

JOB UNCERTAINTY AND DEEP RECESSIONS

MORTEN O. RAVN^{1,2,3} AND VINCENT STERK^{1,2*}

University College London¹, CfM@UCL² and the CEPR³

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Abstract

We study a heterogeneous agents model that combines matching frictions in the labor market with incomplete asset markets and nominal rigidities. Workers can experience job terminations that send them into very short term unemployment or more serious job terminations that a longer search process. We show that an increase in job uncertainty decreases aggregate demand which lowers hiring and therefore produces even more job uncertainty and potentially a deep recession. The amplification mechanism is small when asset markets are complete, prices are flexible or unemployment is predominantly short term. With a moderate and empirically plausible amount of change in the composition of unemployment, the model can account for the amplitude of the increase in unemployment during the Great Recession, for the increase in unemployment duration, and for much of the shift and movement along the Beveridge curve.

Keywords: Job uncertainty, recessions, incomplete markets, heterogeneous agents, nominal rigidities, unemployment

JEL: E21, E24, E31, E32

***Ravn:** Department of Economics, University College London and the CEPR. m.ravn@ucl.ac.uk; **Sterk:** Department of Economics, University College London. v.sterk@ucl.ac.uk. We are grateful to seminar participants at the Federal Reserve Bank of Chicago, Northwestern University, the University of Oslo, University of Vienna and at the Bank of England for comments.

1 Introduction

The Great Recession has witnessed unprecedented increases in the level and the duration of unemployment in the United States. The unemployment rate in September 2012 remains close to 8 percent having surpassed 10 percent in late 2009 and the number of unemployed workers who have been out of work for 6 months or more now accounts for more than 40 percent of total unemployment, see Figure 1.¹ Much attention has already been devoted to examining these developments,² but relatively little work has explored the idea that the labor market crisis itself may have been an important *source* of the recession. In this paper we show that shocks that impact on future job prospects can be amplified significantly when a vicious circle of feedback arises between the labor market and the goods market. We demonstrate that such an amplification mechanism occurs when unemployment risk has a large impact on the desire for household savings, when aggregate demand has a strong impact on firms' hiring decisions, and when at least some fraction of the unemployed face more difficult matching problems. We argue that all of these circumstances are relevant for the Great Recession and thus that the labor market amplification mechanism is key for understanding the depth of the Great Recession.

We examine a model in which a menu of frictions interact. First, households face idiosyncratic unemployment risk and asset markets are incomplete. Specifically, we assume that households have access only to a nominal state-non-contingent bond and must obey a borrowing limit. Actual asset markets may be more sophisticated than this but the key assumption is that households cannot fully insure against idiosyncratic risks originating in the labor market. Market incompleteness induces an incentive for households to engage in precautionary

¹Elsby, Hobijn and Şahin (2010) provide a very insightful discussion of the labor market developments during the Great Recession.

²See Daly et al (2011) for a discussion of the change in the natural rate of unemployment during the Great Recession; Farber (2012) and Sterk (2011) examine the impact of the housing crisis on unemployment; Farber and Valletta (2011) and Rothstein (2011) examine the role of unemployment benefit duration; Atif and Sufi (2011) argue that much of the drop in employment can be accounted for by an adverse aggregate demand shock caused by a deterioration of household balance sheets.

savings as a means of self-insurance. Secondly, firms are monopolistically competitive and face nominal rigidities in price setting which we model using a state-contingent pricing set-up, see Rotemberg (1982). The presence of nominal rigidities implies that changes in aggregate demand are more likely to be reflected in labor demand than when prices are fully flexible. Third, in the tradition of Diamond-Mortensen-Pissarides we assume that the labor market is characterized by matching frictions which prevent unemployed workers and firms with vacancies from being instantaneously and costlessly paired. We extend the standard matching model by allowing for two unemployment states which differ in the search efficiency. In particular, we assume that unemployed workers may either experience very short term unemployment (switch-unemployment), or longer term unemployment³. The first state induces little risk while the second state produces worse labor market prospects.⁴

We investigate the impact of a temporary increase in involuntary job separations.⁵ An important aspect of this labor market deterioration is that currently employed workers face idiosyncratic risk about which matches will be resolved. Employed workers react to this risk by increasing their desired precautionary savings and unemployed workers, realizing the future job finding opportunities have worsened, may likewise increase their savings rate. This mechanism is stronger the larger share of the job separations that are associated with longer term unemployment because these workers face longer unemployment spells and may receive lower benefits. In response to lower demand for their products, firms have an incentive to cut prices which partially offsets the fall in demand but nominal rigidities reduce this incentive. At the same time, when fewer of the unemployed match very efficiently with vacancies, the incentive of firms to post more vacancies is curbed. Fewer vacancies, in turn, decreases the job finding rate and therefore the expected income loss associated with unemployment. Hence, a vicious

³Farber (2012) also highlights the role of the increase of unemployment duration during the Great Recession.

⁴Ravn (2008) and Hornstein (2011) model heterogeneity in search efficiency in a similar fashion.

⁵Elsby, Hobijn and Şahin (2010) document that the inflow rate into unemployment did increase quite substantially during the early parts of the Great Recession. They also show that most of this is due to a spike in the layoff rate. Hall (2010) also stresses the role of job losses in the early parts of the Great Recession.

circle may appear in which deteriorating job prospects trigger lower demand which worsens the labor market outlook even further. Under these circumstances, job uncertainty can produce a deep recession.

To investigate the importance of the amplification mechanism during the Great Recession, we feed into a calibrated version of the model processes for job separations and for the share of job separations that send workers to the two unemployment state which match the US time series for permanent layoffs and the share of unemployed workers who have been out of work for 6 months or more. The resulting calibration of the size of the shock to the share of less search efficient workers is consistent with the estimates of Barlevy (2011) and Şahin et al (2012) regarding the importance of mis-match during the Great Recession.⁶ The model is shown to produce a large recession in response to these shocks and we find an increase in the unemployment rate and a decrease in vacancies that closely resemble their empirical counterparts. This also means that the model produces the movement along and shift in the Beveridge curve observed in the US during the Great Recession.

We conduct an extensive robustness analysis of our results using a simplified model in which we impose a no-borrowing constraint on the households. Although this model features no wealth inequality in equilibrium, we find that it produces results that are quite similar to the benchmark model. The amplification mechanism is neutralized when either prices are flexible or asset markets are complete. In the latter case, an increase in job separations have limited impact on aggregate demand which reduces the adverse impact of the worsening labor market conditions. When prices are flexible, firms have a strong incentive to cut prices when aggregate demand declines which moderates very significantly the recessionary impact of the increase in

⁶Barlevy (2011) estimates the extent mis-match from the shift in the Beveridge curve post-August 2008. Daly et al (2012) instead use the instability of the Beveridge curve during the Great Recession to derive an estimate of the change in the natural rate of unemployment. Şahin et al (2012) posit a model composed of many distinct labor markets and use this to measure mis-match unemployment. In an application to US data, they use dispersion in unemployment across industries, occupations, education and geography to estimate the extent of mis-match.

job separations. We show that it is the interaction between precautionary savings motives and nominal rigidities that produces the amplification mechanism. We also find that it is crucial to allow for a change in the composition of unemployment and in particular for a drying up of very short term unemployment. When unemployment duration is short, an increase in job separations has limited impact on aggregate demand and firms have a strong incentive to post vacancies when unemployment rises which reduces the adverse impact of an adverse shock to job separations.

We also investigate the importance of wage setting. The benchmark model assumes that the real wage is fixed, an assumption that is not obviously contradicted by the data in the last recession at least, see also Shimer (2012). We replace this assumption with a Nash bargaining model and find that the results are sensitive to the assumptions made regarding workers' outside option should the partners disagree during the bargaining process. If a failed bargaining processes can send workers into both unemployment states, nominal wages fall and the amplification mechanism does not arise. Assuming alternatively that a failed bargaining process sends workers into the high search efficiency state only, the results under Nash bargaining are extremely similar to the benchmark model that assumes constant real wages.

The paper is closely related to the seminal contribution of Den Haan, Ramey and Watson (2000). In their analysis, matches are endogenously terminated and they examine how productivity shocks are propagated due to matching frictions and household savings decisions. Our analysis is also related to a number of other recent papers that have examined aggregate fluctuations in incomplete markets settings. Gomes, Greenwood and Rebelo (2002) and Krusell et al (2009) both study models that introduce frictional labor markets in a general equilibrium incomplete markets model with idiosyncratic risk. The former authors compute the welfare costs of business cycles while the latter authors examine how labor market uncertainty influences inequality and how imperfect insurance affects unemployment and other labor market outcomes. Krusell, Mukoyama and Şahin (2011) and Challe and Ragot (2012) instead investigate the role of precautionary savings for aggregate fluctuations in an incomplete markets setting

with unemployment risk.⁷ The latter of these authors show that precautionary savings may be important for understanding business cycle fluctuations. Caggese and Perez (2011) combine the heterogeneous agents model with a heterogeneous firm model where firm heterogeneity arises due to financial frictions and idiosyncratic “profit shocks”. Consistently with the empirical results of Mian and Sufi (2011), they find that financial shocks can have important impact on unemployment and aggregate activity. Guerrieri and Lorenzoni (2011) and McKay and Reis (2012) instead introduce nominal rigidities into an incomplete markets setting but do not model frictions in the labor market. Our analysis combines nominal rigidities, heterogeneous agents, and frictional labor markets and illustrates how the combination of these frictions produces new insights regarding the aggregate and distributional effects of shocks that originate in the labor market.

Also related to our analysis is a stream of recent papers that examine the impact of uncertainty shocks. In our model, asset market incompleteness implies that shocks to job termination rates produce changes in idiosyncratic uncertainty and we find that this is important for understanding the impact of labor market shocks on aggregate demand. Baker, Bloom and Davis (2012) suggest that policy uncertainty may have contributed significantly to the Great Recession. Closer to our analysis, Schalle (2012) investigates the impact of idiosyncratic productivity volatility shocks on unemployment in a directed search model with heterogeneous firms. He finds that the uncertainty effects induced by the volatility shocks can explain some of the rise in unemployment but not its persistence. Basu and Bundick (2012) analyze the effects of aggregate uncertainty shocks dynamic stochastic general equilibrium model with a representative consumer. Consistently with our analysis of the impact of idiosyncratic uncertainty, they find that nominal rigidities amplify the impact of uncertainty shocks. Basu and Bundick (2012) also demonstrate that a zero lower bound on the nominal interest rate may further amplify the impact of uncertainty shocks. Related to this, Rendahl (2012) shows how news shocks can

⁷The latter of these papers do not explicitly model labor market matching. None of these papers allow for nominal rigidities.

generate severe recessions in a zero lower bound environment in a model with frictional labor markets. We will abstract from issues related to a lower bound on the nominal interest rate.

The remainder of the paper is organized as follows. Section 2 sets out the model economy. In Section 3 we study a version of the model in which there is no wealth inequality in equilibrium. Section 4 presents the results from the main model. Section 5 concludes and summarizes.

2 The Model Economy

The economy is inhabited by households, firms which are owned by entrepreneurs, and by a government which is in charge of monetary and fiscal policy. We introduce a number of frictions in goods, asset and labor market by combining three workhorse models in macroeconomics. First, the labor market is characterized by Diamond-Mortensen-Pissarides matching frictions which we extend by introducing differential matching prospects across unemployed workers. Secondly, as in Bewley (1977) and Aiyagari (1994), asset markets are incomplete. Specifically, following Krusell and Smith (1997), the model features idiosyncratic and aggregate risk in an incomplete markets set-up. Third, we assume that firms are monopolistically competitive and set prices in an environment with nominal rigidities in price setting. We adopt the set-up of Rotemberg (1982) where firms face quadratic costs of adjusting nominal prices.

Households. There is a continuum of mass 1 of infinitely lived households indexed by $i \in (0, 1)$. Households have rational expectations and maximize the expected present value of their utility streams. A household is either working (matched with a firm) or unemployed and looking for a job. Unemployed workers differ in the efficiency of the matching technology that they face. This feature produces heterogeneity across unemployed workers in the expected duration of unemployment spells. Asset markets are assumed to be incomplete which produces an incentive for precautionary savings to insure against unemployment and other shocks. Differences in labor market histories and asset market incompleteness in combination produce wealth inequality across households.

Households consume a basket of consumption goods varieties:

$$\mathbf{c}_{i,t} = \left(\int_j (\mathbf{c}_{i,t}^j)^{1-1/\gamma} dj \right)^{1/(1-1/\gamma)} \quad (1)$$

where $\mathbf{c}_{i,t}^j$ denotes household i 's consumption of goods of variety j and $\gamma > 1$ is the elasticity of substitution between consumption goods. Variety j is purchased at the nominal price $\mathbf{P}_{j,t}$. It follows that household i 's demand for variety j is given as:

$$\mathbf{c}_{i,t}^j = \left(\frac{\mathbf{P}_{j,t}}{\mathbf{P}_t} \right)^{-\gamma} \mathbf{c}_{i,t} \quad (2)$$

where \mathbf{P}_t is the price index associated with the consumption basket defined in (1):

$$\mathbf{P}_t = \left(\int_j \mathbf{P}_{j,t}^{1-\gamma} dj \right)^{1/(1-\gamma)} \quad (3)$$

A household that is employed in period t works full-time hours (normalized to one unit), receives a real wage \mathbf{w}_t , pays a lump-sum social security tax \mathbf{T}_t^h , and experiences a job termination with probability $\rho_{x,t} \leq 1$. A fraction $\rho_{r,t}$ of the workers that experience a job termination make a transition to unemployment state r , $r = s, l$, $\rho_{s,t} + \rho_{l,t} = 1$.

An unemployed household in state r receives (gross of taxes) unemployment benefits $\xi_r \geq 0$ and finds a new job with probability $\eta_{r,t}$. We assume that $\eta_{l,t} < \eta_{s,t}$ and will therefore, slightly imprecisely, refer to the two groups of unemployed workers as short- and long-term unemployed.⁸ We will assume that $\xi_s \geq \xi_l$ so that the risk of a longer unemployment spell is not offset by higher benefits. One interpretation of the two types of unemployed workers is that type l unemployed workers are mis-matched and face a more demanding job search process. An alternative interpretation is that type s unemployed are “switch-unemployed” who face very short-term unemployment. The model as such is silent on which of these two interpretations is the right one but our calibration face the latter.

Job terminations occur at the end of the period while new job matches are formed at the beginning of the period. Households are informed about the job loss probabilities at the

⁸In expectations, a type s unemployed worker will face shorter unemployment duration of an unemployment spell than a type l individual. Ex-post, however, some “short-term” unemployed workers may face long unemployment spells and some “long-term” unemployed workers will find jobs fast.

beginning of the period and therefore have within-period perfect foresight about the share of currently employed workers who will lose their jobs and about the share of these that become short- and long-term unemployed. However, households face idiosyncratic uncertainty about both the identity of the workers who lose their jobs, about the unemployment state should they lose their job, and about future job separation rates.

Households have access to a state non-contingent bond which carries a (gross) nominal interest rate of \mathbf{R}_t . We impose a borrowing constraint by assuming that there is a utility cost of holding debt.⁹ Let $\mathbf{b}_{i,t}^h$ denote household i 's holding of bonds at the end of period t . We assume that there is a utility cost function, φ , and a debt limit $\bar{\mathbf{b}} \leq 0$ so that

$$\varphi(\mathbf{b}_{i,t}^h) = \begin{cases} 0 & \text{for } \mathbf{b}_{i,t}^h \geq \bar{\mathbf{b}} \\ \infty & \text{for } \mathbf{b}_{i,t}^h < \bar{\mathbf{b}} \end{cases} \quad (4)$$

This implies that no household chooses $\mathbf{b}_{i,t}^h < \bar{\mathbf{b}}$.

Households face a sequence of budget constraints:

$$\mathbf{c}_{i,t} + \mathbf{b}_{i,t}^h = \mathbf{n}_{i,t} \mathbf{w}_t - \mathbf{T}_t^h + (1 - \mathbf{n}_{i,t}) \sum_{r=s,l} \mathbf{I}_{r,t} \boldsymbol{\xi}_r + \frac{\mathbf{R}_{t-1}}{1 + \boldsymbol{\pi}_t} \mathbf{b}_{i,t-1}^h, \quad t \geq 0 \quad (5)$$

where $\mathbf{b}_{i,-1}^h > \bar{\mathbf{b}}$ is given and $\boldsymbol{\pi}_t$ denotes the net inflation rate in period t , $(1 + \boldsymbol{\pi}_t) = \mathbf{P}_t / \mathbf{P}_{t-1}$. $\mathbf{n}_{i,t}$ indicates the employment status of household i at date t while $\mathbf{I}_{r,t}$ indicates the unemployment state status:

$$\mathbf{n}_{i,t} = \begin{cases} 0 & \text{if individual } i \text{ is unemployed in period } t \\ 1 & \text{if individual } i \text{ is employed in period } t \end{cases}$$

$$\mathbf{I}_{i,t} = \begin{cases} 1 & \text{if individual } i \text{ is unemployed in state } r \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

Let $\mathbf{V}_i^e(\mathbf{b}_{i,t-1}^h, \mathbf{S}_t)$ be the expected present discounted utility of an employed household given

⁹This assumption is computationally appealing.

the households holdings of bonds and an aggregate state vector, \mathbf{S}_t :

$$\begin{aligned} \mathbf{V}_i^e(\mathbf{b}_{i,t-1}^h, \mathbf{S}_t) &= \max_{\mathbf{c}_{i,t}, \mathbf{b}_{i,t}} \{u(\mathbf{c}_{i,t}) - \varphi(\mathbf{b}_{i,t}^h) \\ &\quad + \beta \mathbb{E}_t \left(1 - \sum_{r=s,l} \rho_{x,t} \rho_{r,t} (1 - \eta_{r,t+1}) \right) \mathbf{V}_i^e(\mathbf{b}_{i,t}^h, \mathbf{S}_{t+1}) \\ &\quad + \beta \sum_{r=s,l} \rho_{x,t} \rho_{r,t} \mathbb{E}_t (1 - \eta_{r,t+1}) \mathbf{V}_i^{u,r}(\mathbf{b}_{i,t}^h, \mathbf{S}_{t+1}) \} \end{aligned} \quad (6)$$

Households maximize subject to the budget constraint in equation (5) setting $\mathbf{n}_{i,t} = 1$. $u(\mathbf{c}_{i,t})$ is a concave utility function. $\mathbf{V}_i^{u,r}(\mathbf{b}_{i,t-1}, \mathbf{S}_t)$ is the value for an unemployed household who is faced with state $r = s, l$ unemployment. $\beta \in (0, 1)$ is the subjective discount factor, and \mathbb{E}_t is the conditional expectations operator. $\rho_{x,t} \rho_{r,t}$ is the probability that a worker who is employed at the beginning of the period makes a transition to unemployment state r and $\sum_{r=s,l} \rho_{x,t} \rho_{r,t} (1 - \eta_{r,t+1})$ is the probability of not being employed at the beginning of period $t + 1$.

A type r unemployed worker faces the problem:

$$\begin{aligned} \mathbf{V}_i^{u,r}(\mathbf{b}_{i,t-1}^h, \mathbf{S}_t) &= \max_{\mathbf{c}_{i,t}, \mathbf{b}_{i,t}} \{u(\mathbf{c}_{i,t}) - \varphi(\mathbf{b}_{i,t}^h) + \kappa + \beta \mathbb{E} \eta_{r,t+1} \mathbf{V}_i^e(\mathbf{b}_{i,t}, \mathbf{S}_{t+1}) \\ &\quad + \beta \mathbb{E}_t (1 - \eta_{r,t+1}) \mathbf{V}_i^{u,r}(\mathbf{b}_{i,t}, \mathbf{S}_{t+1}) \}, h = s, l \end{aligned} \quad (7)$$

subject to the budget constraint in equation (5) setting $\mathbf{n}_{i,t} = 0$ and $\mathbf{I}_{r,t} = 1$. $\kappa > 0$ denotes the utility of leisure enjoyed by an unemployed household (having normalized the utility of leisure of employed households to zero). As a matter of consistency, we will make the assumption that $\mathbf{V}_i^e(\mathbf{b}^h, \mathbf{S}) > \mathbf{V}_i^{u,s}(\mathbf{b}^h, \mathbf{S})$ for all \mathbf{b}^h and \mathbf{S} so that no employed household has an incentive to voluntarily leave their current job. Under the condition that $\eta_{s,t+1} > \eta_{l,t+1}$ the condition that $\mathbf{V}_i^e(\mathbf{b}^h, \mathbf{S}) > \mathbf{V}_i^{u,s}(\mathbf{b}^h, \mathbf{S})$ for all \mathbf{b}^h and \mathbf{S} automatically implies that $\mathbf{V}_i^e(\mathbf{b}^h, \mathbf{S}) > \mathbf{V}_i^{u,l}(\mathbf{b}^h, \mathbf{S})$ for all \mathbf{b} and \mathbf{S} .¹⁰

¹⁰The formulation of the unemployed workers' problem in equation (7) assumes that there are no flows between the two unemployment states during unemployment. However, all workers face identical job prospects upon employment. The first of these assumptions is easily relaxed and immaterial for the results as long as the flow out of unemployment is sufficiently small for type l workers relative to type s workers.

Entrepreneurs. Consumption goods are produced by a continuum of monopolistically competitive firms indexed by $j \in (0, 1)$ which are owned by risk neutral entrepreneurs. We let $\Psi < 1$ denote the measure of entrepreneurs. Entrepreneurs discount utility at the rate β and make decisions on the pricing of their goods, on vacancy postings, and on their consumption and savings policies. In return for managing (and owning) the firm, they are the sole claimants to its profits (but will also have to stand ready to cover losses). We assume that entrepreneurs can save but face a no-borrowing constraint. This no-borrowing constraint implies that the entrepreneur finances hiring costs through retained earnings.¹¹

Output is produced by according to a linear technology:

$$\mathbf{y}_{j,t} = \mathbf{n}_{j,t} \tag{8}$$

where $\mathbf{n}_{j,t}$ denotes entrepreneur j 's input of labor which is purchased from the households. Firms are assumed to be sufficiently large (i.e. the mass of entrepreneurs is small relative to the mass of households) that there are no indivisibility problems associated with the full-time hours assumption made earlier.

Following Rotemberg (1982) we assume that there are nominal rigidities in the form of quadratic costs of price adjustment. Given risk neutrality, entrepreneurs set prices to maximize the present discounted value of profits:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\left(\frac{\mathbf{P}_{j,t+s}}{\mathbf{P}_{t+s}} - \mathbf{mc}_{j,t+s} \right) \mathbf{y}_{j,t+s} - \frac{\phi}{2} \left(\frac{\mathbf{P}_{j,t+s} - \mathbf{P}_{j,t+s-1}}{\mathbf{P}_{j,t+s-1}} \right)^2 \mathbf{y}_{t+s} \right) \tag{9}$$

subject to:

$$\mathbf{y}_{jt} = \left(\frac{\mathbf{P}_{jt}}{\mathbf{P}_t} \right)^{-\gamma} \mathbf{y}_t \tag{10}$$

Equation (10) is the demand for goods variety j . \mathbf{y}_t , which is defined formally below, can be interpreted as aggregate real income. In equation (9) $\phi \geq 0$ indicates the size of costs of changing prices with $\phi = 0$ corresponding to flexible prices. $\mathbf{mc}_{j,t}$ denotes real marginal costs.

¹¹In the stationary equilibrium, $\beta < 1/(R/((1 + \pi)))$ so entrepreneurs will be borrowing constrained.

The first-order condition for this problem is given as:

$$\begin{aligned} \left((1 - \gamma) + \gamma \mathbf{mc}_{j,t} \frac{\mathbf{P}_t}{\mathbf{P}_{j,t}} \right) \mathbf{y}_{j,t} &= \phi \frac{\mathbf{P}_t}{\mathbf{P}_{j,t-1}} \left(\frac{\mathbf{P}_{j,t} - \mathbf{P}_{j,t-1}}{\mathbf{P}_{j,t-1}} \right) \mathbf{y}_t \\ &+ \phi \beta \mathbb{E}_t \left[\left(\frac{\mathbf{P}_{j,t+1}}{\mathbf{P}_{j,t}^2} \right) \left(\frac{\mathbf{P}_{j,t+1} - \mathbf{P}_{j,t}}{\mathbf{P}_{j,t}} \right) \mathbf{P}_t \mathbf{y}_{t+1} \right] \end{aligned} \quad (11)$$

In a symmetric equilibrium, which will be the focus of our analysis, this simplifies to:

$$(1 - \gamma) + \gamma \mathbf{mc}_t = \phi \pi_t (1 + \pi_t) - \phi \beta \mathbb{E}_t \left[\pi_{t+1} (1 + \pi_{t+1}) \frac{\mathbf{y}_{t+1}}{\mathbf{y}_t} \right] \quad (12)$$

Firms hire labor in a frictional labor market. The law of motion for employment in firm j is given as:

$$\mathbf{n}_{j,t} = (1 - \rho_{x,t}) \mathbf{n}_{j,t-1} + \mathbf{h}_{j,t}$$

where $\mathbf{h}_{j,t}$ denotes hires made by firm j in period t . The number of hires in turn is given as:

$$\mathbf{h}_{j,t} = \rho_{f,t} \mathbf{v}_{j,t} \quad (13)$$

where $\mathbf{v}_{j,t}$ is the number of vacancies posted by the firm and $\rho_{f,t}$ is the job filling probability. We assume that firms are sufficiently large that $\rho_{f,t}$ can be interpreted as the fraction of vacancies that lead to a match.¹² The cost of posting a vacancy is given by $\mu > 0$. Therefore, real marginal costs are given as:

$$\mathbf{mc}_{j,t} = \frac{\mathbf{w}_t}{\mathbf{A}} + \frac{\mu}{\rho_{f,t}} - \beta \mathbb{E}_t \left[(1 - \rho_{x,t+1}) \frac{\mu}{\rho_{f,t+1}} \right] \quad (14)$$

which incorporates the fact that hiring in period t impacts on future marginal costs through future hiring cost savings.

Finally, the budget constraint of entrepreneurs can be expressed as:

$$\mathbf{d}_{j,t} + \mathbf{b}_{j,t}^e + \mathbf{w}_t \mathbf{n}_{j,t} + \mu \frac{\mathbf{h}_{j,t}}{\rho_{f,t}} = \frac{\mathbf{P}_{j,t}}{\mathbf{P}_t} \mathbf{A} \mathbf{n}_{j,t} - \mathbf{T}_t^e + \frac{\mathbf{R}_{t-1}}{1 + \pi_t} \mathbf{b}_{j,t-1}^e \quad (15)$$

$$\mathbf{b}_{j,t}^e \geq 0 \quad (16)$$

where $\mathbf{b}_{j,-1}^e \geq 0$ is given, $\mathbf{d}_{j,t}$ denotes entrepreneur j 's consumption in period t and $\mathbf{b}_{j,t}^e$ their bond purchases in period t . Condition (16) imposes the no-borrowing constraint on entrepreneurs. \mathbf{T}_t^e are employer contributions to social security.

¹²This assumption can be relaxed which would produce ex-post heterogeneity across firms.

Labor Market. The matching technology is given as:

$$\mathbf{m}_t = \psi (\mathbf{u}_{s,t} + \mathbf{q}\mathbf{u}_{l,t})^\alpha (\mathbf{v}_t)^{1-\alpha} \quad (17)$$

where \mathbf{m}_t denotes the measure of matches between firms (vacancies) and unemployed workers at date t , $\mathbf{u}_{r,t}$ is the measure of type r unemployed workers at date t and \mathbf{v}_t is the measure of vacancies posted by the firms. $\psi > 0$, and $\alpha \in (0, 1)$ are constant parameters. The parameter $\mathbf{q} \in (0, 1]$ is the probability that a type l unemployed worker is searching for a job at date t . When $\mathbf{q} < 1$, type l workers are less likely to find a job than type s unemployed workers and face longer expected unemployment duration.

Given the matching technology, the job filling probability and the job finding probabilities are given as:

$$\rho_{f,t} = \psi (1 - (1 - \mathbf{q}) \mathbf{u}_{l,t}/\mathbf{u}_t)^\alpha \boldsymbol{\theta}_t^{-\alpha} \quad (18)$$

$$\boldsymbol{\eta}_{s,t} = \psi (1 - (1 - q) \mathbf{u}_{l,t}/\mathbf{u}_t)^{\alpha-1} \boldsymbol{\theta}_t^{1-\alpha} \quad (19)$$

$$\boldsymbol{\eta}_{l,t} = \mathbf{q}\boldsymbol{\eta}_{s,t} \quad (20)$$

where $\boldsymbol{\theta}_t = \mathbf{v}_t/\mathbf{u}_t$ denotes labor market tightness, \mathbf{u}_t is the measure of unemployed workers. Importantly, as long as $\mathbf{q} < 1$, the job filling rate depends negatively on the share of the longer term unemployed because these workers match less efficiently with vacancies than short term unemployed workers.¹³

The laws of motion of the stocks of employed and unemployed workers are given as:

$$\mathbf{n}_t = (1 - \boldsymbol{\rho}_{x,t}) \mathbf{n}_{t-1} + \mathbf{m}_t \quad (21)$$

$$\mathbf{u}_{r,t} = (1 - \boldsymbol{\eta}_{r,t}) (\mathbf{u}_{r,t-1} + \boldsymbol{\rho}_{r,t}\boldsymbol{\rho}_{x,t}\mathbf{n}_{t-1}) \quad (22)$$

Our candidates for stochastic shocks to the economy are exogenous changes in the job separation rate, $\boldsymbol{\rho}_{x,t}$, and in $\boldsymbol{\rho}_{s,t}$, which determines the share of workers affected job terminations

¹³When $q = 1$, $1 - (1 - q) u_{l,t}/u_t = 1$ so this term drops out.

that become short-term unemployed. We assume that:

$$\rho_{x,t} = \overline{\rho}_x \exp(\mathbf{z}_{x,t}) \quad (23)$$

$$\rho_{s,t} = \overline{\rho}_s \exp(\mathbf{z}_{s,t}) \quad (24)$$

$$\mathbf{z}_{x,t} = \lambda_x \mathbf{z}_{x,t-1} + \boldsymbol{\varepsilon}_{x,t} \quad (25)$$

$$\mathbf{z}_{s,t} = \lambda_s \mathbf{z}_{s,t-1} + \boldsymbol{\varepsilon}_{s,t} \quad (26)$$

where $\overline{\rho}_x, \overline{\rho}_s \in (0, 1)$ are the long-run levels of job termination and the share of short-term unemployed, respectively, while $\lambda_x, \lambda_s \in (-1, 1)$ denote the persistence of shocks to the job termination rate and to the share of short-term unemployed. It is assumed that $\boldsymbol{\varepsilon}_t \sim N(0, \mathbf{V}_\varepsilon)$ where $\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}_{x,t}, \boldsymbol{\varepsilon}_{s,t})'$.

We experiment with alternative assumptions regarding wage setting. All schemes that we consider are required to be consistent with a non-negative surplus of any worker-firm matches so that none of the partners have an incentive to terminate an existing match voluntarily. In our benchmark model, we assume that real wages are constant, $\mathbf{w}_t = \overline{\mathbf{w}}$.¹⁴ We examine whether the results are robust to assuming that wages are instead determined according to a Nash bargaining model. In the face of wealth heterogeneity, Nash bargaining introduces worker-specific wages which complicates the analysis very significantly.

Government. The government is in charge of monetary and fiscal policies. We assume that the government balances the budget period by period which means that:

$$\mathbf{u}_{s,t} \boldsymbol{\xi}_s + \mathbf{u}_{l,t} \boldsymbol{\xi}_l = \mathbf{T}_t^h + \Psi \mathbf{T}_t^e \quad (27)$$

which reflects the lump-sum nature of the social security taxes.

Monetary policy is specified by a rule for the short-term nominal interest rate. We assume that:

$$\mathbf{R}_t = \overline{\mathbf{R}} \left(\frac{1 + \pi_t}{1 + \overline{\pi}} \right)^\delta \quad (28)$$

¹⁴We have checked that the match surplus is positive for all matches in all the results that we report.

where $\bar{\mathbf{R}}$ is the long-run nominal interest rate target, $\bar{\pi}$ is the inflation target, and δ denotes the elasticity of the nominal interest rate to deviations of inflation from its target.

Equilibrium. We focus upon a recursive equilibrium in which households act competitively taking all prices for given while firms act as monopolistic competitors setting the price of their own variety taking all other prices for given. In equilibrium, firms are symmetric because there are no idiosyncratic productivity shocks, prices are set in a state-contingent manner, and because they are assumed to be sufficiently large that they all hire the same number of workers.¹⁵ We let $\mathbf{p}_{j,t} = \mathbf{P}_{j,t}/\mathbf{P}_t$ denote the relative price of firm j 's product. Symmetry implies that this relative price equals 1 in equilibrium.

Households are instead heterogeneous and differ in their labor market status and in their wealth. In equilibrium, aggregate savings equal zero but, since wealth matters for savings choices, the wealth distribution is an aggregate state variable which impacts both on household and on entrepreneurial choices. We let $\Omega_t = \Omega(\mathbf{b}_{t-1}, \mathbf{e}_t)$ denote the distribution of agents over asset levels and employment states, $\mathbf{b}_t = (\mathbf{b}_t^h, \mathbf{b}_t^e)$, $\mathbf{b}_t^h = (\mathbf{b}_{i,t}^h)_{i=0}^1$, $\mathbf{b}_t^e = \sum_j \mathbf{b}_{j,t}^e$ and $\mathbf{e}_t = (\mathbf{e}_{it})_{i=0}^1$ where e_{it} indicates the labor market status of household i (whether employed or unemployed in state $r = s, l$).¹⁶ We let $\mathbf{d}\Omega_t$ denote the associated density of the joint distribution of assets and labor market status. The relevant state vector is then defined as $\mathbf{S}_t = (\Omega_t, \boldsymbol{\rho}_{x,t}, \boldsymbol{\rho}_{s,t})$.

Definition 1 *A recursive monopolistic competition equilibrium is defined as a distribution of wealth $\Omega(\mathbf{b}, \mathbf{e})$, pricing kernels $(\mathbf{w}(\mathbf{S}), \boldsymbol{\pi}(\mathbf{S}))$, decision rules $(\mathbf{c}_i^h(\mathbf{b}_i^h, \mathbf{S}_t), \mathbf{b}_i^{h'})_{i=0}^1$ and $(\mathbf{p}_j(\mathbf{b}^e, \mathbf{S})_{j=0}^J, \mathbf{n}(\mathbf{b}^e, \mathbf{S}), \mathbf{h}(\mathbf{b}^e, \mathbf{S}))$, value functions $(\mathbf{V}_i^e(\mathbf{b}_i^h, \mathbf{S}), \mathbf{V}_i^{u,s}(\mathbf{b}_i^h, \mathbf{S}), \mathbf{V}_i^{u,l}(\mathbf{b}_i^h, \mathbf{S}))_{i=1}^1$ and $\mathbf{W}(\mathbf{b}^e)$, and government policies $(\mathbf{T}(\mathbf{S}), \mathbf{R}(\mathbf{S}))$ such that*

(i) given the pricing kernel, the government policies, and the aggregate and individual states, the household decision rules solve the households problem;

¹⁵Firms are also assumed to be sufficiently big that there the full time hours assumption does not give rise to any indivisibility problems.

¹⁶The definition of \mathbf{b}_t^e reflects the fact that entrepreneurs are symmetric in equilibrium.

(ii) given the pricing kernel, government policies, and the aggregate state, the entrepreneur decision rules solve the entrepreneurs' problem and $\mathbf{p}_j(\mathbf{b}^e, \mathbf{S})_{j=0}^J = 1$ for all j and all $(\mathbf{b}^e, \mathbf{S})$;

(iii) asset and goods market clear:

$$\begin{aligned} \int \mathbf{b}_i^{h'}(\mathbf{b}_i^h, \mathbf{S}) d\Omega + \Psi \mathbf{b}^e &= 0 \\ \int \mathbf{c}_i^h(\mathbf{b}_i^h, \mathbf{S}) d\Omega + \Psi \mathbf{c}^e &= \mathbf{y} \end{aligned}$$

(iv) the government budget constraint is satisfied and the nominal interest is given by the policy rule in equation (28);

Solution Method. Solving our main model is computationally challenging because of the presence of wealth heterogeneity and aggregate uncertainty. As discussed by Krusell and Smith (1998), the essence of the problem is the presence of a large-dimensional aggregate state that includes the entire wealth distribution. We follow their computational strategy, which involves an update of the individuals' beliefs on the laws of motion of aggregate variables in each step of the algorithm. Thus, the individuals' optimization problem needs to be solved many times. Krusell and Smith (1998) do so by using a global solution algorithm. In our case, this would be prohibitively slow given that - even aside from the wealth distribution- our model has a relatively large number of aggregate state variables that are all continuous.¹⁷

To speed up the solution of the individuals' problem, we develop a perturbation-based method that is hardly slowed down by the presence many state variables. Unlike standard first-order perturbation approaches, however, our method fully preserves the nonlinearity of the individuals' policy rules in individual wealth, which crucially affects precautionary savings behavior. This is achieved by perturbing around the savings path followed by an agent that is hit by a sequence of idiosyncratic shocks that keep her in the same employment state for many

¹⁷Krusell and Smith (1998) reduce the computational burden by assuming that there is only a "good" and a "bad" aggregate state. Moreover, the presence of matching frictions and nominal rigidities increases the number of aggregate state variables relative to their model.

periods.¹⁸ The algorithm allows for straightforward implementation using standard perturbation software and is described in detail in the appendix. The method builds on the algorithm of Reiter (2009), who discretizes an incomplete markets model and applies a first-order perturbation model around a steady state that includes a discretized distribution of wealth.¹⁹ Our method, by contrast, does not require a discretization of the agents' policy rules.²⁰

3 Quantitative Results: No Wealth Inequality

Before turning to the general version of the model, we find it instructive to examine a special case where we impose that $\bar{\mathbf{b}} = 0$. The no-borrowing constraint implies that all households hold zero wealth in equilibrium so that the wealth distribution no longer is an aggregate state variable. We can therefore compute the equilibrium in a simple manner using a standard perturbation method. The relative simplicity of the numerical procedure also means that this version of the model lends itself to a rigorous robustness analysis. Although this model features no equilibrium wealth inequality, its key mechanisms are much the same as in the general version that allows for borrowing apart from the fact that there are no differential impact on agents according to their wealth.

3.1 Calibration

The calibration targets and parameter values are summarized in Tables 1 and 2. One model period corresponds to a calendar month. The household utility function is assumed to be given as:

$$u(\mathbf{c}_{it}) = \frac{\mathbf{c}_{it}^{1-\sigma} - 1}{1-\sigma}, \quad \sigma \geq 0$$

¹⁸The end points of the path are pinned down by steady-state conditions, resembling the extended path algorithm of Gagnon and Taylor (1990).

¹⁹An earlier reference is Campbell (1998), who applies a similar method to a model with heterogeneous firms.

²⁰Another difference is that the algorithm of Reiter (2009) does not require a simulation step.

and we set $\sigma = 1.5$. This value is in the mid-range of empirical estimates of Attanasio and Weber (1995), Eichenbaum, Hansen, and Singleton (1988), and many others who have examined either household data or aggregate time series. σ determines the degree of risk aversion which matters for the household savings response to uncertainty and this is important in our model.

We assume an annual real interest rate of 3 percent and set the subjective discount factor equal to 0.993 for both households and entrepreneurs. This value is low relative to standard representative agent models but because of idiosyncratic risk and incomplete markets, agents have a strong incentive to engage in precautionary savings and a low real interest rate is required to induce zero savings in equilibrium.

We target an unemployment rate of 5 percent and a 15 percent share of long term unemployed (unemployed workers who have been out of work for 6 months or more) in the stationary equilibrium. These targets are close to the mean statistics for the United States in the post-1970 period. Following Rothstein (2011), we target a monthly hazard rate from unemployment to employment for a newly unemployed workers of 40 percent and a 30 percent monthly hazard rate for workers who have been unemployed for 26 weeks or more. These targets imply a steady-state job loss probability, $\overline{\rho}_x$, of 2.95 percent per month, that $\overline{\rho}_s$, the share of workers who experience a job loss that enter the pool of high search efficiency unemployment, equals 35 percent, and that the relative search efficiency of the longer term unemployed, \mathbf{q} , is 50.2 percent. Given the calibration, we favor the interpretation of the multiple unemployment states that associated states s with switch-unemployment, i.e. unemployment that has very short duration.

We assume that the matching function elasticity to unemployment is equal to 50 percent ($\alpha = 0.5$), and normalize $\psi = 1$. $\boldsymbol{\mu}$, the vacancy cost parameter, is calibrated by targeting an average hiring cost of 4.5 percent of the quarterly wage bill. Given other parameters, this implies that $\boldsymbol{\mu} = 0.18$.

We calibrate the benefit levels, $\boldsymbol{\xi}_s$ and $\boldsymbol{\xi}_l$, by targeting estimates of the consumption loss during unemployment reported by Browning and Crossley (2001). Studying Canadian data,

these authors find that consumption drops on average 14 percent upon a permanent job loss. They report that there is a lot of dispersion across workers with 25 percent suffering no loss while the worst hit 10 percent experience a consumption loss of 50 percent. In the the model, the borrowing limit implies that all households hold zero savings in equilibrium and therefore that consumption equals flow (after tax) income. Thus, in the stationary equilibrium, 35 percent (65 percent) of the population would suffer a consumption loss that would correspond to the difference between the real wage and ξ_s (ξ_l). In order to moderate slightly the impact of the no-borrowing constraint, we assume that $\xi_s = 0.925w$ and that $\xi_l = 0.84w$ (where w is the real wage) which imply an average consumption loss upon unemployment of just below 14 percent as estimated by Browning and Crossley (2001) but implies less variance across the unemployed agents.

We set the average mark-up equal to 20 percent which implies that γ , the elasticity of substitution between goods, is equal to 6. ϕ , the parameter that determines the importance of price adjustment costs, is calibrated to match price adjustment frequency of 5 months. This value is conservative but close to the value estimated by Bils and Klenow (2004).²¹ This implies that $\phi = 142.86$. We assume that the government's inflation target $\bar{\pi} = 1$ so that it pursues price stability and we set $\delta = 1.5$, a conventional value in the new Keynesian literature.

Finally, we estimate the parameters of the stochastic processes that determine the persistence and volatility of the job separation rate and of the share of high search efficiency unemployed workers from time-series data for layoffs and the share of workers who have been unemployed for 6 months or more. We find estimates of $\lambda_x = 0.9774$ and $\lambda_s = 0.7926$. this implies a half-life of job separation shocks of 30 months while shocks to the share of short term unemployed die out very fast.

²¹To be precise, we calibrate ϕ by exploiting the equivalence between the log-linearized Phillips curve implied by our model and the Phillips curve implied by the Calvo model.

3.2 Results

The Impact of Shocks. Figures 2 and 3 illustrate the impact of a one standard deviation increase in job separations and a one standard deviation drop in the share of “switch” unemployed, respectively. We report the impact on the unemployment rate, on the share of unemployed workers who have been out of work for 6 months or more, on average unemployment duration, on the job finding rate, on vacancies, and, finally, on the real interest rate and the inflation rate.

An increase in the job separation rate puts upward pressure on the unemployment rate. At the same time, there is a significant drop in the vacancy index which falls by 4 percent on impact and recovers only gradually over time. Thus, the increase in unemployment produced by the increase in layoffs is propagated through fewer vacancies which together imply a significant worsening of the labor market outlook and a large and very persistent decrease in the job finding rate. An important reason for why the increase in job separations spill over to vacancies derives from the impact on households’ savings desire. Recall that employed agents have an incentive to self-insure against unemployment risk because of incomplete markets and because the increase in the job separation rate induces idiosyncratic job risk. We notice that the real interest rate drops very significantly. This reflects mainly that a lower real interest rate is needed to clear the asset market because of the precautionary savings motive just mentioned. In conjunction this produces a large and persistent increase in the level and duration of unemployment. Quantitatively, an increase in the monthly job separation rate from its stationary state value of 2.95 percent to 2.99 percent leads to an increase in the unemployment rate from 5 percent to 5.2 percent and an increase in the average unemployment duration from 2.78 months to 2.83 months.

Figure 3 illustrates the impact of an one standard deviation decrease in the share of job separations that send workers very short term unemployment holding constant the job separation rate. This shock has a large impact on the economy which is qualitatively similar to that of a layoff shock but quantitatively much larger. Less search efficient workers take on average longer to find a job than switch unemployed workers and experience a much larger consumption

cost of unemployment. Hence, the drop in the share of switch unemployment produces a strong desire for precautionary savings amongst employed workers which is reflected in a large drop in the real interest rate. This reduces goods demand and firms realize that vacancies are harder to fill because of the preponderance of low search efficiency workers amongst the unemployed. Hence, vacancies fall very significantly which leads to a large drop in the job finding rate. The decrease in the job finding rate and the lower search efficiency of the stock of unemployed workers jointly imply that the unemployment rate and the average unemployment duration both increase substantially.

In summary, the benchmark model induces a strong interaction between labor, asset and goods market which produces an amplification mechanism. In order to understand the sources of this mechanism better, we find it useful to compare the results of the benchmark economy to two alternative economies. In the first alternative economy we assume that prices are flexible, $\phi = 0$. In this economy, there is a much weaker transmission of shocks from households' demand for goods to firms' demand for labor since firms have a strong incentive to cut prices in response to weak goods demand. It is therefore useful to examine this alternative economy to understand how labor market frictions and idiosyncratic risk interact with goods market frictions.

In the second alternative economy we assume households can insure fully against idiosyncratic shocks. This is equivalent to assuming that households are organized in a single family which insures all idiosyncratic employment risk. The family's budget constraint is given as:

$$\mathbf{c}_t + \mathbf{b}_t^h = \mathbf{n}_t \mathbf{w}_t - \mathbf{T}_t + \mathbf{u}_{s,t} \boldsymbol{\xi}_s + \mathbf{u}_{l,t} \boldsymbol{\xi}_l + \frac{\mathbf{R}_{t-1}}{1 + \boldsymbol{\pi}_t} \mathbf{b}_{t-1}^h, \quad t \geq 0 \quad (29)$$

where we use that intra-household insurance implies that consumption levels are equalized across households. This economy features no idiosyncratic risk and while aggregate shocks in the labor market still impacts on the economy, the precautionary savings channel against idiosyncratic risk is neutralized.

Figure 4 demonstrates the impact of the job separation shock in the benchmark economy and in the economies where either prices are flexible but asset markets are incomplete or asset markets are complete but prices are sticky. The initial increase in unemployment in the

complete markets model is almost the same as in the benchmark model but under complete markets unemployment falls monotonically from the second month after the increase in layoffs. Moreover, the peak increase in unemployment in the complete markets model is less than half of what is observed in the benchmark model. The reason from this is clear from the real interest rate path which shows that the demand channel is neutralized almost immediately when asset markets are complete. Therefore, apart from the first month, firms take advantage of the ease of hiring that results from high unemployment and post more vacancies. This implies that the job finding rate is almost unaffected by the increase in job terminations and the amplification mechanism is almost totally neutralized.

The flexible price cum incomplete markets model produces very similar results to the model with complete markets cum sticky prices with the main difference being that the effects are somewhat more persistent in the flexible price economy than in the complete markets model. In this economy, while the job separation shock induces idiosyncratic risk, firms respond to lower demand by cutting prices. This implies a less significant impact on vacancy postings which induces, in turn, less idiosyncratic risk than in the benchmark model. In equilibrium, we find that the high job separation rate under these circumstances have very limited effects on the economy beyond the direct one that derive from more employed workers losing their jobs.

The amplification mechanism in the benchmark model is even stronger in response to the shock to the share of switch-unemployed, see Figure 5. Here we find almost no impact on unemployment or any other variable when *either* prices are flexible or asset markets are complete. In the complete markets economy, a fall in the share of very search efficient workers increases the expected income loss to the household sector associated with the measure of agents in unemployment state l . This obviously lowers demand from these households and makes it slightly more costly for firms to fill vacancies because of the lower search efficiency of this type of unemployment. Quantitatively, however, the demand channel is of only minor importance since there is no motive for insurance against idiosyncratic risk and this neutralizes most of the effects that arise in the benchmark model. When prices are flexible but asset markets incomplete, the

precautionary savings motive remains but there is little transmission mechanism from goods demand to vacancies which removes most of the amplification mechanism.

We conclude from this that it is the *combination* of incomplete markets and sticky prices that produce an amplification mechanism in which deteriorating labor market prospects produce low demand for goods which in turn leads to weak labor demand thereby creating a vicious circle. When asset markets are complete, there is little impact on goods demand while flexible prices neutralize the channel that goes from goods demand to labor demand.

A Great Recession Experiment. We now turn to a Great Recession experiment. We derive estimates of the sequences of innovations to job termination and to the fraction of workers that flow into high search efficiency unemployment, $(\epsilon_{x,t}, \epsilon_{s,t})_{t=2007:1}^{2012:1}$, by matching the observed US time-series on layoffs and the number of unemployed workers who have been out of work for 6 months or more (relative to the labor force). We back out these shocks for the sample period from January 2007 until January 2012. In order to avoid having too erratic shocks, we smooth both data series with a 4 months moving average filter. We then feed the resulting shock processes into the model economy and simulate the economy in response to this particular sequence of shocks.

The first two panels of Figure 6 show the path of layoffs and the fraction of long term unemployed that we target. The other two panels illustrate the time-series of unemployment and vacancies observed in US data together with the corresponding time-series implied by the benchmark model.²² We also show the corresponding time-series generated by the model when assuming either that there is no change in the composition of the unemployed, prices are flexible, or asset markets are complete. We initialize the unemployment rate at the steady-state and we report vacancies in terms of percentage deviations from steady-state.

The benchmark model reproduces almost exactly the observed unemployment rate. As in the actual data, unemployment starts increasing fast from early 2008 and peaks around 10 percent in late 2009 - early 2010. The model implies a slightly lower unemployment rate in the

²²We show these time-series in terms of deviations from their pre-crisis levels.

post-2011 sample than the actual value observed in the US but the difference is not large. One might think that this success in accounting for the increase in unemployment during the Great Recession derives from the fact that we are feeding the observed layoffs into the model but this intuition is not correct. In fact, when we assume either flexible prices or complete markets, we can account for a maximum 1.5 percentage point increase in unemployment, less than one third of the increase implied by the benchmark model.

The key reason for these differences derive from how the shocks impact on vacancies. In the US, the number of vacancies fell dramatically during the Great Recession and have only recovered mildly since the middle of 2009. The benchmark model captures very well the size of the drop in vacancies (but perhaps not quite so well the dynamics since it implies a much later recovery in the number of vacancies than the US experience). Under the assumptions of either flexible prices or complete markets, we find instead that the model implies little change in vacancies during the early parts of the recession and an *increase* above 2007 levels from 2009 onwards. The lack of a drop in vacancies under these circumstances means that job prospects deteriorate only little beyond the change in job separations and this is the reason why the model fails to produce a strong increase in unemployment when assuming either flexible prices or complete markets.

In order to evaluate the importance of the switch unemployment state we also report the outcome for unemployment and vacancies when we set $(\varepsilon_{s,t})_{t=2007:1}^{2012:1} = 0$ so that the economy is hit by job separation shocks only. In this case, we find a moderate increase in unemployment and a path of vacancies that are very similar to what the outcome under flexible prices. Thus, the increased risk for a low income state with long duration that is key for generating the amplification mechanism.

Much discussion surrounding the Great Recession as focused on the relationship between unemployment and vacancies and the outward shift in the Beveridge curve that seems to have occurred since 2007. We illustrate this in Figure 7. During the early parts of the recession, there was a marked decline in both vacancies and in the unemployment rate but later on the crisis, the

recovery in vacancies has been associated with only a minor decline in unemployment consistent with the view that the Beveridge curve has shifted out. In Figure 7 we also report the joint trends in unemployment and vacancies implied by our model by combining the last two panels of Figure 6. The versions of the model that assume either flexible prices, complete markets, or changes only in job separation rates fail to reproduce the joint decrease in unemployment and vacancies during the early parts of the recession and the implied relationship between unemployment and vacancies bear little resemblance if any to its empirical counterpart. The benchmark model instead performs extremely well at least until the later part of the recession where there is a more significant rise in vacancies and drop in unemployment in the US data than predicted by the model. Nonetheless, the model is remarkably successful in accounting for joint movements in unemployment and vacancies.

3.3 Robustness

Nash Bargaining. We have so far has assumed that the real wage is fixed. The US experience during the Great Recession is by and large consistent with this assumption, see Shimer (2012). Nonetheless, it is interesting to investigate the importance of this assumption. For that reason we now instead assume that wages are fully flexible and determined in a Nash bargaining game between entrepreneurs and workers. We will assume that the bargaining weights equal 0.5.²³

An issue that needs to be addressed relates to proper definition of the outside value of workers which depends on the assumptions made regarding the state of workers should the bargaining process be unsuccessful. We make two alternative assumptions. In the first case (Nash Bargaining I) we assume that an unsuccessful bargaining process sends workers into the two unemployment states with the same probabilities as workers that experience a job separation shock while under Nash Bargaining II we assume that workers flow entirely into switch unemployment should the bargaining process break down.

²³In this economy we set the value of leisure, κ , to match the same targets for unemployment and other central labor market variables as in the benchmark model.

Figure 8 illustrates the impact on the real wage and on unemployment of shocks to the job termination rate and to the share of mis-matched unemployed workers under alternative wage setting assumptions. The impact of an increase in layoffs on the level of unemployment is qualitatively similar under Nash Bargaining to the benchmark model but quantitatively smaller because the real wage falls under Nash bargaining. Moreover, since the share of mis-matched workers remain constant, the results are identical under the two alternative Nash bargaining assumptions. The moderation in the unemployment impact corresponds to a decrease in the peak increase in the unemployment rate of approximately 25 percent which is significant but smaller than the corresponding decrease under flexible prices or complete markets.

The impact on the response to an decrease in the share of switch unemployment is more dramatic. Under Nash Bargaining I, the real wage falls sufficiently much that the unemployment rate actually drops after a decrease in the share of switch unemployed workers.²⁴ The reason is that the less search efficient unemployment state is sufficiently bad that workers are willing to take a pay cut during the bargaining process to minimize the risk of flowing into this state. Lower real wages in turn means that firms post more vacancies which improves labor market prospects and lowers the level of unemployment. Under Nash Bargaining II workers instead feel less need to take a pay cut because they do not face an immediate risk of becoming low search efficient unemployed. In this case, the fall in the real wage is much more moderate and the results are qualitatively similar to the benchmark model.

Figure 9 repeats the Great Recession experiment discussed above for the Nash Bargaining I specification and compares the results with the benchmark model. We report the levels of unemployment and vacancies and the real wage. We find that the Nash Bargaining model fails to account for the persistent rise in the level of unemployment and the long period of very low vacancy postings. In fact, when real wages are flexible, vacancies *rise* above their pre-recession level from 2010 onwards and unemployment starts falling significantly from late

²⁴Quantitatively, the fall in the real wage seems small but this derives from assumptions regarding the value of unemployment.

2009 going below its 2006 level by early 2010. These very counterfactual implications derive from a moderate fall in the real wage.

3.4 Amplification and Local Determinacy

The benchmark model displays a strong amplification mechanism. It seems intuitively clear that the strength of the amplification mechanism depends on the degree of risk aversion since the key channel of amplification arises from the impact of idiosyncratic risk on household demand for goods. The larger is the degree of risk aversion, the stronger is the impact of job uncertainty because the precautionary risk response to labor market shocks is stronger. Another important parameter is the monetary policy response to inflation because this determines the impact of shocks on the real interest rate which in turn impacts on demand.

We will now examine how the strength of the amplification mechanism depends on these two parameters. Figure 10 illustrates the impact effect of a joint shock to job separation and the share of switch-unemployed workers for a wide variety of different values of σ and δ . For each combination of σ and δ , we normalize the impact effect with the impact effect in the flexible price economy assuming the same value of σ . We figure therefore illustrates the amplification effect. As expected, the amplification mechanism is stronger the more risk averse agents are and the weaker is the monetary policy response to inflation. In principle, the model can generate a very large amplification mechanism for a sufficiently high degree of risk aversion and low degree of monetary policy response to inflation. When there is little risk aversion and the nominal interest rate response to inflation is very strong, the model generates the same impact of labor market shocks as under flexible prices. However, standard values of these parameters can generate a very significant amplification mechanism.

In this model, not all parameter combinations are compatible with a unique equilibrium. We find that there is a large zone of parameters for which the deterministic steady-state of the model is locally indeterminate, see Figure 10. In this zone, many other types of equilibria including self-fulfilling sunspot type fluctuations may exist. Intuitively, when the deterministic steady-

state is locally indeterminate, an expectation of future job uncertainty may be self-fulfilling because it can lead to a large drop in household demand for goods which leads firms to decrease vacancy postings therefore spurring job uncertainty. We hesitate to speculate too much about the empirical relevance of such fluctuations but believe that this may be an interesting further issue to explore. The indeterminacy region occurs when either agents are very risk averse or when the nominal interest rate response to variations in the inflation rate is sufficiently weak. The amplification mechanism is strongest the closer the parameters are to the indeterminacy region. We notice that the Taylor principle is far from sufficient for guaranteeing local determinacy in this model.

4 Quantitative Results: General Model

The results discussed above are based upon imposing a no-borrowing constraint so that there is no wealth inequality in equilibrium. We now relax this assumption and allow for a positive debt limit. We retain the calibration of $\beta = 0.993$ which in this economy implies a steady-state real interest rate of 4.33 percent annually.

Allowing for a positive debt level means that consumption losses due to unemployment no longer equal income losses because agents can save to smooth their consumption should they go through an unemployment spell. We therefore calibrate ξ to match benefit levels rather than consumption losses directly. We assume that $\xi_s = \xi_l = 0.4\mathbf{w}$. Agents will still reduce consumption more should they make a transition from the high search efficiency state to the normal unemployment state because of the impact on expected unemployment duration. Moreover, the consumption loss due to unemployment will depend upon wealth. Hence, in the model there is a wide distribution of consumption losses after a job termination shock across agents.

We assume that the debt penalty function is given as:

$$\varphi(\mathbf{b}_{i,t}^h) = \frac{\exp(-\eta_0(\mathbf{b}_{i,t}^h - \eta_1))}{\eta_0}$$

We calibrate the debt limit η_1 so that we match an average consumption loss of 14 percent when an agent experience a job separation. This implies a borrowing limit that corresponds to 1.9 months of salary. We also recalibrate the persistence of unemployment state shock so that it matches the persistence of the share of longer term unemployed workers.

Figure 11 illustrates the consumption policy functions for the model. We plot these against wealth (and recall that the real wage is 0.8). In the model, the consumption loss for workers that experience a job loss is monotonically declining in wealth. A worker with a long employment spell save for precautionary reasons. The consumption loss is larger for households that flow into longer term unemployment than for households that flow to short term unemployment because of the differences in the associated income losses. We also notice that savings provide a lot of insurance against unemployment for richer households.

Figure 12 and 13 illustrate the impulse responses of the model to one standard shocks to the job separation rate and to the share of workers that upon a job termination shock make a transition to the less search efficient unemployment state. As above, we compare with the outcomes under flexible prices. The results are very similar to what we found in the model with a no-borrowing constraint. In particular, we still find a large amplification of both types of shocks. One important difference, though, is that there is a much large impact on the real interest rate (which must fall sufficiently that aggregate savings are zero in equilibrium). Effectively, because of wealth heterogeneity, some households that are in debt are very reluctant not to increase their savings in response to increased uncertainty and worsening labor market conditions. Restoring equilibrium requires a sharp fall in the real interest rate. Hence, we find that idiosyncratic uncertainty has an even more important role in this model than in the simpler model analyzed above.

5 Conclusions and Summary

The great recession have witnessed a long and deep recession. While unemployment may finally showing signs of declining, the level and duration and unemployment are still very high relative

to historic standards. In this paper we have suggested that the uncertainty effects of the job losses in the early part of the recession may have been an important factor behind the depth of the recession. In particular, we have argued that shocks in the labor market may have been amplified significantly due to the combination precautionary savings against the idiosyncratic risk associated with job uncertainty, the risk of longer term unemployment, and nominal rigidities. We have shown that, conditional upon real wage rigidity, these frictions can in conjunction create a vicious circle of feedback that leads to an amplification mechanism that is neutralized when either prices are flexible or asset markets are complete.

In our analysis the risk of longer term unemployment is important for generating a sizeable response to labor market shocks. We have given no further structural interpretation the multiple unemployment states and it would be interesting to dig deeper and to examine the sources of this shock which we find is important quantitatively. In the same vein, it would be interesting to investigate further the sources of the initial increase in job separations.

A key aspect of the model that we examine is that the aggregate shocks to labor market conditions bring about idiosyncratic risk. Because workers do not know *who* will lose their jobs when the job separation rate increases, there is a link between aggregate shocks that impact on job prospects and idiosyncratic uncertainty. We think this is an interesting mechanism which would be worth exploring further.

There are many avenues for possible extensions of our analysis. In our model there is no aggregate savings vehicle. It would be interesting to include accumulable assets which could potentially introduce a propagation mechanism. Moreover, we have allowed only for shocks to job separations and to the share of unemployed workers flowing into the two states while individuals in reality face other shocks that may generate precautionary savings such as earnings uncertainty and health shocks. Furthermore, we have not allowed for idiosyncratic firm risk which would be very interesting to study in our setting because it would potentially generate an alternative amplification mechanism. We leave these and other extensions to future work.

6 References

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7 Appendices

7.1 Krusell-Smith algorithm

To solve the model, we apply the algorithm of Krusell and Smith. In particular we assume that agents take as given a law of motion for the bond price. This law of motion is assumed linear in nine variables: the contemporaneous values of the two shocks, two lags of each shock, the lagged employment rate, the lagged measure of agents in the short-term pool, and the lagged bond price.²⁵ Taking as given this law of motion, we then solve the individuals’ decision

²⁵We verify that this law of motion can accurately predict the contemporaneous bond price in the equilibrium.

problems, simulate a continuum of agents and compute a time series for the bond price that would clear the bond market given decision rules.²⁶ We iterate until the law of motion taken as given coincides with the actual law of motion, up to an accuracy criterion.

7.2 Solving the individuals' problems

Our algorithm for solving the individual worker's problem is the following. First, we decision rules the bond holdings by households in the two employment states, denoted by $\mathbf{b}'(\mathbf{b}_t, \mathbf{S}_t | \mathbf{n}_t = 0, \mathbf{I}_t = 1)$ for the short-term unemployed and $\mathbf{b}'(\mathbf{b}_t, \mathbf{S}_t | \mathbf{n}_t = 0, \mathbf{I}_t = 0)$ for the unemployed in the less search efficient state. Next, consider N *employed* workers in some period t , with N being a finite but reasonably large number (200 in our application). For any of these workers, we are free to set an initial level of bond holdings. The initial level of bond holdings of worker 1 is set to some \mathbf{b}_0 that is likely to be at the lower end of the ergodic set. The level of bond holdings of any agent $n > 1$, is set to be the *exactly* amount of bond holdings *chosen* by agent $n - 1$ in the previous period. Thus, we let period t 's agent n be labeled as agent $n + 1$ in period $t + 1$. Accordingly, $\mathbf{c}_{n+1,t+1}$ is the next period's consumption of agent n and describe the agents by

²⁶We ensure market clearing in the simulation by following Den Haan and Rendahl (2010) who suggest to solve for a transformation of individual bond holdings instead for bond holdings itself. In particular they let agents solve for $\mathbf{x}_{i,t} \equiv \mathbf{b}_{i,t} + \mathbf{q}_t$ where $\mathbf{b}_{i,t}$ denotes individual bond holdings and \mathbf{q}_t denotes the bond price. In each simulation period, one can then compute the market clearing bond price by simply aggregating the choices for $\mathbf{x}_{i,t}$, using the fact that the aggregate net supply of bonds is zero.

the following system of budget constraints and Euler equations:

$$\begin{aligned}
\mathbf{c}_{1,t} &= \mathbf{w}_t - \mathbf{T}_t^h + \frac{\mathbf{R}_{t-1}}{1 + \boldsymbol{\pi}_t} \mathbf{b}_0 - \mathbf{b}_{1,t} \\
u'(\mathbf{c}_{1,t}) &= \beta \mathbb{E}_t \left\{ \frac{\mathbf{R}_t}{1 + \boldsymbol{\pi}_{t+1}} \left(\begin{aligned} &\left(1 - \sum_{r=s,l} \boldsymbol{\rho}_{x,t} \boldsymbol{\rho}_{r,t} (1 - \boldsymbol{\eta}_{r,t+1}) \right) u'(\mathbf{c}_{2,t+1}) \\ &+ \sum_{r=s,l} \boldsymbol{\rho}_{x,t} \boldsymbol{\rho}_{r,t} (1 - \boldsymbol{\eta}_{r,t+1}) u'(\mathbf{c}_{2,r,t+1}^u) \end{aligned} \right) \right\} \\
\mathbf{c}_{2,s,t+1}^u &= \boldsymbol{\xi}_s + \frac{\mathbf{R}_{t-1}}{1 + \boldsymbol{\pi}_t} \mathbf{b}_{1,t} - \mathbf{b}'(\mathbf{b}_{1,t}, \mathbf{S}_t | \mathbf{n}_{t+1} = 0, \mathbf{I}_{t+1} = 1) \\
\mathbf{c}_{2,l,t+1}^u &= \boldsymbol{\xi}_l + \frac{\mathbf{R}_{t-1}}{1 + \boldsymbol{\pi}_t} \mathbf{b}_{1,t} - \mathbf{b}'(\mathbf{b}_{1,t}, \mathbf{S}_t | \mathbf{n}_{t+1} = 0, \mathbf{I}_{t+1} = 0) \\
\mathbf{c}_{2,t} &= \mathbf{w}_t - \mathbf{T}_t^h + \frac{\mathbf{R}_{t-1}}{1 + \boldsymbol{\pi}_t} \mathbf{b}_{1,t} - \mathbf{b}_{2,t} \\
u'(\mathbf{c}_{2,t}) &= \beta \mathbb{E}_t \left\{ \frac{\mathbf{R}_t}{1 + \boldsymbol{\pi}_{t+1}} \left(\begin{aligned} &\left(1 - \sum_{r=s,l} \boldsymbol{\rho}_{x,t} \boldsymbol{\rho}_{r,t} (1 - \boldsymbol{\eta}_{r,t+1}) \right) u'(\mathbf{c}_{3,t+1}) \\ &+ \sum_{r=s,l} \boldsymbol{\rho}_{x,t} \boldsymbol{\rho}_{r,t} (1 - \boldsymbol{\eta}_{r,t+1}) u'(\mathbf{c}_{3,r,t+1}^u) \end{aligned} \right) \right\} \\
&\vdots \\
\mathbf{c}_{N,t} &= \mathbf{w}_t - \mathbf{T}_t^h + \frac{\mathbf{R}_{t-1}}{1 + \boldsymbol{\pi}_t} \mathbf{b}_{N-1,t-1} - \mathbf{b}_{N,t} \\
u'(\mathbf{c}_{N,t}) &= \beta \mathbb{E}_t \left\{ \frac{\mathbf{R}_t}{1 + \boldsymbol{\pi}_{t+1}} \left(\begin{aligned} &\left(1 - \sum_{r=s,l} \boldsymbol{\rho}_{x,t} \boldsymbol{\rho}_{r,t} (1 - \boldsymbol{\eta}_{r,t+1}) \right) u'(\mathbf{c}_{N+1,t+1}) \\ &+ \sum_{r=s,l} \boldsymbol{\rho}_{x,t} \boldsymbol{\rho}_{r,t} (1 - \boldsymbol{\eta}_{r,t+1}) u'(\mathbf{c}_{N+1,r,t+1}^u) \end{aligned} \right) \right\} \\
\mathbf{c}_{N+1,s,t+1}^u &= \boldsymbol{\xi}_s + \frac{\mathbf{R}_{t-1}}{1 + \boldsymbol{\pi}_t} \mathbf{b}_{N-1,t} - \mathbf{b}'(\mathbf{b}_{N-1,t}, \mathbf{S}_t | \mathbf{n}_{t+1} = 0, \mathbf{I}_{t+1} = 1) \\
\mathbf{c}_{N+1,l,t+1}^u &= \boldsymbol{\xi}_l + \frac{\mathbf{R}_{t-1}}{1 + \boldsymbol{\pi}_t} \mathbf{b}_{N-1,t} - \mathbf{b}'(\mathbf{b}_{N-1,t}, \mathbf{S}_t | \mathbf{n}_{t+1} = 0, \mathbf{I}_{t+1} = 0)
\end{aligned}$$

Taking the aggregate variables as given, this is a dynamic system of $4N$ equations in $4N + 1$ endogenous variables ($\mathbf{c}_{2,s,t}^u, \dots, \mathbf{c}_{N+1,s,t}, \mathbf{c}_{2,l,t}^u, \dots, \mathbf{c}_{N+1,l,t}, \mathbf{b}_{1,t}, \dots, \mathbf{b}_{N,t}, \mathbf{c}_{1,t}, \dots, \mathbf{c}_{N,t}$ and $\mathbf{c}_{N+1,t}$). How to close the system? Note that agent N was agent 1 in period $t - N$. Thus, we can think of this agent as a very lucky one who has started off with wealth b_0 and has since remained employed for N consecutive periods. For any amount of b_0 that is part of the ergodic set, the agent is likely to be among the very richest agents in the economy and will be extremely close to the amount of bond holdings that any agent will maximally choose. We can solve for the maximum amount of bond holdings and the associated level of consumption from the following equations,

which resemble steady-state conditions:

$$\begin{aligned}
\mathbf{c}_{\max,t} &= \mathbf{w}_t - \mathbf{T}_t^h + \frac{\mathbf{R}_{t-1}}{1 + \boldsymbol{\pi}_t} \mathbf{b}_{\max,t-1} - \mathbf{b}_{\max,t} \\
u'(\mathbf{c}_{\max,t}) &= \beta \mathbb{E}_t \left\{ \frac{\mathbf{R}_t}{1 + \boldsymbol{\pi}_{t+1}} \left(\begin{aligned} &\left(1 - \sum_{r=s,l} \boldsymbol{\rho}_{x,t} \boldsymbol{\rho}_{r,t} (1 - \boldsymbol{\eta}_{r,t+1}) \right) u'(\mathbf{c}_{\max,t+1}) \\ &+ \sum_{r=s,l} \boldsymbol{\rho}_{x,t} \boldsymbol{\rho}_{r,t} (1 - \boldsymbol{\eta}_{r,t+1}) u'(\mathbf{c}_{\max,r,t+1}^u) \end{aligned} \right) \right\} \\
\mathbf{c}_{\max,s,t+1}^u &= \boldsymbol{\xi}_s + \frac{\mathbf{R}_t}{1 + \boldsymbol{\pi}_{t+1}} \mathbf{b}_{\max,t} - \mathbf{b}'(\mathbf{b}_{\max,t}, \mathbf{S}_t | \mathbf{n}_{t+1} = 0, \mathbf{I}_{t+1} = 1) \\
\mathbf{c}_{\max,l,t+1}^u &= \boldsymbol{\xi}_l + \frac{\mathbf{R}_t}{1 + \boldsymbol{\pi}_{t+1}} \mathbf{b}_{\max,t} - \mathbf{b}'(\mathbf{b}_{\max,t}, \mathbf{S}_t | \mathbf{n}_{t+1} = 0, \mathbf{I}_{t+1} = 0)
\end{aligned}$$

The above system represents the choices of a worker who happens to remain employed forever. The difference between this agent, and an agent who has remained employed for at least N periods can be made arbitrarily small by letting N grow large. Hence, we pin down $c_{N+1,t+1}^e$ by setting

$$\mathbf{c}_{N+1,t+1} = \mathbf{c}_{\max,t+1}.$$

We now have a dynamic system of $4N + 5$ equations in $4N + 5$ variables, which we solve using standard perturbation software.

Once the dynamic system is solved, we fit a (9th-order) polynomial that represents the decision rule of the employed worker, denoted by $\mathbf{b}'(\mathbf{b}_t, \mathbf{S}_t | \mathbf{n}_t = 1)$. Next we turn to the two types of unemployed agents and solve systems similar to the one above. But now we take the decision rule of an employed agent for given and set the initial wealth of unemployment agent 1 to b_{\max} . Agent N is now an agent who has been unemployed for at at least N periods and will be at the very lower end of the wealth distribution. Hence, we pin down the system by solving for $c_{\min,t}^u$, the consumption choice of an agent who has been unemployed for an infinitely large number of time periods.

After the perturbation step, we update the policy rules for the unemployed agent and return to the employed agent's problem. We keep on iterating on the agents until the maximum change in coefficients of the fitted policy rules drops below a certain convergence criterion.

7.3 Tables and Figures

Table 1: Targets and Parameter Values

Targets	
0.05	unemployment rate
0.4	average job finding rate among newly unemployed
0.3	average job finding rate among unemployed > 6m
0.045	hiring cost as a fraction of the quarterly wage
0.15	fraction of long-term unemployed
0	inflation rate
Parameters	
0.03	s.s. job termination rate ($\bar{\rho}_x$)
0.65	s.s. fraction into mismatch ($\bar{\eta}$)
0.19	$\frac{\theta}{\mu}$ ratio of vacancy cost over matching efficiency parameter
0.50	prob. of search mismatched (γ)
0.84	real wage (\bar{w})
0.0024	s.s. nominal interest rate (\bar{r})

Table 2: Stationary State Values

Parameter	Value	Meaning
c_e	0.829	consumption employed
$c_{u,s}$	0.767	consumption regular unemployed
$c_{u,l}$	0.696	consumption mismatch unemployed
r	0.00246	nominal interest rate (monthly)
ρ_x	0.0295	job termination rate
ρ_s	0.350	fraction of terminations that flow to regular unemployment
ρ_f	0.592	job finding rate regular unemployed
q	0.502	probability that mismatched can search
u_s	0.00677	mass of regular unemployed
u_l	0.04322	mass of mismatch unemployed

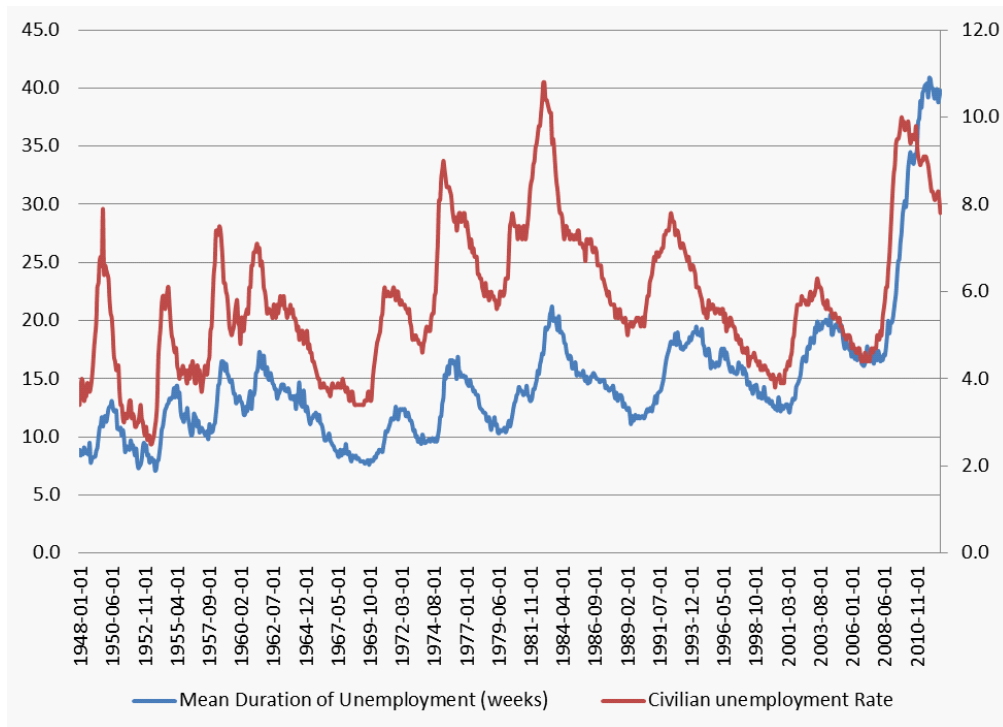


Figure 1. US Labor Market Conditions

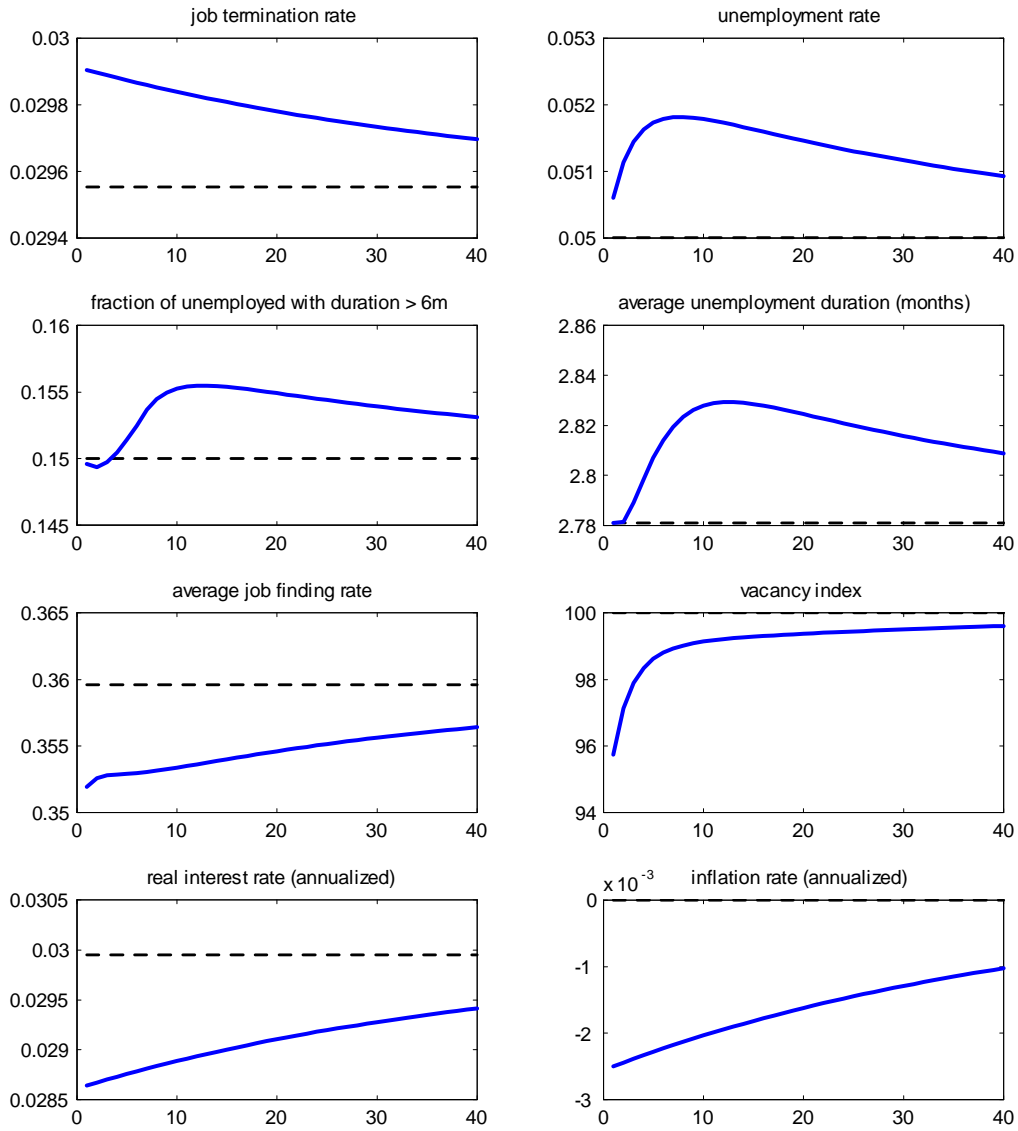


Figure 2. Benchmark Model: Impact of Job Separation Shock

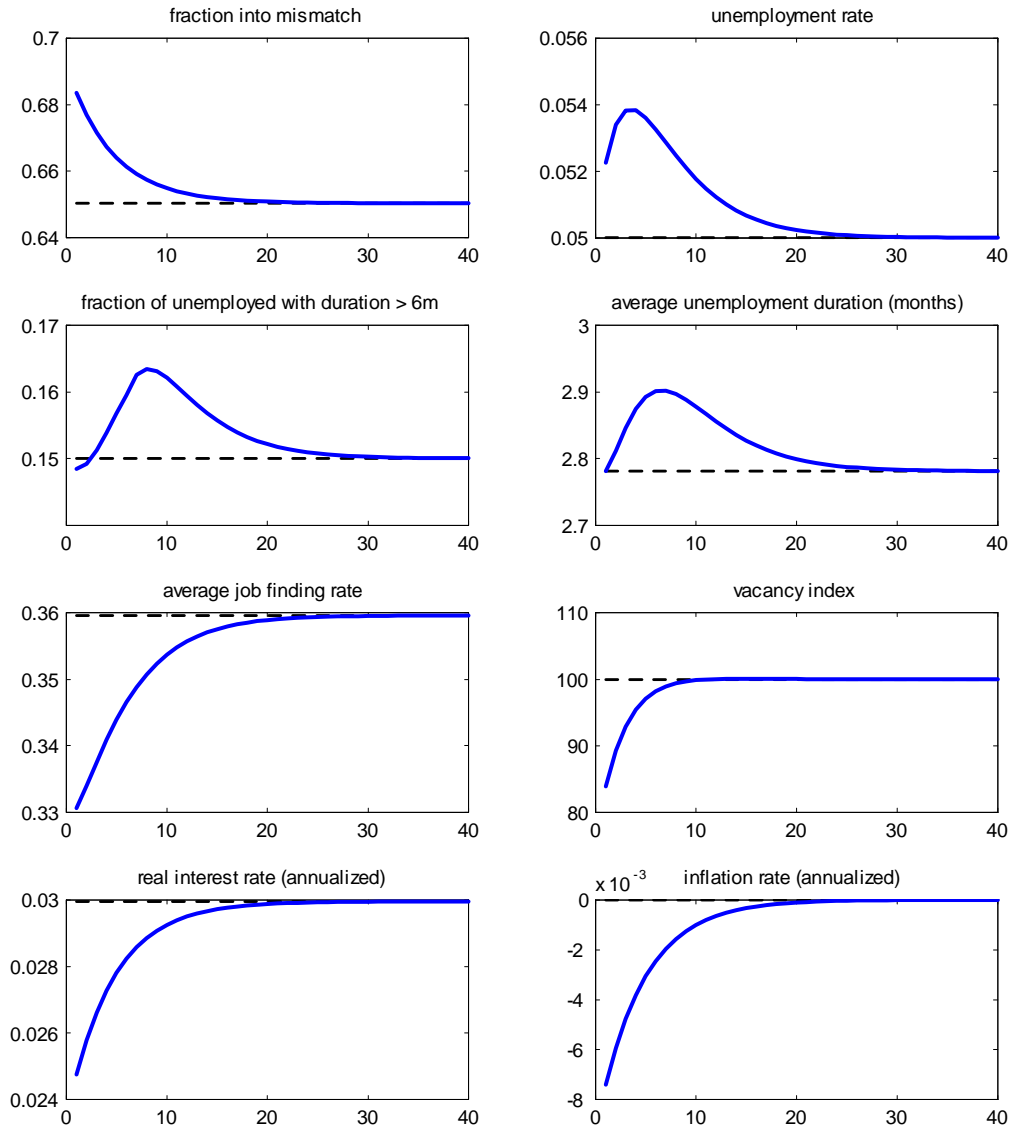


Figure 3. Benchmark Model: Impact of Mis-Match Shock

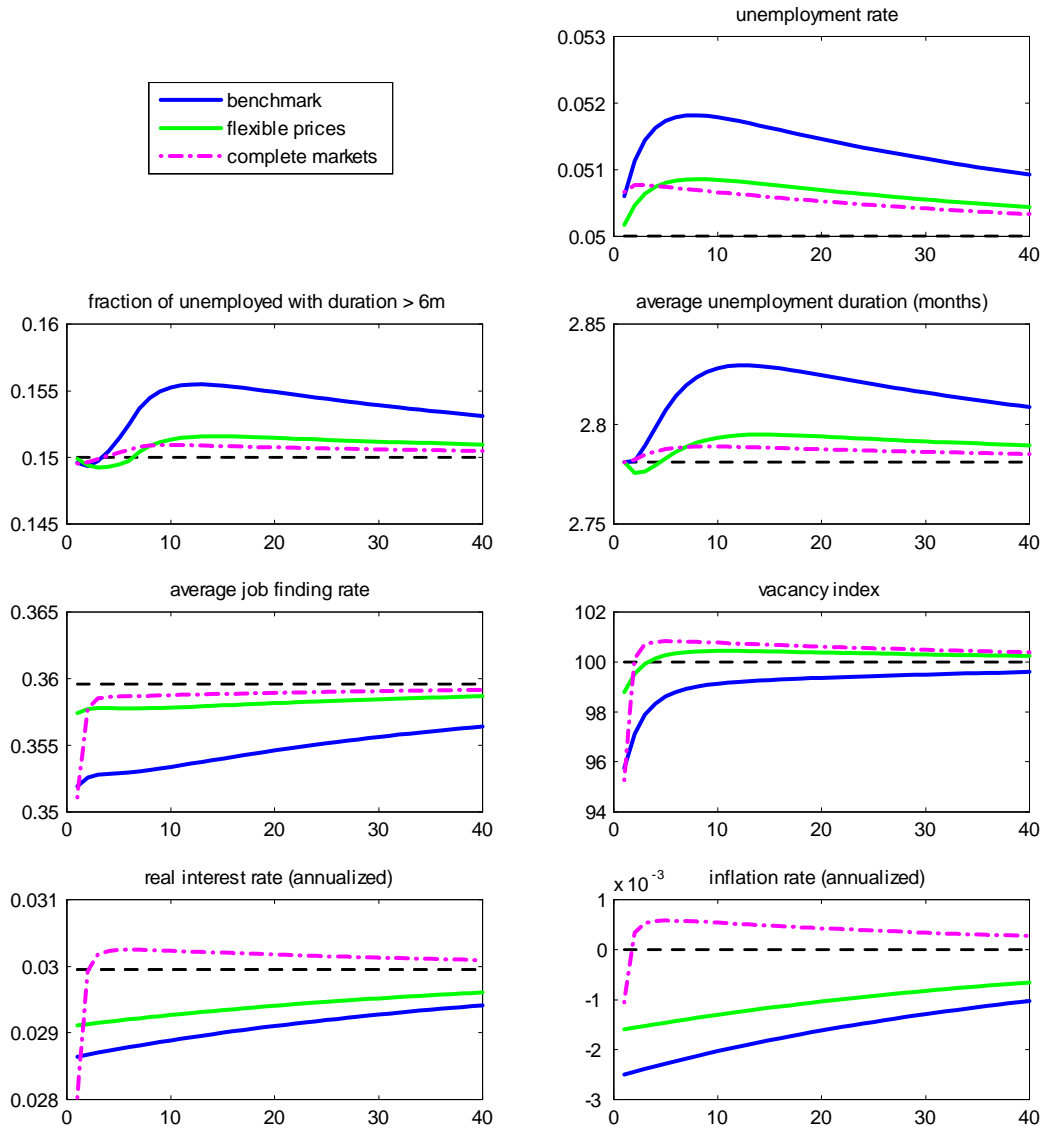


Figure 4. Impact of Job Separation Shock under Alternative Assumptions

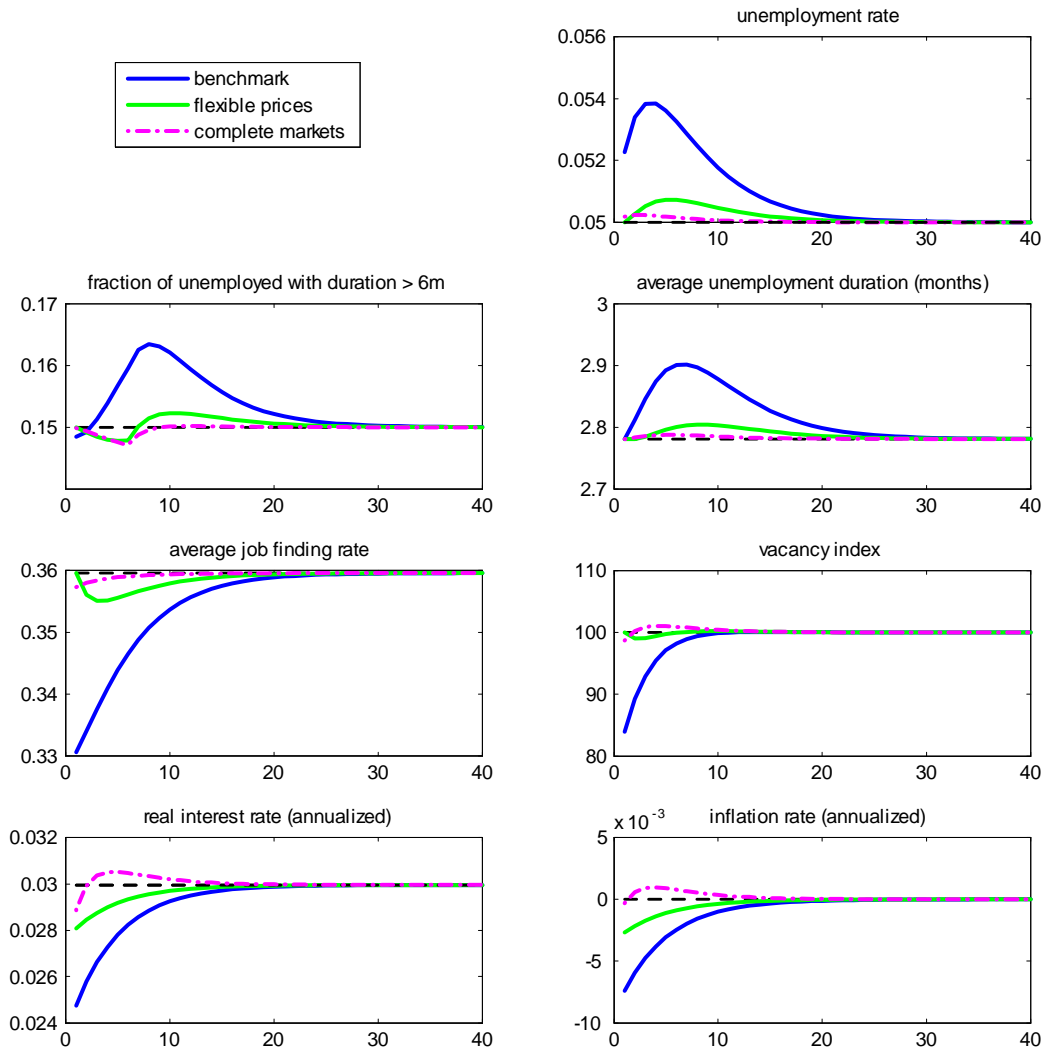


Figure 5. Impact of Mis-Match Shock under Alternative Assumptions

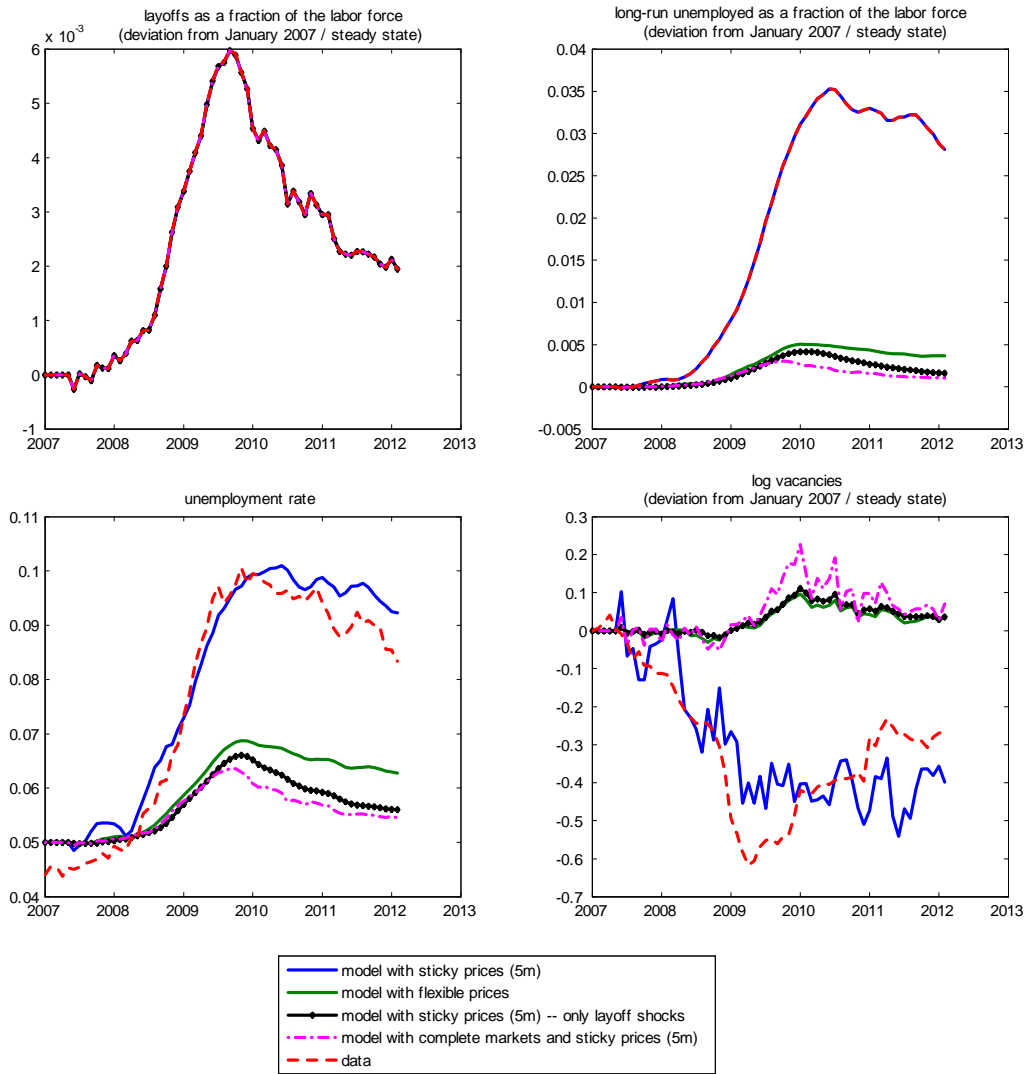


Figure 6. Great Recession Experiment

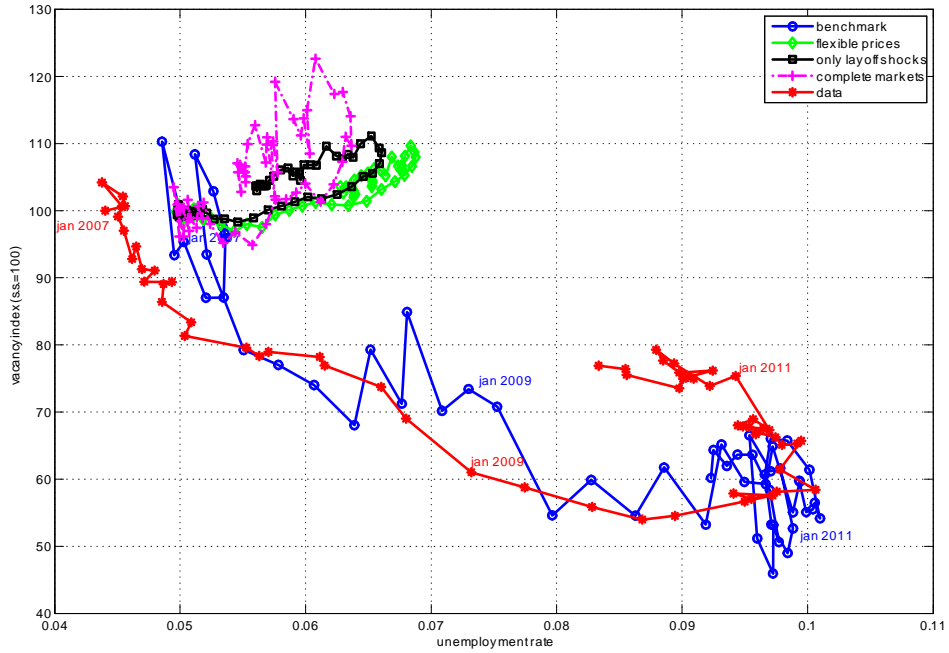


Figure 7. The Beveridge Curve

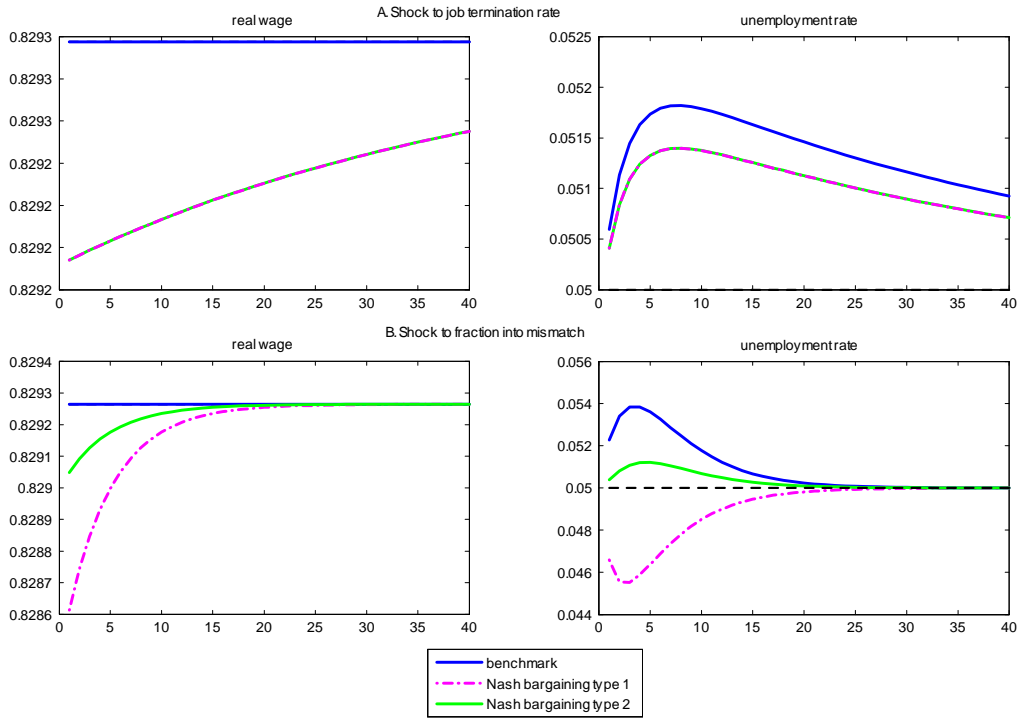


Figure 8. The Impact of Labor Market Shocks under Nash Bargaining

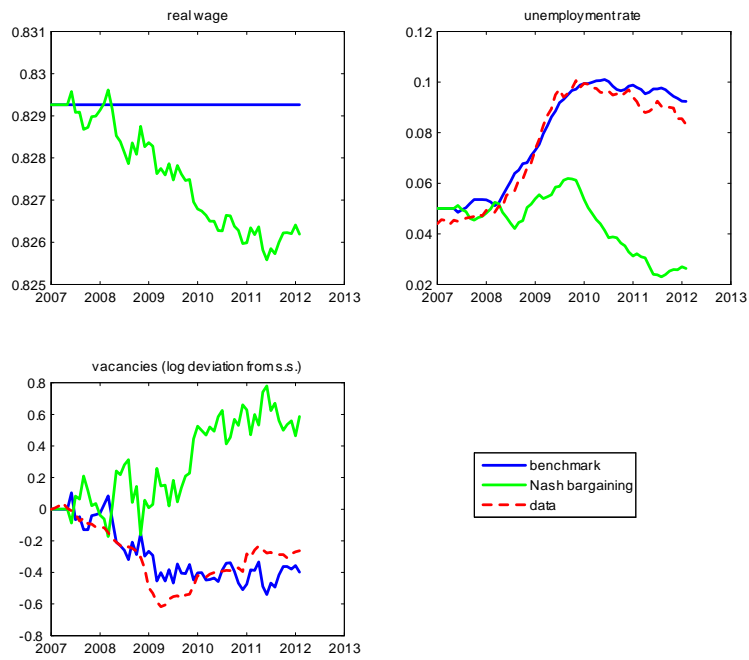


Figure 9. The Great Recession and Nash Bargaining

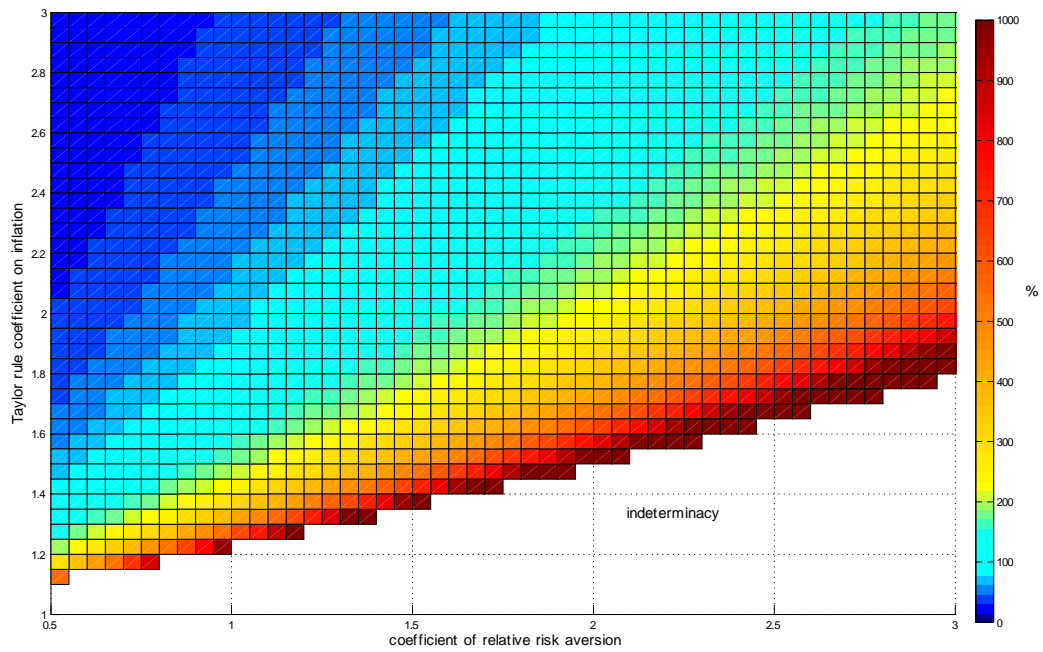


Figure 10. The Strength of the Amplification Mechanism and Local Indeterminacy

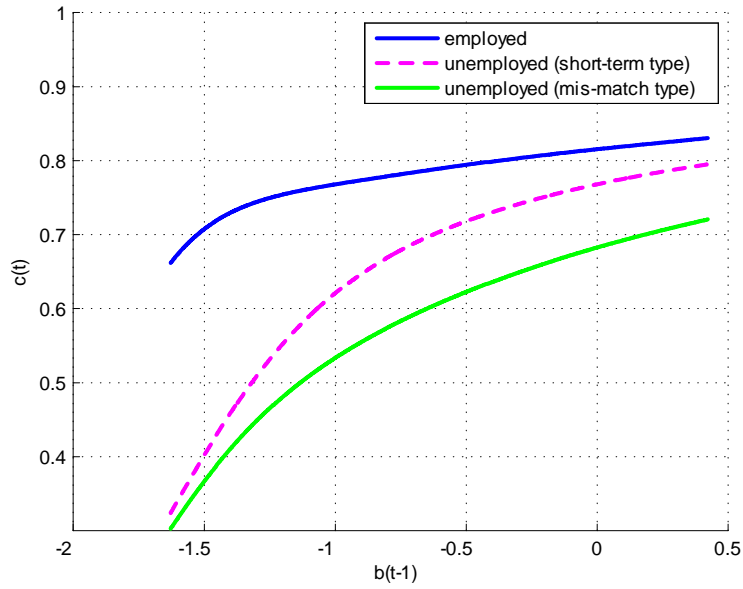


Figure 11. Consumption Policy Functions: General Model

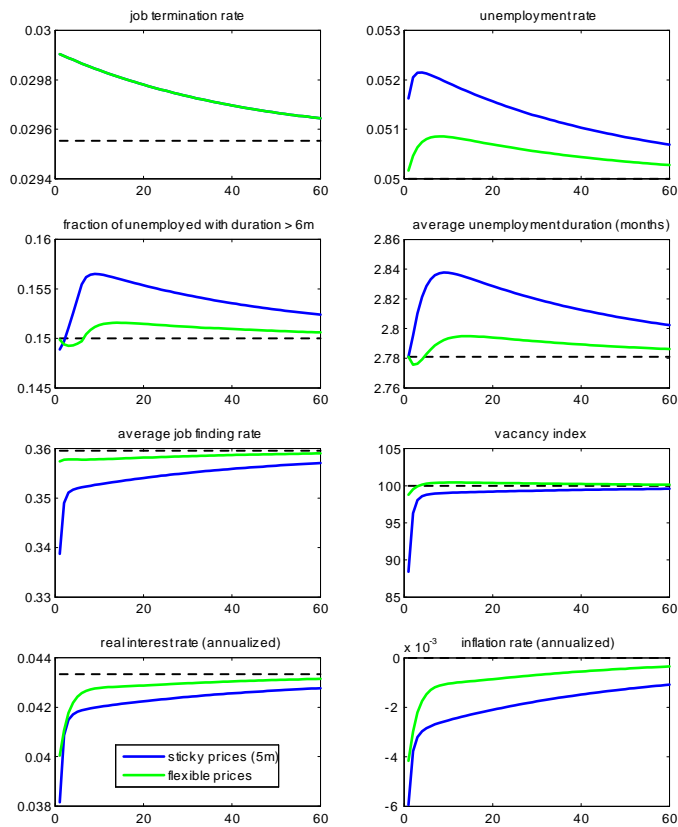


Figure 12. The Impact of a Job Separation Shock: General Model

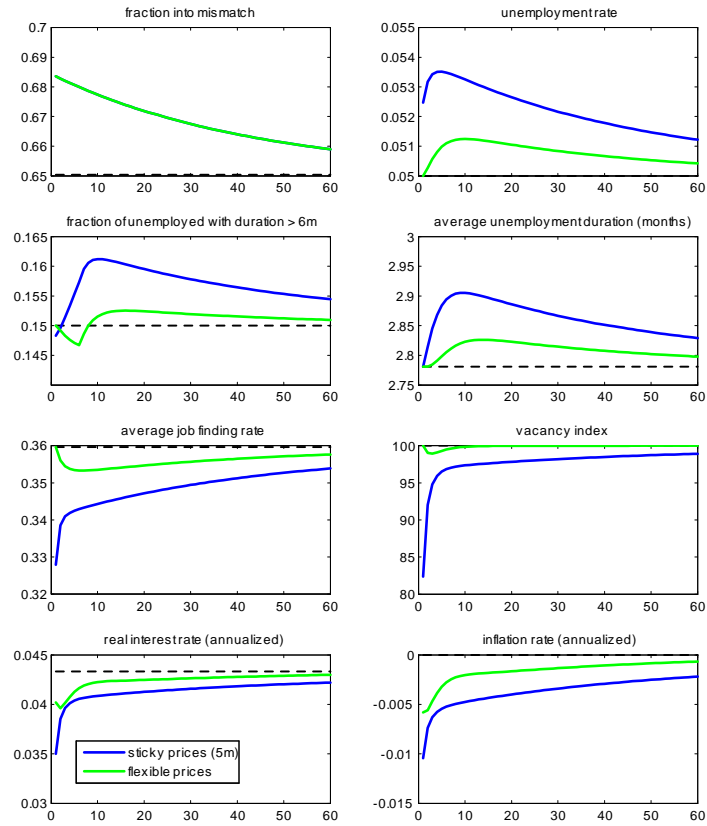


Figure 13. Impact of a Mis-Match Shock: General Model