders a large quantity to hold a sale. This means that in equilibrium retailers are unable to perfectly anticipate the choices of their rivals—even if there is complete information about aggregate demand and all costs. Residual demand uncertainty arises then endogenously for each individual retailer as a consequence of the optimal strategy of rivals, even if there is no aggregate demand uncertainty.[2]

[1] Padmanabhan and Png (2004) responded to Wang (2004) by reintroducing demand uncertainty in the model. They concluded that returns can help to "manage price competition between retailers but that this effect holds only in the presence of demand uncertainty." Indeed, the potential benefit of returns converges to zero as the uncertainty vanishes. With logistic and administrative costs, it becomes hard to justify returns unless there is significant demand uncertainty.

[2] This finding is reminiscent of Varian's (1980) explanation of retail price dispersion in a model with loyal consumers and shoppers. Like here, there the aggregate demand is deterministic but the realized sales of each store are not since they depend on the prices chosen by rival retailers, and these cannot be perfectly anticipated. Retail price dispersion is a well documented fact—see e.g. Berk et al. (2008).

Our first result is that full returns intensify retail competition but may still be unprofitable. The intuition is the following. Higher returns reduce the retailers' downside of being left with unsold stock and so induce retailers to order larger stocks. As retailers try to sell these larger stocks, this results in more intense price competition. In the limit case of full returns the "retailers choose not be constrained by their stocks" and set prices at the wholesale price level, as in Bertrand retail competition—just like PP suggested. However the manufacturer also needs to pay up-front for the production cost of the excess stock that will be returned. The profitability of full returns is then naturally determined by the balance of these two effects, which depends on wether the manufacturing marginal cost is high or low.

Our second result is to characterize the optimal distribution policy. This requires solving a trade-off between the benefit of more intense retail competition and the higher cost of excess stocks. We find that the optimal wholesale price increases with the manufacturing marginal cost and the optimal return price (as a fraction of the wholesale price) decreases with that cost. Note, for example, that the manufacturing marginal costs of books, CDs and software are small in comparison to their market price. Consistent with this result, generous return policies are common in the distribution of those products.

Our third and final result connects our analysis with that of Wang (2004) by studying if observability of the stocks benefits or hurts the manufacturer. Wang showed that when retailers' stock levels are observed before prices are chosen, in equilibrium all stocks are eventually sold to consumers and in that case the manufacturer cannot squeeze the retail margins. We show that when they are unobservable we can have more intense retail competition but an associated cost of excess stocks. It follows that the manufacturer benefits from unobservable retail stocks when the manufacturing marginal cost is low, but this hurts him when that cost is high.

2 Literature review

In addition to the work described above, there is a large literature in economics and marketing that explains the use of returns by an upstream monopolist. Yet, all those theories crucially rely on demand uncertainty.

With a *single retailer*, if demand is uncertain, it has been shown that returns can for example transfer risk from a retailer to the manufacturer (e.g. Kandel 1996 and Marvel and Peck 1995). In that case a return policy and a linear wholesale price can also coordinate a supply chain or improve profits over outright sales (e.g. Pasternack 1985, Emmons and Gilbert 1998 and Marvel and Peck 1995). If demand is realized after production takes place but before the wholesale price is determined, then returns allow a manufacturer to achieve the same profit weather she remains the owner of the stock until demand is realized or if she transfers that ownership to the retailer—which would otherwise decrease the manufacturer's profit, despite increasing the industry total surplus (Biyalogorsky and Koenigsberg 2010). If a retailer needs to invest on a marketing-mix activity that affects consumer demand but the terms of trade may be renegotiated once demand is realized, then transferring the ownership of the stock to the retailer and allowing for returns can reduce double marginalization by converting what would be an ex-post negotiation over the wholesale price into a negotiation over the return price (Iyer and Villas-Boas 2003).

When *retailing is perfectly competitive*, but demand is still uncertain, buybacks can also lead to optimal levels of inventory and price dispersion (Marvel and Wang 2007). They may also be profitably used as a substitute for a price floor. By inhibiting the price cutting that would otherwise occur when demand is low, returns protect the retailers margins in those states. This increases the retailers' incentives to hold larger inventories in equilibrium, even if retail margins are lower when demand is high (Deneckere et al. 1996). Such logic, that returns policies attenuate retail price competition when demand is low but intensify competition when demand is high, also extends to a retail oligopoly (Butz 1997 and Padmanabhan and Png 2004).

In a *retail oligopoly*, contracts with a linear wholesale and return prices can coordinate the supply chain only if the manufacturer retains controls over the retail price (Pasternack 1985). Yet supply chain coordination can still be achieved if the wholesale and return price depend on the prices chosen by retailers (Bernstein and Federgruen 2005), or if in addition a fixed fee is charged and demand uncertainty or retailer differentiation are large (Narayanan et al. 2005 and Krishnan and Winter 2007).[3]

There is also work in the context of *asymmetric information* on an uncertain demand. For example, returns may be used by a manufacturer to signal to retailers private information on demand (Kandel 1996), to elicit from retailers their private information on demand or provide them with incentives, or to acquire such information (Arya and Mittendorf 2004 and Taylor and Xiao 2009). The role of returns in learning demand of a new product has also been studied, assuming that the retail price is fixed and demand is random but identical across periods (Sarvary and Padmanabhan 2001).

[3] In Narayanan et al. (2005) the uncertainty is over the level of demand, i.e. any level of demand between zero and some higher level is equally likely. In Krishnan and Winter (2007) there is uncertainty over both the level of demand and consumers' preferences over retailers while using a first-order approach—which means the results do not apply to cases without sufficient retail differentiation.

While most of the literature has focused on models with a single manufacturer, it has also been shown that, with an *upstream oligopoly* and demand uncertainty, a manufacturer may offer returns to improve her sales relative to competing manufacturers (e.g. Pellegrini 1986 and Bandyopadhyay and Paul 2010).

3 The unobservable stocks model

We study a model similar to Padmananabhan and Png (1997) and Wang (2004) with the exception of the information structure on stocks. There is one manufacturer (she), two retailers (both he), and all players are risk neutral. There is a constant manufacturing marginal cost c. We consider the same three stage game:

First stage: the manufacturer sets a distribution policy with a linear wholesale price w and return price r at which retailers can buy and return any quantity.[4]

Second stage: retailers observe the distribution policy and each retailer i simultaneously decides how much stock, s_i, to order. The only cost of retailing is the payment of the wholesale price w.

Third stage: each retailer i chooses a price p_i and consumers, after observing both prices, decide to purchase or not. Unsold stocks are returned to the manufacturer and refunded at the return price r per unit.

One important element of the information structure is still missing: do retailers observe or not the stocks ordered by the other retailer before they choose their prices?

Following Kreps and Scheinkman (1983), both PP and Wang assumed that retailers observe the stocks of each other before moving to the third stage. Here we focus instead on those situations where each retailer does not observe the stocks of the other retailer before choosing his price. As we argued in the introduction, in a retail setting the latter may be a more realistic assumption than the former.

All our assumptions about demand are similar to those in Kreps and Scheinkman (1983). The consumer demand $D(p)$, with $D' < 0$, has a choke price $c < \overline{p} < \infty$ with $\overline{p} = \inf(p \,|D(p) = 0)$. Market revenue $pD(p)$ is strictly concave on $[0, \overline{p}]$, so first-order conditions on the market profit $(p - c)D(p)$ give the market monopoly price $p^m(c)$.

[4] PP found that no manufacturer used two-part pricing in the ten product categories they studied empirically. One reason can be that arbitrage by retailers across multiple markets served by the same manufacturer restricts the use of non-linear policies.

Retailers are undifferentiated.[5] Therefore all consumers would like to purchase from the retailer with the lowest price. If both firms charge the same price then they share the market equally. If price are different then, for any given pairs (s_1, p_1) and (s_2, p_2), if $p_i < p_j$ retailer i faces the total consumer demand, while if $p_i > p_j$ then i faces a residual demand from those consumers who were unable to buy from retailer j because of a stockout, i.e. since $D(p_j) > s_j$. Assuming efficient rationing, the sales of retailer i are[6]

$$q_i = \begin{cases} \min\{s_i, D(p_i)\} \text{ if } p_i < p_j \\ \min\{s_i, \max\{0, D(p_i) - s_j\}\} \text{ if } p_i > p_j \\ \min\{s_i, D(p_i) - \min\{s_j, D(p_i)/2\}\} \text{ if } p_i = p_j \end{cases}$$

A pure-strategy for the manufacturer is a pair (w, r), with $w \geq 0$ and $0 \leq r < w$. The restriction $r < w$ alleviates notation without loss of generality. A pure-strategy for retailer i is a pair (s_i, p_i), where $p_i \geq 0$ and $s_i \geq 0$. To focus on the payoff relevant strategies, and make it possible to define uniquely the equilibrium, we make the innocuous assumption that each retailer i can only choose a price for which there is positive demand if he has some stock to sell, i.e. if retailer i chooses a pair (s_i, p_i) and $D(p_i) > 0$ then $s_i > 0$.

The manufacturer's profit is

$$\Pi = \sum_i [(w - c)s_i - r(s_i - q_i)]$$

and the profit of retailer i is

$$\pi_i = (p_i - r)q_i - (w - r)s_i.$$

When retailers do not observe stocks before choosing prices, the second and third stages of the game become a simultaneous move game where retailers chose a pair (s_i, p_i) after observing the manufacturer's first stage choice of a policy (w, r). So we need to characterize the Nash equilibria of all retail competition subgames with a generic policy (w, r).

From a retailer's perspective, $w - r$ is a sunk cost while r is the non-sunk part. In related ongoing research, we show that in this case the retail subgame has a unique Nash equilibrium (see supplemental file).[7] This equilibrium is symmetric and involves mixed-strategies that are simple

[5] This technical assumption makes it possible to characterize retail equilibria (unfortunately, the techniques used in this proof do not apply to product differentiation). We are more general than PP in terms of demand specification for homogenous goods, but less general when it comes to differentiation.

[6] Efficient rationing can be interpreted as a situation where if there is price dispersion then those consumers with the highest valuation enter the market first and purchase from the lowest priced retailer.

[7] This extends two studies of related situations where firms choose prices without observing rival's binding

to characterize. Specifically, for any distribution policy (w, r), each retailer i orders $s_i(p_i) = D(p_i)$ while choosing his price using a cdf (with pdf f)[8]

$$F(p_i) = \begin{cases} \frac{p_i - w}{p_i - r} \text{ for all } p_i \in [w, \overline{p}] \\ 1 \text{ for } p_i > \overline{p} \end{cases} \quad \text{and} \; .$$

Moreover both retailers make, in expectation, zero profit.

In equilibrium each retailer i is indifferent between ordering a low quantity and choose a high price (in which case i is likely to be undercut by his rival j and not sell his stock, but if he does sell it then he will make a high margin on this small quantity), and ordering a large quantity to hold a sales promotion (in which case i is likely to undercut j and sell all his stock, but then make a low margin on this large quantity). While on any given date, retailer i cannot predict if retailer j will hold a promotion or set a high price, i can still predict the frequency and intensity of j's promotions—an idea and equilibrium structure which is also reminiscent of Varian's (1980) theory of retail price dispersion.

In the absence of a pure-strategy equilibrium, retailers are unable to predict the exact choice of stock or price made by his rival—even if there is complete information about aggregate demand and all costs involved. This highlights two different sources of demand uncertainty at the store level: One source concerns uncertainty about the aggregate market demand—which is absent from the model. Another source concerns strategic uncertainty about rivals' actions, which also makes the residual demand of a retailer uncertain, and is a consequence of rivals' optimal strategies.

When returns are low the retailers need to charge positive margins to make up for the loss they make when they are unable to sell their stocks, and therefore buy on average small stocks. For any given wholesale price w, an increase in the return price r intensifies retail competition since it reduces the downside of being left with unsold stock and this leads retailers to (on average) order larger stocks and charge lower margins, which is captured by a shift in F leftward.

With full returns, i.e. as $r \to w$, there is no cost of holding stocks and therefore each retailer orders enough stock to serve the whole market alone, i.e., $F(w) \to 1$. As "retailers will not be constrained by their stocks" the price of both retailers converges to the wholesale price w and their margins to zero as if there was Bertrand competition in the retail segment—just like PP suggested.

capacity choices, Levitan and Shubik (1972) and Gertner (1985).

[8] With a residual probability each retailer i also orders $s_i = 0$, i.e. chooses to delist the manufacturer's good from its store.

4 Full returns versus no returns

If the manufacturer offers full returns, in the limit as $r \to w$, each retailer i sets $p_i = w$ and orders $s_i = D(w)$ with probability one. It follows that in this case the manufacturer's profit is

$$\Pi_{FR}(w, c) = (w - 2c)D(w), \tag{1}$$

where FR stands for *full returns.* Maximizing the expression above with respect to w we obtain $w^*_{FR}(c)$ and $\Pi^*_{FR}(c)$. Notice that the manufacturer makes the profit of a monopolist that sells directly to consumer at a price of w but having a marginal cost $2c$ instead of c. Therefore the optimal wholesale price exceeds the monopoly price.

The benefit from intense retail competition created by full returns is countered by the production cost of excess stocks. Only when $c = 0$ can the manufacturer extract the monopoly profit of the vertical chain, i.e. $\Pi^*_{FR}(0) = \Pi^m(0)$.

In the case of no returns, $r = 0$, the manufacturer's (expected) profit is given by the wholesale margin multiplied by the expected stocks to be sold to each of the two symmetric retailers, i.e.,

$$\Pi_{NR}(w, c) = 2(w - c)\int_w^{\overline{p}} f(p)D(p)dp = 2(w - c)\int_w^{\overline{p}} \frac{w}{p^2}D(p)dp, \tag{2}$$

where NR stands for *no returns.*

The trade-off between the benefit of more intense retail competition induced by the larger stocks and the cost of producing those stocks leads to the following result:

Proposition 1. *There exist critical values of the manufacturing marginal cost* $0 < \underline{c} \leq \overline{c} < \overline{p}$ *such that the manufacturer profit is higher with full returns than with no returns if* $c < \underline{c}$, *and the opposite holds if* $c > \overline{c}$. *Moreover the optimal wholesale price with no returns is lower than with full returns.*

Proof. All proofs are in Appendix A.

A prediction of this model is that if a manufacturer transitions from a policy without returns to a policy with full returns then the wholesale price should increase. Moreover a manufacturer should only choose to do this if c is sufficiently low. So we should observe a positive relationship between such transitions and reductions in the manufacturing cost.

To illustrate these results, suppose that demand is described by a general linear form $D(p) = a - bp$. Thus a is the measure of market size, $\overline{p} = a/b$ is the choke price, and the monopoly price lies at half the distance between c and the choke price, i.e. $p^m(c) = (\overline{p} + c)/2$.

In the case of full returns the optimal wholesale price coincides with the monopoly price of a vertical chain with a cost of $2c$ instead of c, i.e.,

$$w^*_{FR}(c) = \frac{\overline{p}}{2} + c \text{ if } c < \frac{\overline{p}}{2} \text{ and } w^*_{FR} = \overline{p} \text{ if } c \geq \frac{\overline{p}}{2}.$$

In the case of no returns the manufacturer's objective function simplifies to

$$\Pi_{NR}(w, c) = 2\,(w - c)\left(\overline{p} - w(1 - (\ln \frac{w}{\overline{p}})\right),$$

and $w^*_{NR}(c) \approx \lambda\overline{p} + (1 - \lambda)c$ where $\lambda = 0.4/1.4$. So $w^*_{FR}(c) > w^*_{NR}(c)$, since the latter is slightly below a third of the distance between the marginal cost and the choke price.

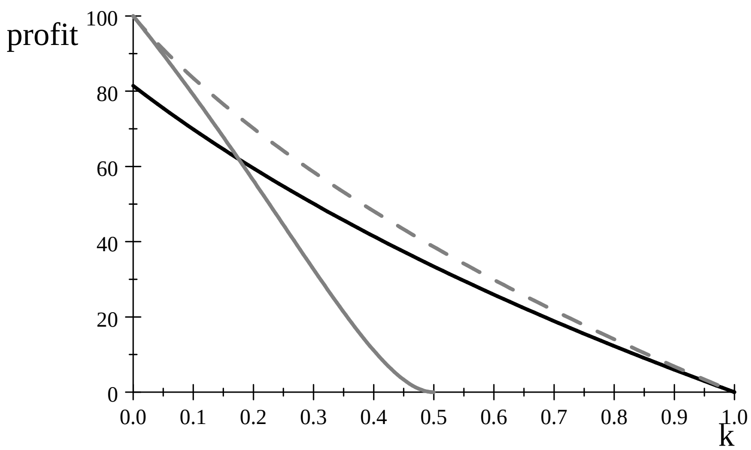


Fig 1. Π^*_{FR} (grey) and Π^*_{NR} (black) as a percentage of Π^m.

The comparison between the equilibrium profits is illustrated in fig. 1 above, where k represents c as a share of the choke price, i.e., $k = c/\overline{p}$. Note that a move from no returns to full returns can increase manufacturer profitability by up to 25%, since if $c = 0$ the profit increases from 80% to 100%. So reductions in the marginal cost c or an increase in the consumers' willingness to pay, captured by an increase in $\overline{p}$, are likely to make full returns more profitable than no returns.

The dashed line in fig. 1 above represents the manufacturer's expected profit when she uses the optimal return price, some amount between zero and the wholesale price. We see that this profit is in general larger than both the profit with full and no returns. We derive and study this optimal policy in the next section.

5 The optimal return policy

Given the retailers' optimal responses, the equilibrium price paid by consumers and the quantity sold to consumers are

$$\underline{p} = \min\{p_i, p_j\} \text{ and } \sum_i E\left[q_i | \underline{p}\right] = D(\underline{p}).$$

The manufacturer makes $(w-r)$ on each unit that is purchased by consumers and gets $(w-c-r)$ on each unit that is produced but ends being returned—this margin can be either positive or negative. The expected quantity returned when $\underline{p}$ is the lowest price is

$$\sum_i E\left[s_i - q_i | \underline{p}\right] = \int_{\underline{p}}^{\overline{p}} \frac{f(p')}{1-F(\underline{p})} D(p')dp'.$$

The probability that the lowest of the two prices is $\underline{p}$ is $2f(\underline{p})(1-F(\underline{p}))$, so the manufacturer's (expected) profit for a general distribution policy (w, r) is

$$\Pi_{UN} = \int_w^{\overline{p}} 2f(\underline{p})(1-F(\underline{p})) \left[(w-c)D(\underline{p}) + (w-c-r)\int_{\underline{p}}^{\overline{p}} \frac{f(p')}{1-F(\underline{p})} D(p')dp'\right] d\underline{p},$$

where UN stands for "unobservable stocks".

To generate tractable analytical results, we consider a situation where demand is linear.

Proposition 2. *When demand is linear, the optimal distribution policy is*

$$r^*(c) = \frac{\overline{p}}{2} \text{ and } w^*(c) = \frac{\overline{p}+c}{2} = p^m(c).$$

So the optimal wholesale price is the monopoly price and the manufacturer makes $(w^* - c) = (\overline{p} - c)/2$ on each unit that is purchased by consumers and a loss of $(w^* - c - r^*) = c/2$ on each unit that is eventually returned. The optimal fraction of the wholesale price offered as a return is then

$$\frac{r^*(c)}{w^*(c)} = \frac{\overline{p}}{\overline{p}+c}.$$

This ratio decreases with c, from 100% when c is zero to 50% as c converges to the choke price. These percentages would be lower if there were some additional variable costs associated with returns, like transportation costs—see the conclusion for a discussion of this topic.

In this case the manufacturer's (expected) profit with the optimal distribution policy simplifies to

$$\Pi_{UN}^{*} = \frac{\overline{p}-c}{2} \underbrace{\left[\frac{a}{2}(1-k)^2\right]}_{\text{Expect. quantity sold}} - \frac{c}{2} \underbrace{\left[\frac{a}{2}(1+2k\ln k-k^2)\right]}_{\text{Expect. quantity returned}}. \tag{3}$$

When the manufacturer's marginal cost is low, the optimal distribution policy involves large returns, which mean that a large fraction of the total production is also returned. In that case the cost associated to excess stocking is insignificant but the quantity sold to consumers is close to the monopoly quantity. Therefore the manufacturer's profit is also close to the monopoly profit—and equal to it when $k = 0$.

As the marginal cost increases, the manufacturer finds it optimal to reduce the percentage of the wholesale price that is returned because this reduces the fraction of the total quantity produced that is expected to be returned—note from (3) that the ratio of the total quantity returned to the quantity sold decreases as k increases and converges to zero as c converges to the choke price, i.e. as $k \to 1$.

5.1 Observable vs unobservable stocks

Comparing a situation where stocks are observed before price competition to a situation where they are not will help us understand the incentives manufacturers have to promote or not this information exchange.

As Wang (2004) showed, if retailers observe each others' stocks before choosing prices then for any return price the retail market outcome coincides with the Cournot outcome with a wholesale price w. So, while there is no excess stocks, the manufacturer is unable to squeeze the retail margins since retailers use stocks as a quantity precommitment to soften price competition and protect those margins.

When demand is linear the manufacturer's profit with observable stocks is maximized for a wholesale price equal to $p^m(c)$—so the optimal wholesale price is the same when stocks are observed and when they are not. At that wholesale price, each retailer purchases one third of the monopoly quantity. Retailers charge a positive margin and the manufacturer's equilibrium profit when stocks are observed is only 2/3 of the monopoly profit, i.e.,

$$\Pi_{OB}^{*} = \frac{2}{3}(p^m(c)-c)D(p^m(c)) = \frac{\overline{p}-c}{2}\frac{a}{3}(1-k). \tag{4}$$

Recall, looking at the dotted line in figure 1, that with unobservable stocks the manufacturer's profit as a fraction of the monopoly profit decreases as k increases, from the full monopoly profit when $k = 0$ to zero when $k = 1$. Naturally, the net effect depends on the manufacturing marginal cost.

Proposition 3. *When demand is linear and the manufacturer uses optimal distribution policies, there is a critical value* $\widetilde{k}$, where $k = c/\overline{p}$ and $\widetilde{k} \simeq 0.228$, *such that for* $k \leq \widetilde{k}$ *the manufacturer's profit when stocks are not observable before retailers choose their prices is larger than when they are observable, and the opposite holds for* $k \geq \widetilde{k}$.

When stocks are unobservable the manufacturer can squeeze retail margins at the expense of overstocking. If the manufacturing marginal costs is low, the manufacturer can use a generous return policy to intensify retail competition without suffering much from the cost of producing excess stocks. In the limit case where k is zero, unobservable retail stocks can increase the manufacturer's profit up to 50%—since it increases from two thirds to the full monopoly profit. The opposite argument and result holds when the cost is high.

6 Conclusion

In practice retailers are unlikely to perfectly observe the stocks of their rivals before choosing their prices. The main message of this paper is that in this situation a manufacturer can profitably use a return policy to induce retailers to order excess stocks and compete more aggressively in prices, thus squeezing the retail margins. In our model some level of returns (as a percentage of the wholesale price) always increase manufacturer profitability if retailers do not observe the stocks of their rivals—while returns would have no effect on profit if instead those stocks were observable.

This insight naturally extend to contracts with per unit royalties. Indeed, the return policies studied here are also equivalent to contracts with a linear wholesale price and a royalty per unit sold—in both cases the retailer needs to pay a price per unit sold and a lower price per unit not sold.

In our analysis we have assumed that there were no retailing costs in addition to the wholesale price. We could however have introduced two types of retail costs: a cost of selling the product (e.g. sales and post-sales service) and a cost of stocking the product. With the former type of cost the analyzes and intuition remain unchanged, since it can be thought as a inward shift of the demand curve—changing the critical values but not the insights.

Retail stocking costs would on the other hand make the manufacturer choose to offer less generous return policies since using returns to intensify retail competition becomes then less effective. The reason is that in this case the wholesale price is only a fraction of a retailer's cost of holding stocks, which means that even a full return policy cannot completely eliminate the retailer's cost of holding excessive stocks.

Returned stocks may also be a source of additional revenue (e.g. scrap value) or create additional costs (e.g. transportation and disposal). For example, we see generous return policies in milk distribution as a manufacturer can still use it to make some dairy products but there is not much supermarket can do with unsold milk,

We have not considered explicitly such benefits or costs in our model, but it is simple to introduce those elements as well. If the scrap value of excess stock is a fraction of the manufacturing cost then the manufacturer will offer more generous return policies to intensify retail competition—and the opposite in the case where returned stocks create additional costs.

This effect can motivate a manufacturer to develop secondary uses for her products, such as exporting returned apparel goods to developing countries or for a milk producer to expand her dairy line. Even if those products were to be sold below their original production cost—i.e., even if those alternative are in isolation unprofitable—, the additional revenue generated by those alternative uses make return policies less onerous. This allows a manufacturer to offer more generous return policies to intensify retail competition in the primary market, which can in turn increase the manufacturer's overall profitability.

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Appendix A

Proof of Proposition 1. In step 1 we show the first part of the Proposition, in step 2 we explain that $w^*_{NR}(c)$ is unique, and in step 3 we show that $w^*_{NR}(c) \leq w^*_{FR}(c)$.

Step 1. $\Pi_{FR}(w^*_{FR}, c)$ and $\Pi_{NR}(w^*_{NR}, c)$ are continuous in c and a revealed preference argument shows that both are also strictly decreasing in c. As

$$\Pi_{FR}(w^*_{FR}, 0) = \Pi^m(0) > \Pi_{NR}(w^*_{NR}, 0),$$

and

$$\Pi_{FR}(w^*_{FR}, \frac{\overline{p}}{2}) = 0 < \Pi_{NR}(w^*_{NR}, \frac{\overline{p}}{2}),$$

we have that there must exist $0 < \underline{c} \leq \overline{c} < \overline{p}$ such that the manufacturer profit is higher with full returns than with no returns if $c < \underline{c}$, and the opposite holds if $c > \overline{c}$.

Step 2. Notice that Π_{NR} is differentiable and strictly positive for all $w \in (c, \overline{p})$ and that $\Pi_{NR}(c, c) = \Pi_{NR}(\overline{p}, c) = 0$. We can therefore use first-order conditions to determine $w^*_{NR}(c)$. Moreover $w^*_{NR}(c)$ is unique since $\Pi_{NR}(w, c)$ has a single inflection point—using the second and third derivative of $\Pi_{NR}(w, c)$, we find that the third derivative with respect to w is positive for every $w \in [c, \overline{p}]$ and that the second derivative with respect to w is negative for w close to c and positive for w close to $\overline{p}$.

Step 3. With the FOC we find that $w^*_{NR}(c)$ must satisfy

$$(2w^*_{NR} - c) \int_{w^*_{NR}}^{\overline{p}} f(p)D(p) - (w^*_{NR} - c)D(w^*_{NR}) = 0.$$

So w^*_{NR} is the single w where the revenue made with the stock expected to be sold to both retailers minus the cost of producing the expected stock sold to one retailer is equal to the profit made by selling directly to consumers at w^*_{NR}.

Suppose that $w^*_{NR} > w^*_{FR}$. Then we should have that at w^*_{FR} the FOC of $\Pi_{NR}(w, c)$ is positive, i.e.,

$$(2w^*_{FR} - c) \int_{w^*_{FR}}^{\overline{p}} f(p)D(p) - (w^*_{FR} - c)D(w^*_{FR}) > 0,$$

in which case its second derivative must be negative, i.e.,

$$2 \int_{w^*_{FR}}^{\overline{p}} f(p)D(p) - 2D(w^*_{FR}) - (w^*_{FR} - c)D'(w^*_{FR}) < 0,$$

and finally that the FOC of $\Pi_{FR}(w,c)$ is zero, i.e.,

$$D(w^*_{FR}) + (w^*_{FR} - 2c)D'(w^*_{FR}) = 0.$$

From the second and third of these conditions we get that

$$-D'(w^*_{FR})(w^*_{FR} - 3c) > 2\int_{w^*_{FR}}^{\overline{p}} f(p)D(p).$$

Replacing in the first condition we get that

$$\begin{aligned} 2(2w^*_{FR} - c)\int_{w^*_{FR}}^{\overline{p}} f(p)D(p) &> 2(w^*_{FR} - c)D(w^*_{FR}) \Rightarrow \\ -D'(w^*_{FR})(w^*_{FR} - 3c)(2w^*_{FR} - c) &> -2(w^*_{FR} - c)(w^*_{FR} - 2c)D'(w^*_{FR}) \Leftrightarrow \\ (w^*_{FR} - 3c)(2w^*_{FR} - c) &> 2(w^*_{FR} - c)(w^*_{FR} - 2c) \Leftrightarrow -c\,(c + w^*_{FR}) > 0, \end{aligned}$$

which is an impossibility. Thus we get a contradiction, and therefore it must be that $w^*_{NR} \leq w^*_{FR}$.

Proof of Proposition 2. In step 1 we determine the expected profit. In step 2 we take the FOC with respect to r and w. In step 3 we discuss second order and global conditions.

Step 1. As explained in the text, the manufacturer's (expected) profit for a general distribution policy (w,r) is

$$\Pi_{UN} = \int_w^{\overline{p}} 2f(\underline{p})(1 - F(\underline{p}))\left[(w - c)D(\underline{p}) + (w - c - r)\int_{\underline{p}}^{\overline{p}} \frac{f(p')}{1 - F(\underline{p})}D(p')dp'\right]d\underline{p}$$

Suppose $D(p) = a - bp$ and the choke price is therefore $\overline{p} = a/b$. From Lemma 1, $F(p) = \frac{p-w}{p-r}$, $f(p) = \frac{w-r}{(p-r)^2}$ and $1 - F(p) = \frac{w-r}{p-r}$. Replacing, we have that

$$\int f(p')D(p')dp' = -(w - r)b\left(\frac{\overline{p} - r}{p' - r} + \ln\left(p' - r\right)\right)$$

and thus

$$\begin{aligned} \int_{\underline{p}}^{\overline{p}} \frac{f(p')}{1 - F(\underline{p})}D(p')dp' &= -\left(\frac{\underline{p} - r}{w - r}\right)(w - r)b\left[(1 + \ln(\overline{p} - r)) - \left(\frac{\overline{p} - r}{\underline{p} - r} + \ln(\underline{p} - r)\right)\right] = \\ & -(\underline{p} - r)b\left(1 - \frac{\overline{p} - r}{\underline{p} - r} + \ln\left(\frac{\overline{p} - r}{\underline{p} - r}\right)\right). \end{aligned}$$

Using below the fact that $y = \frac{\overline{p}-r}{\underline{p}-r}$ so $\frac{dy}{d\underline{p}} = -\frac{\overline{p}-r}{(\underline{p}-r)^2}$ or $dy = -\frac{\overline{p}-r}{(\underline{p}-r)^2} d\underline{p}$, we will have that

$$\begin{aligned}
& \int_w^{\overline{p}} 2f(\underline{p})(1-F(\underline{p})) \int_{\underline{p}}^{\overline{p}} \frac{f(p')}{1-F(\underline{p})} D(p')dp'd\underline{p} \\
= & \int_w^{\overline{p}} 2b\frac{(w-r)^2}{(\underline{p}-r)^2}\left(\frac{\overline{p}-r}{\underline{p}-r} - 1 - \ln\left(\frac{\overline{p}-r}{\underline{p}-r}\right)\right)d\underline{p} \\
= & -2b\frac{(w-r)^2}{\overline{p}-r}\left[\int (y-1-\ln y)dy\right]_{\underline{p}=w}^{\underline{p}=\overline{p}} \\
= & -b(\overline{p}-r)\left(\left(\frac{w-r}{\overline{p}-r}\right)^2 - 1 - 2\left(\frac{w-r}{\overline{p}-r}\right)\ln\left(\frac{w-r}{\overline{p}-r}\right)\right)
\end{aligned}$$

In addition, setting $z = \frac{w-r}{\overline{p}-r}$ and since $\overline{p} = a/b$, we also have that

$$\begin{aligned}
& \int_w^{\overline{p}} 2f(\underline{p})(1-F(\underline{p}))(w-c)D(\underline{p})d\underline{p} \\
= & (w-c)\int_w^{\overline{p}} 2\frac{(w-r)^2}{(\underline{p}-r)^3}(a-b\underline{p})d\underline{p} \\
= & (w-c)b\frac{(\overline{p}-w)^2}{(\overline{p}-r)} \\
= & b(1-2z+z^2)(\overline{p}-r)((\overline{p}-r)z-(c-r))
\end{aligned}$$

So we finally have

$$\begin{aligned}
\Pi_{UN} &= b(1-2z+z^2)(\overline{p}-r)((\overline{p}-r)z-(c-r)) - (w-c-r)b(\overline{p}-r)(z^2-1-2z\ln z) \\
&= b(\overline{p}-r)(2(1-z+z\ln z)(z(\overline{p}-r)-c) + r(1-z)^2)
\end{aligned}$$

Step 2. To simplify further, we normalize the problem by setting $\overline{p} = 1$, so $k = c$ and

$$\Pi_{UN} = b(1-r)(2(1-z+z\ln z)(z(1-r)-k) + r(1-z)^2)$$

We take FOC with respect to r and solve it to find

$$r(z,k) = \frac{1}{-8z-4z^2\ln z+6z^2+2}\left(2k-6z-4z^2\ln z-2kz+5z^2+2kz\ln z+1\right)$$

Given the length of the expressions, we only report here the reasoning and result: Take the FOC with respect to z and replace $r(z,k)$ in that FOC. Solve the expression and find that the only solution that lies in the admissible interval $z \in (0,1)$ is $z^* = k$. Replace back in $r(z,k)$ and simplify to find $r^* = 1/2$.

Step 3. (Again, given the length of the expressions, we have not included the expressions and only report here the procedure and result.) Take the SOC and compute the Hessian matrix. Evaluate the Hessian matrix at $z^* = k$ and $r^* = 1/2$ to find that it is negative-definite and therefore this point is a local maximum. Since this is the only interior solution that satisfies FOC, to check that it is a global maximum we only need to look at the corner solutions. Solutions along the plane boundary where $z = 1$ and $r \in (0, 1)$ generate zero profit and are therefore dominated. Solutions along that boundary where $r = w$ correspond to $z = 0$ and therefore to the case of full returns, which is also dominated (see Fig. 1)—only equal if $k = 0$. Finally those solutions on the boundary where $z \in (0, 1)$ and $r = 0$ correspond to the case of no returns also studied above, and are also dominated (see Fig. 1). Finally, since we normalized the units with $\overline{p}$ and $z = \frac{w-r}{\overline{p}-r}$, we multiply the previous solution by $\overline{p}$ and conclude that the pair $(w^*, r^*) = (\frac{\overline{p}+c}{2}, \frac{\overline{p}}{2})$ is a global maximum of Π_{UN}.

Proof of Proposition 3. Follows directly from comparing (4) with (3). We solve the inequalities to find that there exists a $\widetilde{k} \simeq 0.228\,47$ such that

$$\Pi^*_{OB} \begin{cases} > \Pi^*_{UN} \text{ if } 1 > k > \widetilde{k} \\ = \Pi^*_{UN} \text{ if } k = \widetilde{k} \\ < \Pi^*_{UN} \text{ if } k < \widetilde{k} \end{cases}$$

Appendix B

Proof of Lemma 1. In step 1 we show that there can not exit a pure-strategy equilibrium of the subgame. In step 2 we show that there can not either exist a mixed strategy equilibrium where the expected profit of both retailers is strictly positive, while in step 3 we show that in any equilibrium the retailers' expected profit must be zero. In step 4 we show that the strategies of the Lemma form a symmetric mixed-strategy Nash equilibrium, and in step 5 we show that this equilibrium of the subgame is unique. In step 6 we perform some comparative statics on F to substantiate the claims made in the text following Lemma 1.

Step 1. For a given distribution policy (w, r), the Nash equilibrium of the retail subgame is a pair (s_1^*, p_1^*) and (s_2^*, p_2^*) such that for each retailer i we have that (s_i^*, p_i^*) maximizes π_i given that $(s_j, p_j) = (s_j^*, p_j^*)$ (for simplicity we have dropped the dependence of (s_i^*, p_i^*) on w and r). In a mixed strategy the exact (s_i^*, p_i^*) can be a random vector. We denote the mixed strategy of retailer i by a c.d.f. F_i of prices and a c.d.f. G_i^p of stocks when his price is p. Given the strategy of retailer j, the expected profit of retailer i if he chooses a pair (s_i^*, p_i^*) is

$$\pi_i^e(s_i^*, p_i^*) = \int_0^{\overline{p}} \int_0^{D(p)} \pi_i(p_i^*, s_i^*, p, s) dG_j^p(q) dF_j(p).$$

If i uses a mixed strategy, then his expected profit must be equal for all pairs (s_i^*, p_i^*) that are chosen with a positive probability.

It is shown by contradiction that there is no pure-strategy equilibrium. (The formal proof is tedious and not essential for the remainder of the argument, therefore we here provide only the intuition but a formal proof is available on request.) If both retailers charge the same price and make positive profits, then one retailer would find it profitable to deviate by slightly undercutting his rival and order a stock equal to the market demand at that price. Also, there cannot be an equilibrium where both firms earn zero profits and charge the same price because, with the assumption that $s_i > 0$ if $D(p_i) > 0$, one firm could deviate to a slightly higher price and make a positive profit. We can show in a similar way that pure-strategy equilibria with different prices also cannot exist because it creates similar profitable deviations.

Step 2. We now show that there cannot exist a mixed strategy equilibrium where the expected profit of *both* retailers is strictly positive. Define $\underline{p}_i$ as the lowest price that retailer i uses with a positive probability. If $\underline{p}_i = \underline{p}_j$ then only one of them can have a mass point at that price—otherwise one of the retailers could increase his profit by charging a slightly lower price and serve the whole demand at that price and increase his profit. We refer to the lowest priced retailer as retailer 1, or if the the lowest price is the same for both then the one with a mass point at that price is firm 1—if neither has a mass point at that price then either firm can be retailer 1. Suppose that $\pi_1^e > 0$. At $\underline{p}_1$ retailer 1 should order $D(\underline{p}_1)$, since $\underline{p}_1$ must exceed w and therefore he makes a positive margin on each unit. So at the bottom of the joint price the whole demand is supplied, weather firm 1 has a mass point at $\underline{p}_1$ or not. Define now $\overline{p}_i$ as the highest price charged by retailer i with a positive probability and by $\overline{p}^0$ the highest of those two prices. If both firms earn positive profits then $\overline{p}^0 < \overline{p}$ for each firm to be able to make a positive expected profit with each price that he uses with a positive probability. At $\overline{p}^0$ the probability this price is strictly lower than p_j is by definition zero and therefore the probability of facing the entire industry demand is also zero. Thus the stock of a firm that chooses that price must be less than $D(\overline{p}^0)$. We conclude that if an equilibrium where both retailers make a positive profit exists then it must be that at the low end of the price distribution of that equilibrium the entire industry demand is produced, while at the high end of that distribution less than the entire industry demand is produced.

In this case there must also exist some price p^0, with $\underline{p}_1 < p^0 < \overline{p}^0$, below which the entire demand is produced, thus p^0 is the infimum price such that less than $D(p)$ is produced by at least one of the retailers. Suppose that p^0 is determined by the price distribution of firm i. If retailer i chooses p^0 then he only makes a positive profit if $p^0 < p_j$ since, by p^0's definition, j produces the entire demand for any lower price leaving in that case no residual demand for retailer i. Then the profit of retailer i if he chooses that price is

$$(1 - F_j(p^0))(p^0 - w)s_i(p^0) - F_j(p^0)(w - r)s_i(p^0).$$

If positive, it increase with $s_i(p^0)$. In that case firm i would be able to increase his profit by increasing his stock order from the proposed $s_i(p^0)$ to $D(p^0)$. This contradicts that p^0 is the infimum price such that less than $D(p)$ is produced by at least one of the retailers. This shows that such an equilibrium where the profit of both retailers is positive cannot exist.

Step 3. To show that both retailers must earn zero profits we need in addition to show that we cannot have an equilibrium where firm i makes a strictly positive profit and firm j has zero profit. This is true because firm i must earn a positive profit even if he charges $\underline{p}_i$, and for that profit to be positive it must also be that $\underline{p}_i > w$. But then firm j could deviate to a price slightly below $\underline{p}_i$ and order a stock equal to the whole demand at that price to make a strictly positive profit—thus proving that in any mixed strategy equilibrium it must be the case that the profit of both retailers is zero.

Step 4. Now we check that the strategies in Lemma 1 form a Nash equilibrium of the game where both firms earn zero profit. First note that, given the strategy of his rival, ordering a positive stock and setting a price below w or above $\overline{p}$ yields a negative profit. Moreover, for any price $p \in [w, \overline{p})$, the profit of firm i is

$$(1 - F(p))(p - w)D(p) - F(p)(w - r)D(p) = 0,$$

so no firm can find a profitable deviation from his strategy against the strategy of his rival.

Step 5. To show that this equilibrium is also unique, let $\theta_i(p)$ denote the unconditional probability that $s_i(p_i)$ is equal to $D(p_i)$ for each price $p_i < p$. We must show that $F(p) = \theta_i(p)$ for all $p \in [w, \overline{p}]$ and i. Suppose not. Then there exists some $p' \in [w, \overline{p}]$ such that $\theta_i(p') < F(p')$ or $\theta_i(p') > F(p')$. If the former is verified, then retailer j can earn a positive profit by ordering some stock $0 < \epsilon \leq D(p')$ and charging p' since the profit is then

$$\pi_j^e(p', \epsilon) = (1 - \theta_i(p'))(p' - w)\epsilon + (\theta_i(p'))(w - r)\epsilon > 0$$

given the way F was obtained above. Therefore this cannot happen in an equilibrium. If $\theta_i(p') > F(p')$ and j chooses $p^{'}$ he will make a negative profit, therefore in that case $f_j(p') = 0$. In a equilibrium firm i must earn zero profits and be indifferent between charging each price in its support. However this cannot be verified if $f_j(p') = 0$ for some p' since in that case i can make a strictly positive profit by choosing to order a arbitrarily small stock and charge p'—as he must make zero profit for any price below p'. Again we obtain a contradiction. We conclude that $\theta_i(p) = F(p)$ for all i and $p \in [w, \overline{p}]$, and therefore the strategies in Lemma 1 form the unique Nash equilibrium of the retail subgame.

Step 6. Since $s_i(p_i) = D(p_i)$, the expected stock and retail margin of i for a given pair (w, r) are respectively

$$\int_w^{\overline{p}} D(x)f(x)dx \text{ and } \int_w^{\overline{p}} (x - w)f(x)dx.$$

Note that $\frac{\partial F(p_i)}{\partial r} > 0$ for all $p \in (w, \overline{p}]$, and $D(p)$ decreases with p while $(p - w)$ increases in p. Thus, for a given p, we have that the expected stock and retail margin respectively decrease and increase with r. In the limit as $r \to w$ we have the expected stock and retail margin respectively converging to $D(w)$ and 0.