The Dynamics of Sovereign Debt Crises and Bailouts

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VERY PRELMINARY
COMMENTS WELCOME

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Abstract

*The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management. This is preliminary indeed. The results have not yet been checked with sufficient care, despite multiple presentations of earlier drafts. We are thankful to our discussants so far. We are particularly thankful for an insightful discussion by Fernando Broner at a CREI conference in December 2011, which led to some important changes regarding the bailout analysis in the paper. Address: Francisco Roch, IMF, email: froch@uchicago.edu. Harald Uhlig, Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, IL 60637, U.S.A, email: huhlig@uchicago.edu. The research of Harald Uhlig has been supported by the NSF grant SES-0922550. Harald Uhlig has an ongoing consulting relationship with a Federal Reserve Bank, the Bundesbank and the ECB.

Inspired by the European debt crisis since 2010, this paper provides a theoretical framework to analyze the dynamics of a sovereign debt crisis and bailouts. To do so, we draw on three sets of literatures. First, Arellano (2008) has shown how "bad luck" can lead to a sovereign debt crisis. Second, Cole-Kehoe (1996,2000) have shown how multiplicity of equilibria can lead to a buyers strike. Finally, the impatience of policy makers as in Beetsma and Uhlig (1999) provides a reason why a country would be in a crisis zone in the first place. We introduce a bailout agency or large investor, and characterize the minimal actuarily fair intervention that restores the "good" equilibrium of Cole-Kehoe, relying on the market to provide residual financing.

Keywords:

JEL codes:

1 Introduction

Warnings and analyses of future sovereign debt crises in Europe and their impact on European monetary policy such as Uhlig (2003) seemed to have received little echo during an episode where such fears were judged unfounded. That has changed. Since 2010, doubts persist on financial markets that a number of countries such as Greece, Portugal, Spain or Italy will be able to repay their sovereign debt. Various bailouts and interventions have been proposed or been executed, with mixed success. This paper is motivated by these developments and seeks to understand the dynamics of sovereign debt crises in a single country, when there is an outside intervention by some bail-out agency. We characterize the minimal actuarily fair intervention that restores the "good" equilibrium of Cole-Kehoe (2000), relying on the market to provide residual financing. "Fair value" here means that the resources provided by the bail-out fund earn the market return in expectation. We believe this is an important benchmark.

The analysis of the dynamics of a sovereign debt crisis builds on and moderately extends three branches of the literature in particular. First, Arellano (2008) has analyzed the dynamics of sovereign default under fluctuations in income, and shown that defaults are more likely when income is low[1]. Second, Cole and Kehoe (1996,2000) have pointed out that debt crises may be

[1]That may sound unsurprising, but is actually not trivial. Indeed the recursive contract literature typically implies incentive issues for contract continuation at high rather than low income states, see e.g. Ljungqvist-Sargent (2004).

self-fulfilling: the fear of a future default may trigger a current rise in default premia on sovereign debt and thereby raise the probability of a default in the first place. Both theories imply, however, that countries would have a strong incentive to avoid default-triggering scenarios in the first place. We therefore build on the political economy theories of the need for debt contraints in a monetary union of short-sighted fiscal policy makers as in to provide a rationale for a default-prone scenario, see e.g. Beetsma and Uhlig (1999) or Cooper, Kempf and Peled (2010).

We consider a bailout agency, modeled as a particularly large and infinitely lived investor and who is committed to rule out the sunspot-driven defaults of Cole-Kehoe (2000) per debt purchases, even if all other investors do not. We assume that this bailout agency seeks an actuarily fair return, and characterize the minimal intervention. The bailout agency will not prevent defaults due to fundamental reasons as in Arellano (2008) nor impose additional policy constraints such as conditionality as in e.g. Fink and Scholl (2011).

2 A model of sovereign default dynamics: no bailout agency.

This section closely follows Cole-Kehoe (2000) and Arellano (2008) and serves mainly to fix notation and assumptions. We assume that there is a single fiscal authority, which finances government consumption $c_t \geq 0$ with tax receipts $y_t \geq 0$ and assets $B_t \in \mathbf{R}$ (with positive values denoting debt, in reverse of the notation used in Arellano (2008)), in order to maximize its utility

$$U = \sum_{t=0}^{\infty} \beta^t (u(c_t) - \chi_t \delta_t) \tag{1}$$

where β is the discount factor of the policy maker, $u(\cdot)$ is a strictly increasing, strictly concave and twice differentiable felicity function, χ_t is an exogenous one-time utility cost of default and $\delta_t \in \{0, 1\}$ is the decision to default in period t. We shall assume that tax receipts y_t are exogenous[2], while consumption, the level of debt and the default decisions are endogenous and chosen by the government.

In Arellano (2008) as well as Cole and Kehoe (2000), this is the utility of the representative household, y_t is total output and c_t is the consumption of the household, i.e. the fiscal authority is assumed to maximize welfare. The structure assumed here is mathematically the same, and consistent with that interpretation. It is also consistent with our preferred interpretation, where the utility function represents the preferences of the policy maker. For example, given the uncertainty of re-election, a policy maker may discount the future more steeply than would the private sector. Spending may be on groups that are particularly effective in lobbying the government. Finally, y_t should then be viewed as tax receipts, not national income.

A more subtle difference is the cost of a default, modeled here as a one-time utility cost χ_t, while it is modelled as a fractional loss in output in Arellano (2008) with Cole and Kehoe (2000). Note, however, that $c_t = y_t$ in default, and that at least for log-preferences, $u(c_t) = \log(c_t)$, a proportional decline in consumption each period following the default can equivalently be written as a one-time loss in utility. The utility cost formulation provides a free parameter to fine-tune the quantitative implications of the baseline specification of the model: a feature that we shall exploit in the numerical analysis.

In each period, the government enters with some debt level B_t and the tax receipts y_t as well as some other random variables are realized. Traders on financial markets are assumed to be risk neutral and discount future

[2]It may be interesting to endogenize tax collection!

repayments of debt at some return R, and price new debt B_{t+1} according to some market pricing schedule $q_t(B_{t+1})$. Given the pricing schedule, the government then first makes a decision whether or not to default on its existing debt. If so, it will experience the one-time exogenously given default utility loss χ_t, be excluded from debt markets until re-entry, and simply consume its output, $c_t = y_t$ in this as well as all future periods, while excluded from debt markets. We assume that re-entry to the debt market happens with probability $0 \leq \alpha < 1$, drawn iid each period, and that re-entry starts with a debt level of zero. If the government does not default, it will choose consumption and the new debt level according to the budget constraint

$$c_t + (1-\theta)B_t = y_t + q_t(B_{t+1})(B_{t+1} - \theta B_t) \tag{2}$$

where $0 < \theta \leq 1$ is a parameter, denoting the fraction of debt that currently needs to be repaid. The parameter θ allows to study the effect of altering the maturity structure: the lower θ, the longer the maturity of government debt. The remainder of the debt θB_t will be carried forward, with the government issuing the new debt $B_{t+1} - \theta B_t$.

2.1 State space representation

We shall restrict attention to the following state-space representations of the equilibrium. At the beginning of a period, the aggregate state

$$s = (B, d, z) \tag{3}$$

describes the endogenous level of debt B, the default status d and some exogenous variable $z \in Z$. We assume that z follows a Markov process and that all decisions can be described in terms of the state s. The probability measure describing the transition for z to z' shall be denoted with $\mu(dz' \mid z)$. More specifically, we shall assume that z is given by

$$z = (y, \chi, \zeta) \tag{4}$$

We assume that $y \in [y_L, y_H]$ with $0 < y_L \leq y_H$ either has a strictly positive and continuous density $f(y \mid z_{\text{prev}})$, given the previous Markov state z_{prev}. We assume that $\chi \in \{\chi_L, \chi_H\}$ takes one of two possible values, with $0 = \chi_L \leq \chi_H$. We assume that $\zeta \in [0,1]$ is uniformly distributed and denotes a "crisis" sunspot. We assume that the three entries in z are independent of each other, given the previous state. For most parts, we shall assume that z is iid, and that therefore the distributions for y and χ also do not depend on z_{prev}. For notation, we shall use $y(s)$ to denote the entry y in the state s, etc..

If the government does not default ($\delta = 0$), the period-per-period budget constraint is

$$c + (1-\theta)B(s) = y(s) + q(B'; s)(B' - \theta B(s)) \tag{5}$$

where B' is the new debt level chosen by the government and where $q(B'; s)$ is the pricing function for the new debt B'.

If the government defaults ($\delta = 1$), the budget constraint is

$$c = y(s) \tag{6}$$

We assume that the government will be excluded from debt markets until it is given the possibility for re-entry. We assume that re-entry to the debt market happens with probability $0 \leq \alpha < 1$, drawn iid each period[3], and that re-entry starts with a debt level of zero. In that case, "good standing" $d = 0$ in the state s will be turned to "bad standing" or "in default" $d = 1$ in the state s' following a default, and that $d = 1$ is followed by $d = 1$ with probability $1 - \alpha$ and with $d = 0$ with probability α. There is no other role for d. The default decision of the government is endogenous and (assumed to be) a function of the state s, $\delta = \delta(s)$.

[3]Technically, assume that re-entry happens if $\zeta \leq \alpha$, in order to achieve dependence on the state z.

We can now provide a recursive formulation of the decision problem for the government. The value function in the default state and after the initial default utility loss is given by

$$v_D(z) = u(y(z)) + \beta(1-\alpha)E\left[v_D(z') \mid z\right] + \alpha E\left[v_N D(s' = (0, 0, z')) \mid z\right] \quad (7)$$

Given the debt pricing schedule $q(B; s)$, the value from not defaulting is

$$\begin{aligned} v_{ND}(s) = \max_{c,B'} \{ & u(c) + \beta E\left[v(s') \mid z\right] \mid \\ & c + (1-\theta)B(s) = y(s) + q(B'; s)(B' - \theta B(s)) \\ & s' = (B', d(s), z')\} \end{aligned}$$

The overall value function is given by

$$v(s) = \max_{\delta \in \{0,1\}} (1-\delta)v_{ND}(s) + \delta(v_D(z(s)) - \chi(s)) \quad (8)$$

Given parameters, a law of motion for z, an **equilibrium** is defined as measurable mappings $q(B'; s)$ in B' and s as well as $c(s), \delta(s)$ and $B'(s)$ in s, such that

1. Given the pricing function $q(B'; s)$, the government maximizes its utility with the choices $c(s), \delta(s)$ and $B'(s)$, subject to the budget constraint (5) and subject to the exclusion from financial markets for all periods, following a default.

2. The market pricing function $q(B'; s)$ is consistent with risk-neutral pricing of government debt and discounting at the risk free return R.

2.2 Debt pricing

This subsection of the analysis follows closely the analysis in Cole and Kehoe (2000) and Arellano (2008), adapted to the model at hand. Given a level of debt B and "good standing" $d = 0$, let

$$D(B) = \{z \mid \delta(s) = 1 \text{ for } s = (B, 0, z)\} \quad (9)$$

be the default set, and let

$$A(B) = \{z \mid \delta(s) = 0 \text{ for } s = (B, 0, z)\} \tag{10}$$

be the set of all z, such that the government will not default and instead, continue to honor its debt obligations: both are (restricted to be) a measurable set, according to our equilibrium definition. The disjoint union of $D(B)$ and $A(B)$ is the entire set Z. Define the market price for debt, in case of no current default, i.e.

$$\bar{q}(B'; s) = \frac{1}{R} \int_{z' \in A(B)} (1 - \theta + \theta q(B(s' = (B', 0, z')))) \, \mu(dz' \mid z) \tag{11}$$

Here and below, we use the notation $B(s' = (B', 0, z'))$ to denote the new debt level $B(s')$, given the new state $s' = (B', 0, z')$. A shorter, more accurate, but perhaps more confusing notation would simply be $B((B', 0, z'))$. Due to risk neutral discounting, this is the market price of debt, if there is no default "today". Define the probability of a continuation **next period** per

$$P(B'; s) = \text{Prob}(z' \in A(B') \mid s) = E\left[1_{\delta(s')=0} \mid s\right] \tag{12}$$

If $\theta = 0$, i.e., if all debt has the maturity of one period only, then

$$\bar{q}(B'; s) = \frac{1}{R} P(B'; s) \tag{13}$$

We need to check, whether there could be a default "today". We shall impose the following assumption.

Assumption A. 1 *Given a state s, either $q(B'; s) = \bar{q}(B'; s)$ for all B' or $q(B'; s) = 0$ for all B'.*

This assumption rules out equilibria, where, say, the market expects a current default, if the government tries to finance some future debt level B', but not for others[4]

[4] Cole and Kehoe (2000) finesse this issue with more within-period detail, having the government first sell new debt at some pricing schedule, before taking the default decision.

We now turn to analyzing the possibility for a self-fulfilling expectation of a default. Define the value of not defaulting, if the market prices are consistent with current debt repayment,

$$\begin{aligned}\bar{v}_{ND}(s) = \max_{c,B'} \{ & u(c) + \beta E\left[v(s') \mid z\right] \mid \\ & c + (1-\theta)B(s) = y(s) + \bar{q}(B'; s)(B' - \theta B(s)) \\ & s' = (B', d(s), z')\}\end{aligned}$$

where it should be noted that the continuation value function is as before, i.e. given by (8). Define the value of not defaulting, if the market prices are consistent with a current default,

$$\begin{aligned}\underline{v}_{ND}(s) = \max_{c,B'} \{ & u(c) + \beta E\left[v(s') \mid z\right] \mid \\ & c + (1-\theta)B(s) = y(s) \\ & s' = (B', d(s), z')\}\end{aligned}$$

With that, define two bounds for the current debt levels B, see also figure 18. Above the upper bound $B \geq \bar{B}(z)$, the government finds it optimal to default today, even if the market was willing to finance future debt in the absence of a default now, i.e. even if $q(B'; s) = \bar{q}(B'; s)$. Above the lower bound $B \geq \underline{B}(z)$, the government finds it optimal to default, if the market thinks it will do so and therefore is unwilling to finance further debt, $q(B'; s) = 0$. I.e., let

$$\bar{B}(z) = \inf\{B \mid \bar{v}_{ND}(s = (B, 1, z)) \leq v_D(z(s)) - \chi(s = (B, 1, z))\} \qquad (14)$$

as well as

$$\underline{B}(z) = \inf\{B \mid \underline{v}_{ND}(s = (B, 0, z)) \leq v_D(z(s)) - \chi(s = (B, 0, z))\} \qquad (15)$$

Whether or not there will be a default at some debt level B between these bounds will be governed by the sunspot random variable ζ. As in Cole-Kehoe (2000), we shall assume that the probability of a default in this range is some exogenously given probability π.

Assumption A. 2 *For some parameter $\pi \in [0,1]$, and all s with $\underline{B}(z) \leq B(s) \leq \bar{B}(z)$, we have $q(B';s) = \bar{q}(B';s)$, if $\zeta(s) \geq \pi$ and $q(B';s) = 0$, if $\zeta < \pi$.*

Note that the assumption relates endogenous objects to each other.

The equilibrium will therefore look as follows (up to breaking indifference at the boundary points):

1. If $B > \bar{B}(z)$, the government will default now and not be able to sell any debt. The market price for new debt will be zero.
2. If $\underline{B}(z) \leq B \leq \bar{B}(z)$, the government will
   (a) default with probability π (more precisely, for $\zeta(z) < \pi$), and the market price for new debt will be zero,
   (b) continue with probability $1-\pi$ (more precisely, for $\zeta(z) \geq \pi$), and the market price for new debt will be $\bar{q}(B';s)$.
3. If $B < \underline{B}(z)$, the government will not default, and the market price for debt will be given by $\bar{q}(B';s)$.

Following Cole and Kehoe (2000), we shall use the term "crisis zone" for the maximal range for new debt, for which there might be a "sunspot" default next period, i.e. for

$$B' \in \mathcal{B} = \left[\min \underline{B}(z), \max \bar{B}(z)\right]$$

Note that safe debt will be priced at q^* satisfying

$$q^* = \frac{1}{R}(1 - \theta + \theta q^*)$$

and is therefore given by

$$q^* = \frac{1-\theta}{R-\theta} \tag{16}$$

Conversely, given some price q, one can infer the implicit equivalent safe rate

$$R(q) = \theta + \frac{1-\theta}{q} \tag{17}$$

To denote the dependence of the equilibrium on the sunspot parameter π or the dependence on the debt duration parameter θ, we shall use them as superscripts, if needed. Some analysis for the no-bailout case and some insights into the stationary distribution of debt and their dependence on the discount factor are in appendix B.

3 Bailouts

We now introduce the possibility for a bailout per a large and infinitely lived, risk neutral outside investor. More precisely, we envision a facility with sufficiently deep pockets (backed by, say, governments other than the one under consideration here), which aims at ensuring the selection of the "good" equilibrium, while earning the market rate of return in expectation on its bond holdings. I.e., we imagine that this bailout facility insists on actuarily fair pricing. It may well be that actual policy interventions amount to a subsidy or perhaps even a penalty. We view the actuarily fair "restoration-of-the-good-equilibrium" as an important benchmark. It might be interesting to consider other mechanisms, which are not actuarily fair, as well, and we do so in the appendix C. An alternative is to examine the conditionality of such bailouts, combining help with insistence on fiscal discipline, see Fink-Scholl (2011).

If the bailout facility buys the entire debt, then the solution is easy in principle. It should calculate the $\pi = 0$-equilibrium described above, price debt accordingly, and let the country choose the debt level it wants, given this pricing schedule. Since the bailout facility is always there, also in the future, to guarantee the "good" equilibrium, the pricing is actuarily fair.

There is generally no need to buy the entire debt, however, in order to assure the $\pi = 0$ equilibrium. We therefore assume a minimal bailout facility. I.e., we characterize the minimal level of debt $B'_a(s)$ such a facility needs to guarantee buying at the $\pi = 0$ equilibrium price, so that markets must coordinate on this equilibrium. We assume that the facility buys at the $\pi = 0$ equilibrium price, even if the rest of the market does not buy at all: this is only relevant "off-equilibrium". It is important in this construction, that the debt held by the facility is treated the same as the debt held by market participants. The country is indifferent between purchasing this debt from the facility or from the market, and so is the market. The guarantee just needs to be there, in the (now hypothetical) case that the market coordinates on the default outcome.

To characterize the minimal guarantee level $B'_a(s)$, we need to re-examine and slightly modify the value function of the government. We need an assumption about the continuation in the case that the market does not buy, and whether the buyers' strike persists or not. In order to truly characterize the minimal intervention, we make the "optimistic" assumption that a potential buyer's strike only lasts for one period, i.e., given the presence of the large investor, the continuation value following a no-default today shall be given by the value function valid for the $\pi = 0$ equilibrium. Given the policy $B'_a(s)$, define the no default value under assistance (and current buyers' strike, except for the large investor) as

$$\begin{aligned}
\bar{v}_{ND;a}(s) = \max_{c,B'} \{ & u(c) + \beta E\left[v^{(\pi=0)}(s') \mid z\right] \mid \\
& c + (1-\theta)B(s) = y(s) + q^{(\pi=0)}(B';s)(B' - \theta B(s)) \\
& B' \leq B'_a(s) \\
& s' = (B', d(s), z')\}
\end{aligned} \tag{18}$$

Note the second constraint, encapsulating the limit of the assistance. Let $\epsilon > 0$ be a parameter and small number to break indifference. Given $q^{(\pi=0)}$

and $v^{(\pi=0)}$, one can therefore solve for $B_a'(s)$ "state by state" such that

$$\underline{v}_{ND}(s=(B,0,z)) = v_D(z(s)) - \chi(s=(B,0,z)) + \epsilon \text{ for all } 0 \leq B \leq \bar{B}(z) \tag{19}$$

where $\bar{B}(z)$ is the maximum level of current debt consistent with no default in the $\pi = 0$ equilibrium. For $B > \bar{B}(z)$, define $B_a'(s) = 0$, but do note, that $q(B'; s) = 0$ for any $B' > 0$ per definition of $\bar{B}(z)$. In other words, the facility could also provide the (meaningless) guarantee of willing to buy any positive level of debt $B_a'(s)$ at a zero price.

Proposition 1 *Suppose $B_a'(s)$ satisfies (19). Then, $\underline{B}(z) = \bar{B}(z)$, i.e, there will not be a default, unless debt exceeds $\bar{B}(z)$.*

Proof: *to be completed.* •

In the iid case and with a constant embarrassment utility costs $\chi > 0$ of defaulting, a bit more can be said. In that case, some constant value $\beta\tilde{v}_D$

$$\beta E[v_D(z')] \equiv \beta\tilde{v}_D$$

is the continuation value from defaulting. Likewise, when receiving the full guarantee $B_a'(s)$, the continuation value of not defaulting is $\beta\tilde{v}_{ND}(B_a'(s))$, given by

$$\beta E[v(B_a'(s), 0, z')] = \beta\tilde{v}_{ND}(B_a'(s))$$

Criterion (19) becomes

$$\begin{aligned} u(y(s)) - u\left(y(s) + q^{(\pi=0)}(B_a'(s); s)\left(B_a'(s) - \theta B(s)\right) - (1-\theta)B(s)\right) \\ = \beta\tilde{v}_{ND}(B_a'(s)) - \beta\tilde{v}_D - \chi - \epsilon \end{aligned} \tag{20}$$

comparing the current utility gain from defaulting to the utility continuation loss from defaulting, including the embarrassment cost χ.

Proposition 2 *In the iid and constant-χ case, we have*

1. *For two states s_1, s_2, if $B(s_1) > B(s_2)$, then $B'_a(s_1) \geq B'_a(s_2)$.*
2. *If $B(s) > 0$, then*

$$q^{(\pi=0)}(B'_a(s); s)\,(B'_a(s) - \theta B(s)) < (1-\theta)B(s)$$

3. *For two states s_1, s_2, if $y(s_1) > y(s_2)$, then $B'_a(s_1) \leq B'_a(s_2)$.*
4. *For two states s_1, s_2, if $\chi(s_1) > \chi(s_2)$, then $B'_a(s_1) \leq B'_a(s_2)$.*

Proof: *To be completed (and perhaps modified). The first part appears to be obvious. The second part is a version of proposition 2 in Arellano (2008), but may need some additional assumptions. The third follows from the second.* •

4 A numerical example

This section presents the results of a numerical exercise, where the model is solved using value function iteration, see appendix A for more details. First we discuss the functional forms and parametrization, and then we give the results.

The government's within period utility function has the CRRA form

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

We assume that the income process is a log-normal autoregressive process with unconditional mean μ

$$\log(y_{t+1}) = (1-\rho)\,\mu + \rho \log(y_t) + \varepsilon_{t+1}$$

Government's risk aversion	σ	1/2
Interest rate	r	3.0%
Income autocorrelation coefficient	ρ	0.945
Standard deviation of innovations	σ_ϵ	3.4%
Mean log income	μ	$(-1/2)\sigma_\epsilon^2$
Exclusion	α	0.2
Maturity structure	θ	0.8
Discount factor	β	0.4
Cost	χ_L	0
Cost	χ_H	0.5
SFC sunspot probability	π	0.05
Income grid	$y_1, \ldots, y_{20}$	$[0.73, \ldots, 1.37]$
debt grid	$B_1, \ldots, B_{1000}$	

Table 1: Parameter values for the calibration. One period is one year.

	Target	$\theta = 0.8$
Debt/Tax ratio	2 .. 3	2.4
Default rate	5% .. 8%	6.6%

Table 2: *Targets and numerical results for the debt/tax ratio and the default rate*

with $E(\varepsilon) = 0, E(\varepsilon^2) = \sigma_\varepsilon^2$.

A period in the model refers to a year. Table 1 summarizes the key parameters used in this exercise. Additionally, as transition matrix between the two χ-states, we choose

$$\begin{bmatrix} 0 & 1 \\ 0.04 & 0.96 \end{bmatrix}$$

Both the value for χ_H as well as the transition probability from χ_H to χ_L was chosen after some experimentation to hit two target properties. First, we aimed at a debt-to-tax ratio somewhere between two and three. Second, we aimed at default rates between 5 and 8 percent. While it tends to be hard to hit these numerical targets with, say, the assumption that the only penalty to default is higher consumption variability, it is comparatively easy to do it here, with these two additional free parameters, see table 2.

Table 3 shows the "anatomy" of defaults. One can see that 12 percent of the defaults happen due to fundamental problems, even with a "responsible" χ_H government and despite buyers willing to buy the bonds in principle. However, nearly half of all defaults occur due to a buyers' strike: it is these occurrences which the bailout agency shall help to avoid.

Figure 1 shows the resulting crisis zones. Figure 2 shows the debt purchase assistance policy by the bailout agency. Over a fairly narrow range, the guaranteed purchases quickly rise until they reach 100%. At that point, the

	Buyers present	Buyers' strike
χ_L	38%	2%
χ_H	12%	48%

Table 3: *The structure of defaults.*

risk and incentive of a default due to fundamental reasons tomorrow is so large, that the failure to sell a small fraction of the new debt will be enough to trigger a default. If the current debt is even higher, the fundamental debt price collapses all the way to zero, and so does the bailout guarantee. The country will not be willing to repay or will be unable to repay in the future, and purchasing debt at any positive price will result in expected losses. Figure 3 shows the dependence of this policy on income. With currently higher income, it may well be worth guaranteeing debt purchases, that would lead to default at lower income levels. In other words, the bailout agency should rather support the country during a boom than a recession. This result may be counterintuitive from a policy perspective, but surely makes sense from the perspective of a risk-neutral investor.

Table 4 shows the impact of varying the maturity of debt. As the maturity of debt is increased, the threat from a buyers strike in any given period declines, as an ever smaller fraction of the debt needs to be rolled over. As a result, the incentive to maintain higher debt levels rises, and not much changes with the default rates, as the overall result, while the length of the crisis zones shrink. These results are graphically represented in figures 4,5 and 6. The corresponding shift in the debt purchase assistance policy is shown in 7.

Table 5 shows that the change in the sunspot probability π for a buyers strike has only a modest impact on the overall default probability, while the debt level increases. With the fear of a default due to buyer's strike

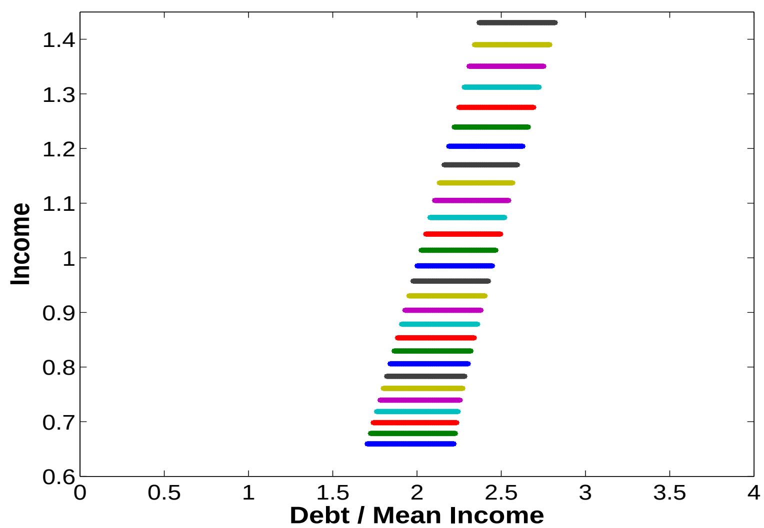

Figure 1: *Crisis zones*

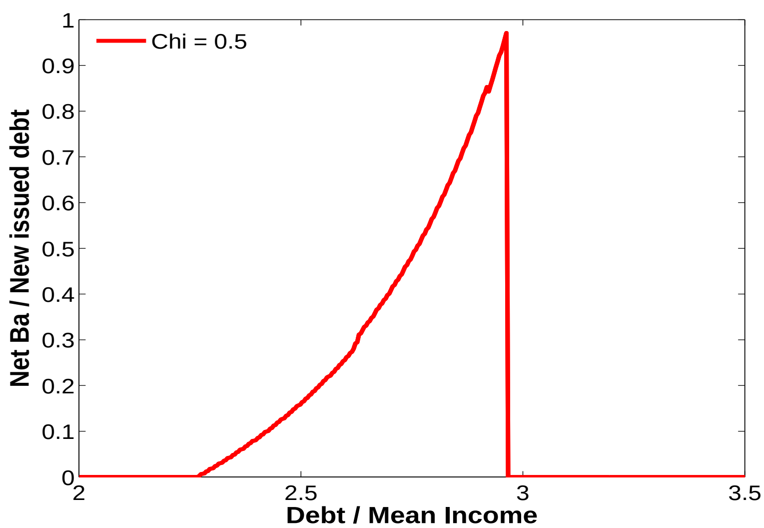

Figure 2: *Debt purchase assistance policy by the bailout agency.*

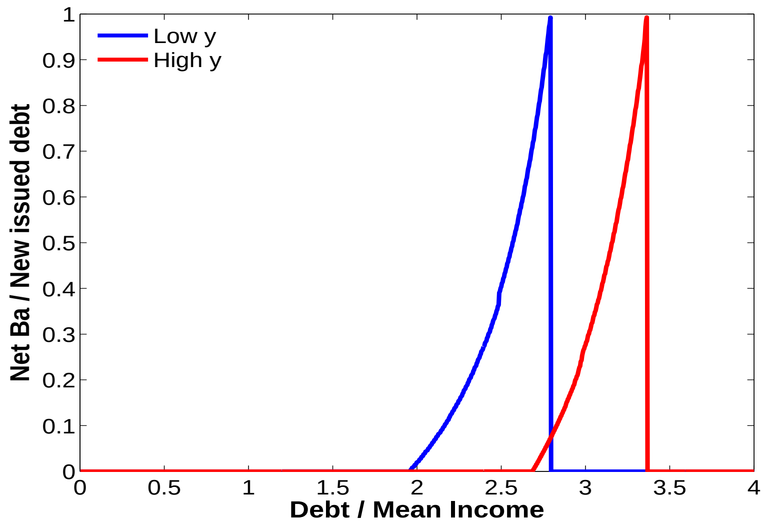

Figure 3: Income and debt purchase assistance

Targets:

	Target	$\theta = 0.9$	$\theta = \mathbf{0.8}$	$\theta = 0.5$	$\theta = 0$
Debt/Tax ratio	2 .. 3	3.3	**2.4**	1.8	1.6
Default rate	5% .. 8%	6.6%	**6.6%**	6.2%	6.2%

Defaults: $\theta = 0.9$:

	Buyers present	Buyers' strike
χ_L	38%	2%
χ_H	16%	44%

Defaults: $\theta = 0$:

	Buyers present	Buyers' strike
χ_L	42%	2%
χ_H	2%	54%

Table 4: *Variations in maturity and their impact on defaults.* $\theta = 0$ *is one-period debt, whereas* $\theta = 0.9$ *is essentially 10-period debt.*

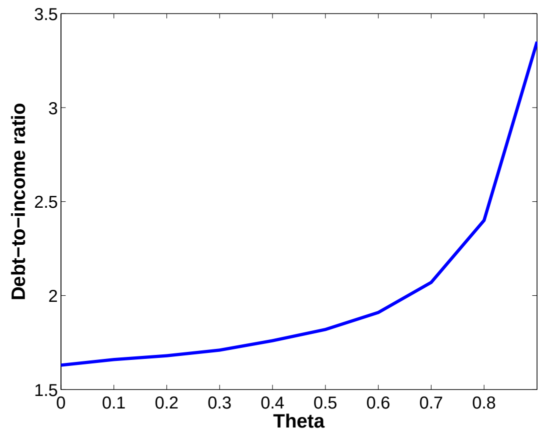

Figure 4: *Debt and* θ

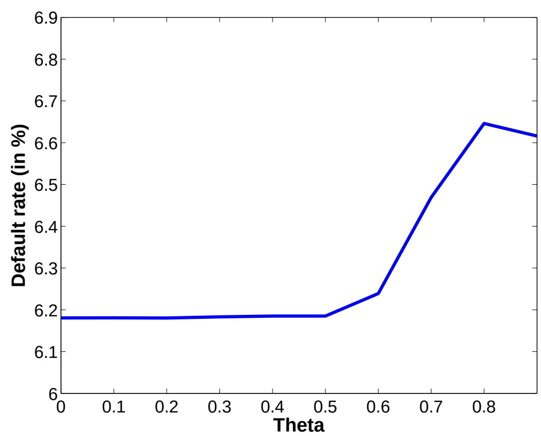

Figure 5: *Default and* θ

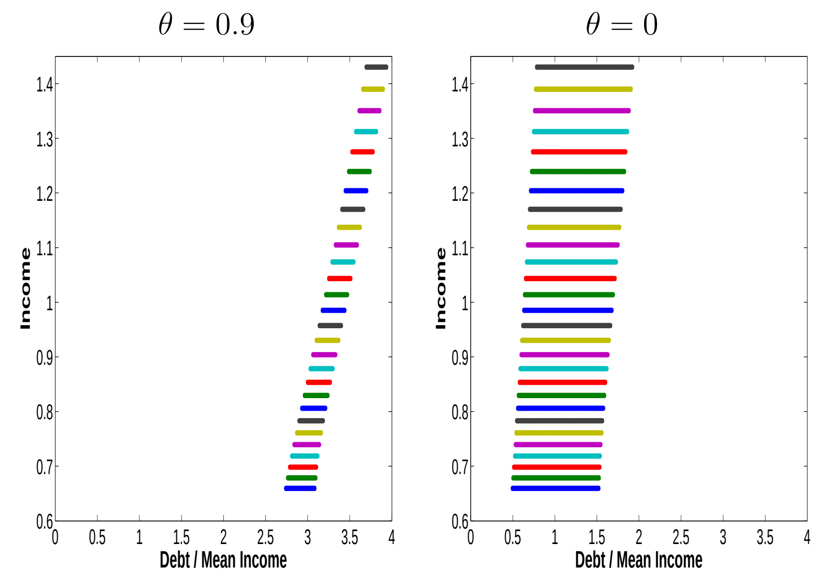

Figure 6: *Maturity and Crisis Zones*

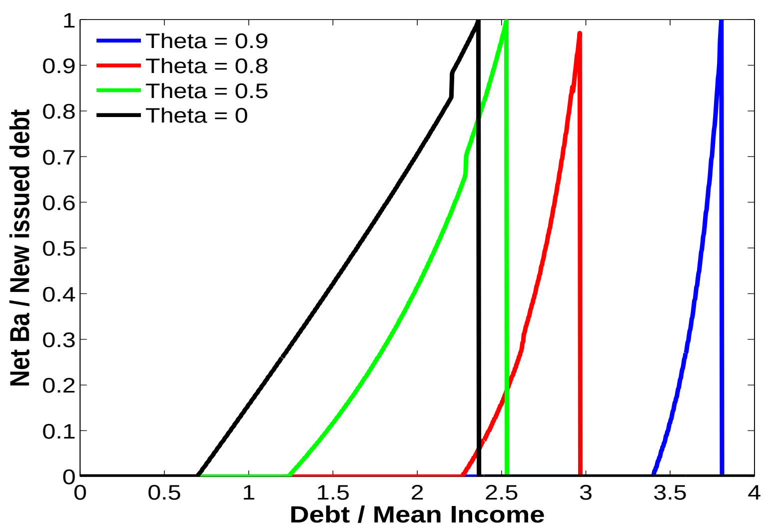

Figure 7: *Maturity and debt purchase assistance*

	Target	$\pi = 0.2$	$\pi = 0.1$	$\pi = \mathbf{0.05}$	$\pi = 0$
Debt/Tax ratio	2 .. 3	1.8	2.1	**2.4**	2.9
Default rate	5% .. 8%	5%	8%	**6.6%**	4%

Table 5: *Sunspot probabilities and debt levels*

Defaults for $\pi = 0.1$: total prob = 8%:

	Buyers present	Buyers' strike
χ_L	27%	3%
χ_H	8%	62%

Defaults for $\pi = 0.05$ (Benchmark): total prob = 6.6%:

	Buyers present	Buyers' strike
χ_L	38%	2%
χ_H	12%	48%

Defaults for $\pi = 0$:total prob = 4%:

	Buyers present	Buyers' strike
χ_L	81%	0%
χ_H	19%	0%

Table 6: *Sunspot probabilities and default details*

gone, debt becomes more attractive. Indeed, as table 6 shows, the default probability mass now shifts from the "buyer strike" scenario to the default due to fundamental reasons. Graphical representations of these relationships are in figures 8 and 9. There is a conundrum for the bailout agency here. As that agency is successful in reducing the sunspot default probability from, say, 20 percent to zero percent, the overall default rates only decline modestly from 5% to 4%. In some ways, the problem gets postponed: the government gets a bit more time to accumulate more debt. As far as default rates are then concerned after this transition, not much will have changed.

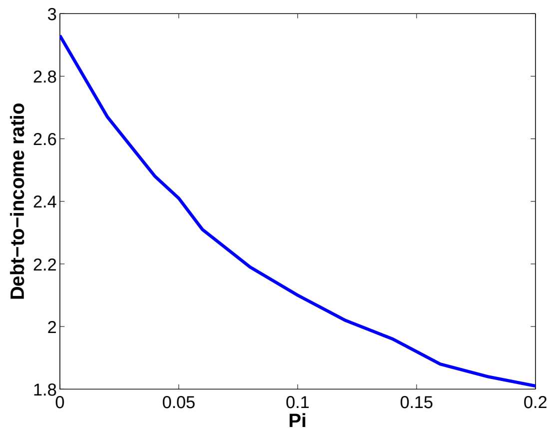

Figure 8: *Debt and* π

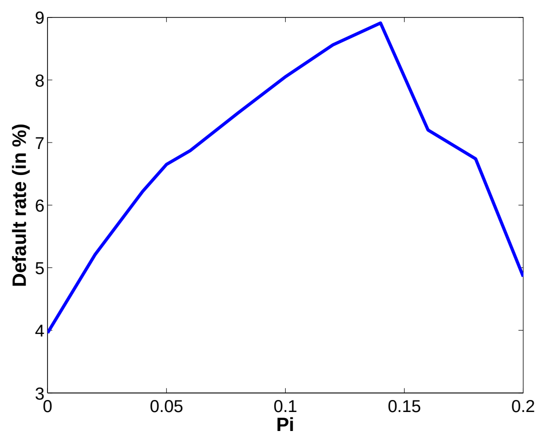

Figure 9: *Default and* π

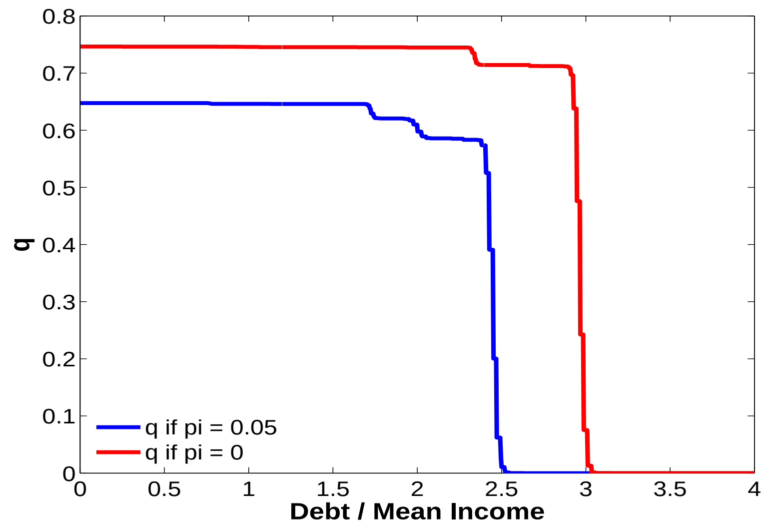

Figure 10: *Debt pricing function,* $\pi = 0.05$ *vs* $\pi = 0$.

Figure 10 shows the pricing function for debt at our benchmark value for θ, while 11 shows the pricing function for the somewhat more intuitive case of $\theta = 0$, i.e. one-period debt. Indeed, debt prices rise and thus yields decline, as the bailout agency assures the $\pi = 0$ equilibrium through its purchase guarantees. The resulting debt buildup is rather fast, as figure 12 shows. Figures 13, 14 and 15 show how the stationary debt distribution is shifted to the right, inducing the higher occurrences of defaults due to fundamental reasons. A graphical representation of the decision rules underlying the increased debt accumulation under debt purchase assistance is shown in figure 16: the decision rule shifts upwards, indicating a larger willingness of the government to incur debt.

5 Conclusions

We have analyzed the dynamics of sovereign debt defaults, drawing on insights from three literatures, particularly Arellano (2008), Cole-Kehoe (2000)

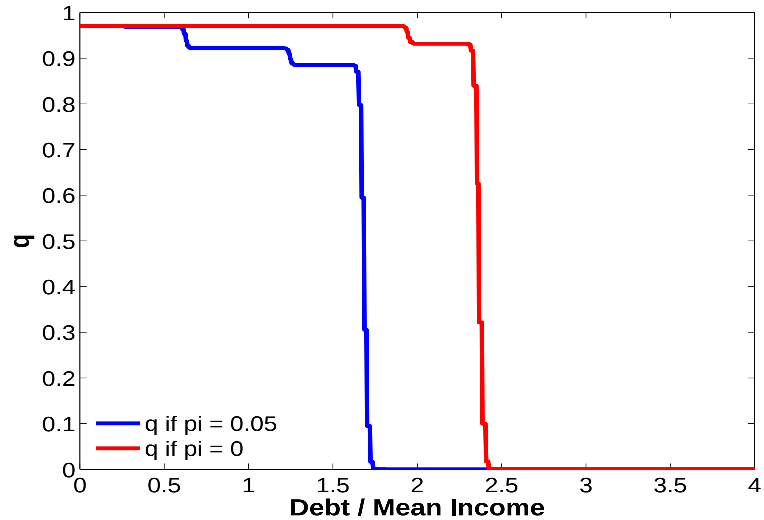

Figure 11: *Debt pricing function,* $\pi = 0.05$ *vs* $\pi = 0$*, when* $\theta = 0$*.*

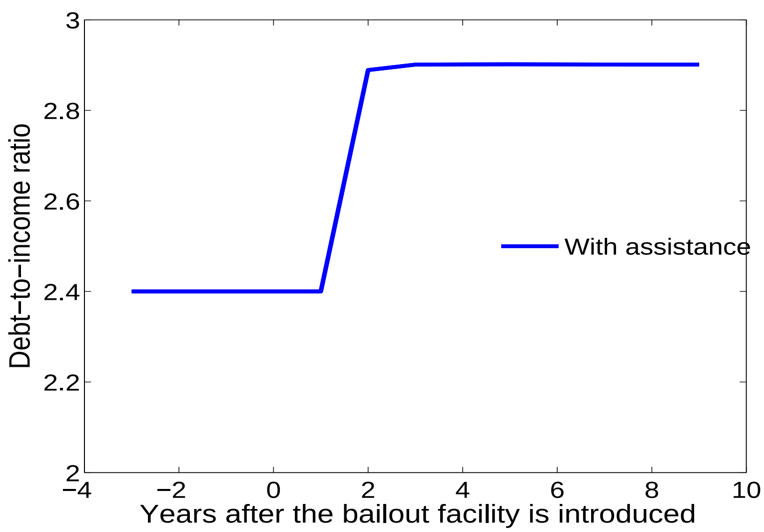

Figure 12: *Debt dynamics after the assistance facility is introduced. Starting point:* $\pi = 0.05$*, mean income, mean debt/gdp ratio.*

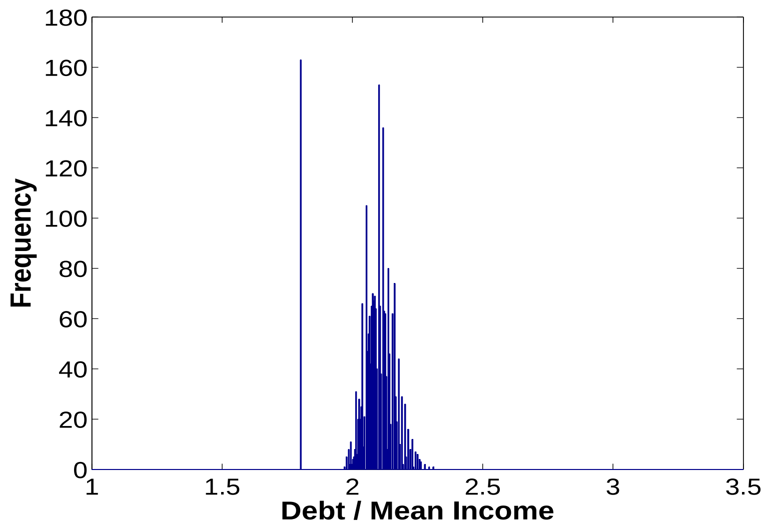

Figure 13: *Debt Distribution with sunspots:* $\pi = 0.1$

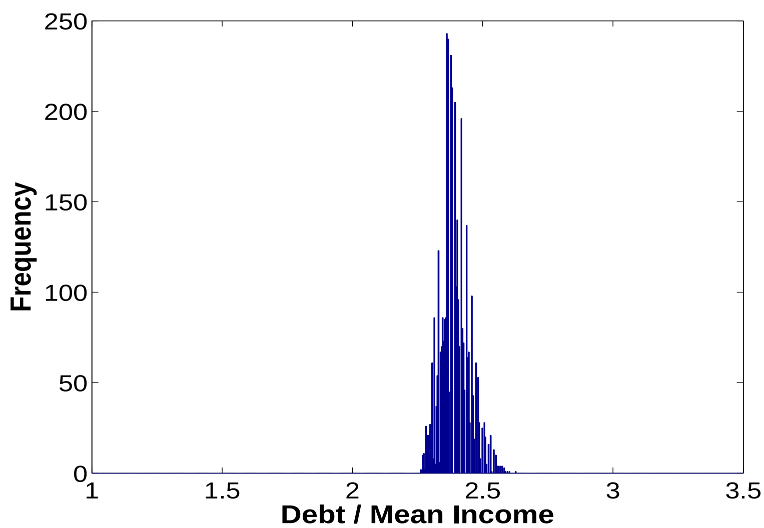

Figure 14: *Debt Distribution with sunspots:* $\pi = 0.05$

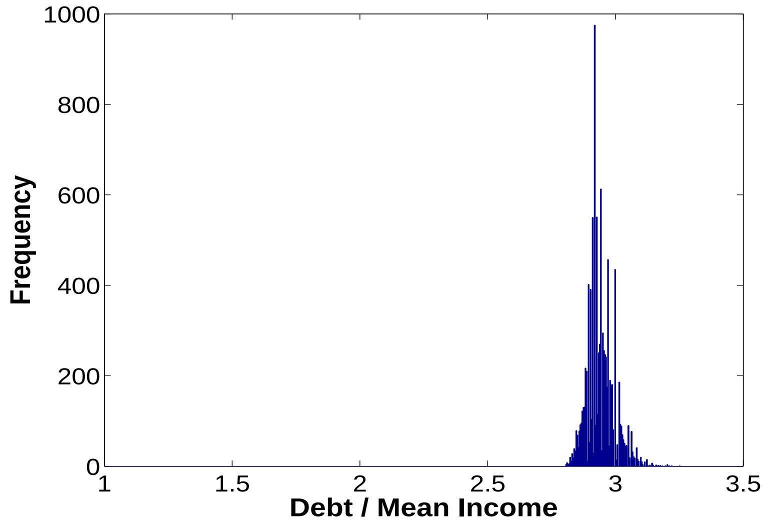

Figure 15: *Debt Distribution without sunspots or with debt purchase assistance:* $\pi = 0$

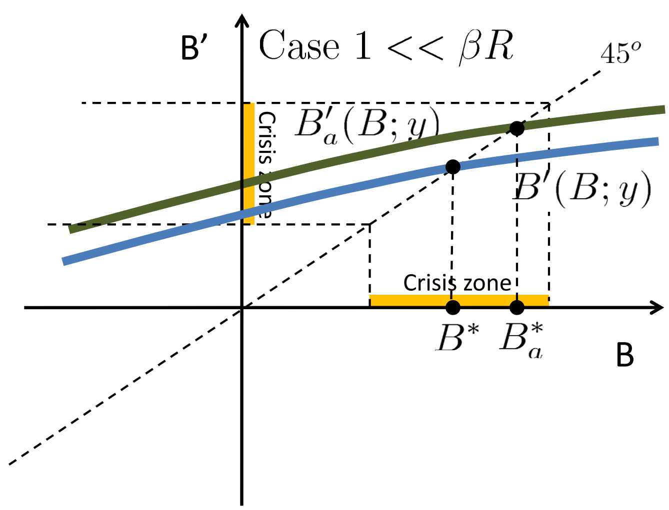

Figure 16: *Stationary debt dynamics, permanent assistance*

and Beetsma-Uhlig (1999). More precisely, we have analyzed the dynamics of sovereign debt, when politicians discount the future considerably more than private markets and when there are possibilities for both a "sunspot-"driven default as well as a default driven by worsening of economic conditions or weakening of the resolve to continue with repaying the country debt.

We have shown how this can lead to a scenario, where the country perches itself in a precarious position, with the possibility of defaults imminent. We characterized the minimal actuarily fair intervention that restores the "good" equilibrium of Cole-Kehoe, relying on the market to provide residual financing.

Three messages and conclusions emerge. First, an actuarily fair bailout agency may be able to restore the "fundamentals-only" equilibrium, by issuing debt purchase guarantees and without incurring losses in expectation. Second, these guarantees need to go far enough, but not too far. Defaults due to fundamental reasons still lurk around the corner, and excessive debt purchase guarantees would then invariably lead to losses for the bailout agency. Third, the overall default rates may not change much, as the higher guarantees and the lower yields mean that the current government can relax a bit in its efforts to repay its debt level and incur more deficits instead. The resulting higher debt levels in the future will then make future defaults inevitable on occasions, but this time due to fundamental reasons rather than buyers' strike.

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A Computational algorithm

We solve the model numerically using a discrete state space method similar to Aguiar and Gopinath (2006). I discretize the endowment space into 15 equally spaced grids, and the asset space into 300 grids.

The computational algorithm consists of the following value function iteration:

1. Assume an initial bond price schedule $q^0 = \frac{1}{1+r}$.
2. Use this price function and initial guess for the value functions to solve for the optimal value functions and policy functions.
3. Update the bond price schedule and repeat the previous steps until the price functions converge.

B No bailouts: analysis

In this section, we exclude assisted debt issuance, i.e. we assume that $q_a(B'; s) \equiv 0$. We therefore furthermore assume, that the bailout sunspot $\psi(s)$ is "irrelevant", i.e. all functions are independent of ψ: it may not be necessary to assume so, but it seems unnecessary to consider it. We finally shall assume that z is iid.

The following results are essentially in Arellano (2008) and states that default incentives increase with higher debt.

Proposition 3 *Suppose z is iid and that all functions are independent of ψ. If default is optimal for $s^{(1)} = (B^{(1)}, 0, z)$, then default is optimal for $s^{(2)} = (B^{(2)}, 0, z)$, whenever $B^{(2)} > B^{(1)}$.*

This is proposition 1 in Arellano (2008).

The next proposition states that lower tax receipts y increases default incentives.

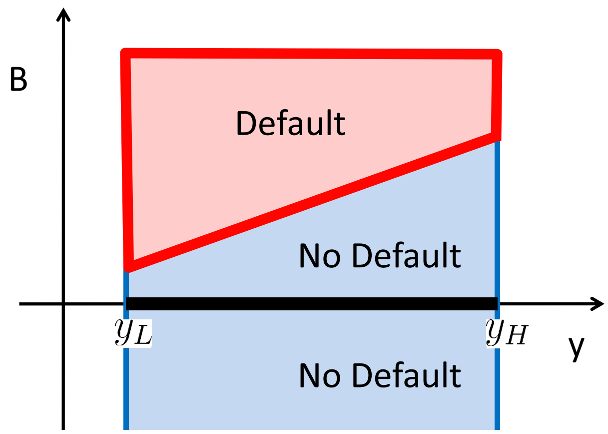


Figure 17: *Relationship between debt, income and the default decision, at a given pricing function* $q(B'; s)$

Proposition 4 *Suppose z is iid and that all functions are independent of ψ. Default incentives are stronger, the lower are tax receipts. I.e., for all $y^{(1)} \leq y^{(2)}$, if $z^{(2)} = (y^{(2)}, \chi, \zeta, \psi) \in D(B)$, then so is $z^{(1)} = (y^{(1)}, \chi, \zeta, \psi) \in D(B)$.*

This is the non-trivial insight and proposition 3 in Arellano (2008) and follows similarly from the concavity of $u(\cdot)$. A graphical representation is in figure 17. In that figure, a pricing function $q(B'; s)$ is taken as given. We are typicallyk considering two pricing functions in particular. Due to the possibility of a sunspot, the pricing function may be $q = \bar{q}_m(B'; s)$ or $q \equiv 0$. The latter results in a larger default set in the latter case. A graphical representation is in figure 18.

By comparison to proposition 4, the next proposition is certainly more trivial and obvious, and states that less "shame" χ of defaulting results in higher incentives to default.

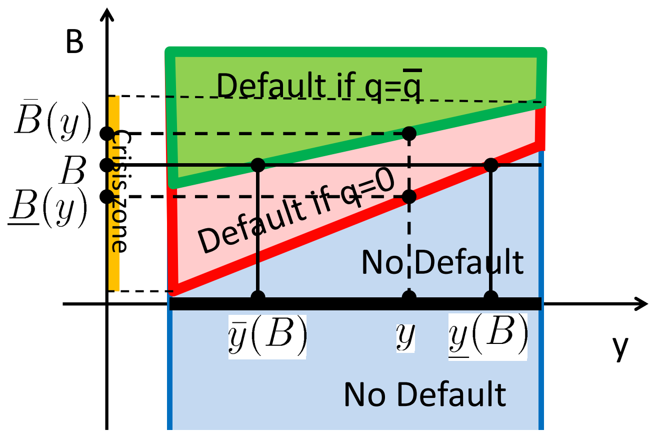


Figure 18: *Relationship between debt, income and the default decision, for the two pricing functions* $q = \bar{q}_m(B'; s)$ *and* $q \equiv 0$

Proposition 5 *Suppose* z *is iid and that all functions are independent of* ψ*. Default incentives are stronger, the lower is the utility penalty from defaulting. I.e., for all* $\chi^{(1)} \leq \chi^{(2)}$*, if* $z^{(2)} = (y, \chi^{(2)}, \zeta, \psi) \in D(B)$*, then so is* $z^{(1)} = (y, \chi^{(1)}, \zeta, \psi) \in D(B)$*.*

With these results, we can derive the dependence of the pricing function on the debt level.

Proposition 6 *Suppose that* $q_a(B'; s) \equiv 0$*, i.e. no bailouts. Then* $q(B'; s)$ *is decreasing in the debt level* B'*. If* y *and/or* χ *is random with a strictly positive and continuous density, then* $q(B'; s)$ *is continuous in* B' *with a nonpositive derivative in* B'*, except for finitely many points.*

Proof: *To be completed. Note, that changes in* B' *"smoothly" move into the default areas, when* y *and/or* χ *is random with a strictly positive and continuous density.* •

A graphical representation of the pricing function $q = \bar{q}_m(B'; s)$ is in figure 19 for the case of $\theta = 0$, i.e. one-period bonds. If the next period debt level is below the lowest level, at which a default could possibly be expected, $B' \leq \min \underline{B}(z)$, then the debt is safe and will be discounted at R. As B' increases beyond this level, there will be some states of nature in the future, for which a default may occur: these defaults become gradually more likely with increases in B', as one can infer from figure 18. Once the debt level is so high, that a default must surely occur tomorrow, then the current price level must be zero as well. The pricing function depends on the sunspot default probability tomorrow in a subtle way, as figure 20 shows. With a zero probability of a "sunspot" default, the debt B' needs to exceed $\min \bar{B}(z)$ in order for the price $\bar{q}_m(B'; s)$ to decline. Indeed, $\bar{B}(z)$ itself depends on π and should intuitively rise, as π falls (since q is shifting upwards): this is indicated by the shift also of $\max \bar{B}(z)$ in that figure.

It is useful to analyze the first-order condition of the government, when considering its choice for the future debt level B', assuming that the debt pricing rule is "sufficiently nice". Define the level of consumption, resulting from a particular debt choice B',

$$c(B'; s) = y(s) + q(B'; s)(B' - \theta B(s)) - (1 - \theta)B(s) \tag{21}$$

At the optimal choice, $B' = B'(s)$ and $c(B'; s) = c(s)$. From there, consider marginally increasing the amount of debt B' . This yields a current utility gain

$$\left(\frac{\partial U}{\partial B'}\right)_{(I)} = u'(c(s)) \left(q(B'; s) + q_1(B'; s)B'\right) \tag{22}$$

Per the envelope theorem for v_{ND}, i.e. conditional on a state s' of no default, the utility loss tomorrow is given by

$$\frac{\partial v_{ND}(s')}{\partial B'} = \beta u'(c(s'))(\theta - 1 - \theta q(B''(s'); s')) \tag{23}$$

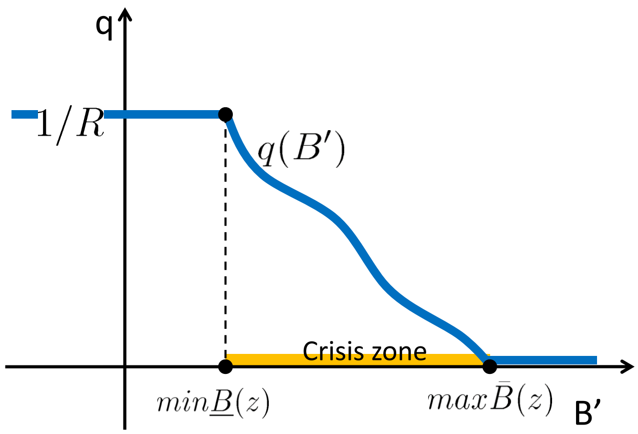


Figure 19: *The market price* $q(B') = \bar{q}_m(B'; s)$ *as a function of future debt* B'.

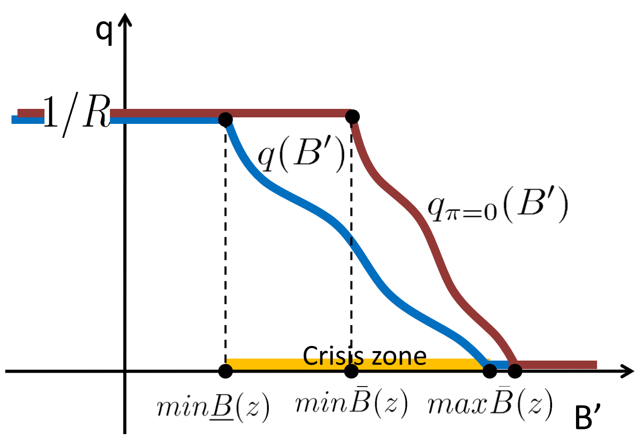


Figure 20: *The market price* $q(B') = \bar{q}_m(B'; s)$ *for nonzero "sunspot" default probability* π *as well as for* $\pi = 0$.

where we have used the hopefully intuitive notation $B''(s')$ to denote the debt choice next period, given next periods state s', instead the of the formally correct but possibly confusing notation $B'(s')$. Integrating the losses given by (23) yields

$$\left(\frac{\partial U}{\partial B'}\right)_{(II)} = \beta\pi \int_{\{z|B' \leq \underline{B}(z)\}} u'(c(s' = (B', 0, y, \chi, 0, 0)))(1 - \theta + \theta q(B''(s'); s'))\mu(dz)$$
$$+\beta(1-\pi) \int_{\{z|B' \leq \bar{B}(z)\}} u'(c(s' = (B', 0, y, \chi, 1, 0)))(1 - \theta + \theta q(B''(s'); s'))\mu(dz)$$
$$= \quad \beta E\left[u'(c(s'))(1 - \theta + \theta q(B''(s'); s'))1_{\delta(s')=0}\right] \tag{24}$$

where we have set $\zeta = 0$ and $\zeta = 1$ for the two crisis sunspot situations, and arbitrarily fixed $\psi = 0$.

However, the set of default states changes. To keep the analysis tractable, suppose that χ is not random but constant, while the distribution for y has a nontrivial, strictly positive and bounded density $f(y) = F'(y)$ on $[y_L, y_H]$. With the help of proposition 4, the condition $B \leq \underline{B}(z)$ can equivalently written as $y \geq \underline{y}(B)$, while the condition $B \leq \bar{B}(z)$ can equivalently written as $y \geq \bar{y}(B)$ for some bounds $\bar{y}(B) \leq \underline{y}(B)$. Additionally, there is then the net loss in utility due to increasing the risk of default (or, technically, the differentiation with respect to the boundary of the integral),

$$\left(\frac{\partial U}{\partial B'}\right)_{(III)} = \beta\pi \left(v_{ND}(B', 0, \underline{y}(B'), \chi, 0, 0) + \chi - v_D(\underline{y}(B'), \chi, 0, 0)\right) f(\underline{y}(B'))\frac{d\underline{y}(B')}{dB'}$$
$$+\beta(1-\pi)\left(v_{ND}(B', 0, \bar{y}(B'), \chi, 1, 0) + \chi - v_D(\bar{y}(B'), \chi, 0, 0)\right) f(\bar{y}(B'))\frac{d\bar{y}(B')}{dB'}$$

Note now, though, that the boundaries are defined by the condition that the expression in brackets equals zero, unless we are at the boundary of the interval $[y_L, y_H]$ and therefore the derivative of $\underline{y}(B')$ or of $\bar{y}(B')$ with respect to B' is zero.

The argument regarding this third part generalizes, in case χ is random too. we note this result as follows.

Proposition 7 *If the condition for optimality can be written as a first-order condition, it is*

$$\left(\frac{\partial U}{\partial B'}\right)_{(I)} = \left(\frac{\partial U}{\partial B'}\right)_{(II)} \tag{25}$$

where the two pieces are given by (22) and (24). Put differently,

$$q(B';s) + q_1(B';s)B' = \beta E\left[\frac{u'(c(s'))}{u'(c(s))}(1-\theta+\theta q(B''(s');s'))1_{\delta(s')=0}\right] \tag{26}$$

If $\theta = 0$ *(only short-term debt), then*

$$q(B';s) + q_1(B';s)B' = \beta E\left[\frac{u'(c(s'))}{u'(c(s))}1_{\delta(s')=0}\right] \tag{27}$$

or

$$1 - h(B';s)B' = \beta R E\left[\frac{u'(c(s'))}{u'(c(s)} \mid \delta(s')=0\right] \tag{28}$$

where the hazard rate $h(B';s)$ *is given by*

$$h(B';s) = -\frac{\partial E\left[\delta(s')=0\right]/\partial B'}{E\left[\delta(s')=0\right]} \tag{29}$$

Proof: *For equation (27), note that* $q(B';s) = E\left[\delta(s')=0\right]/R$. •

There is an important tension here. Consider $\theta = 0$ and the first order condition (27). When increasing the debt level, the usual "consumption-versus-savings" first-order effect ought to be an increase in current consumption and a decrease in future consumption, leading to an decrease in current marginal utility and an increase in future marginal utility, resulting in some optimal level. This is offset by the decrease in resources gained per additional unit of debt on the left-hand side, due to the decrease in q_1 and the decrease in the no-default region on the right-hand side. It is not a priori clear, that there is a unique solution. Put differently, it is not a priori clear and perhaps even unlikely, that the budget set (5) is convex in the choices (c, B').

This issue and the first-order condition (27) are examined in figure 21: we shall focus entirely on the case $\theta = 0$, though this discussion can probably be generalized. The left column shows the "benign" case. In the upper left panel, the market price for new debt $q(B')$ declines at a reasonably even pace, so that the left hand side in equation (27) is monotonously decreasing, and even becomes negative, until debt reaches $\max_z \bar{B}(z)$. That left hand side is then compared to the rhs of (27) in the lower left panel. For the figure, it has been assumed that the rhs is rising in B': as discussed, even that may not be the case. The two curves intersect at a unique point. The right column shows one possible scenario, where multiple solutions to the first-order condition may emerge. Start from the upper right panel: there, $q(B')$ becomes rather flat for a portion of the new debt levels, implying a jump upwards in the left-hand side of (27). As a result, the right-hand side of equation (27) may now intersect the left-hand side of (27) multiple times, as shown in the bottom right panel.

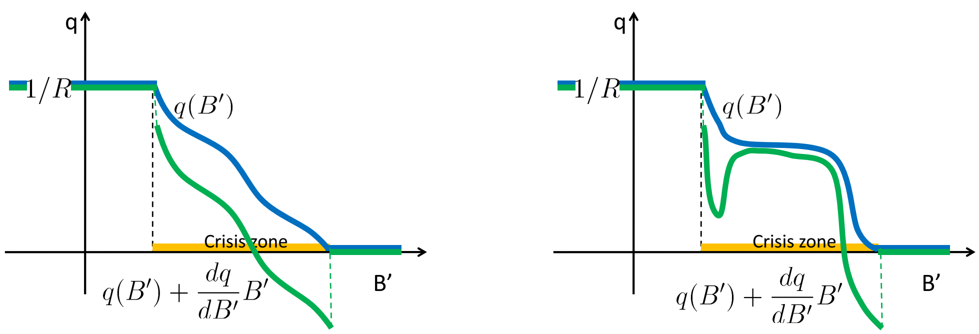
Nonetheless, for the purpose of some discussions, it may be illuminating to proceed with examining the first-order condition, and assuming that it provides the unique solution, while keeping the caveat in mind, that this may not be right. We shall state this as an explicit assumption, in case it is necessary to make an explicit reference to it.

Assumption A. 3 *The first-order conditions given in proposition 7 characterize the solution, and the solution is unique.*

With that assumption, some further comparative statics is possible, as shown in figure 22. For lower y or for higher B, one obtains a lower level of current consumption, keeping future debt B' the same. This results in higher marginal utility $u'(c)$ or a lower rhs of the first-order condition (27).

Consider now the case, where χ is constant and where the fluctuations in

The market price $q(B')$ vs the lhs of (27):



The two sides of (27):

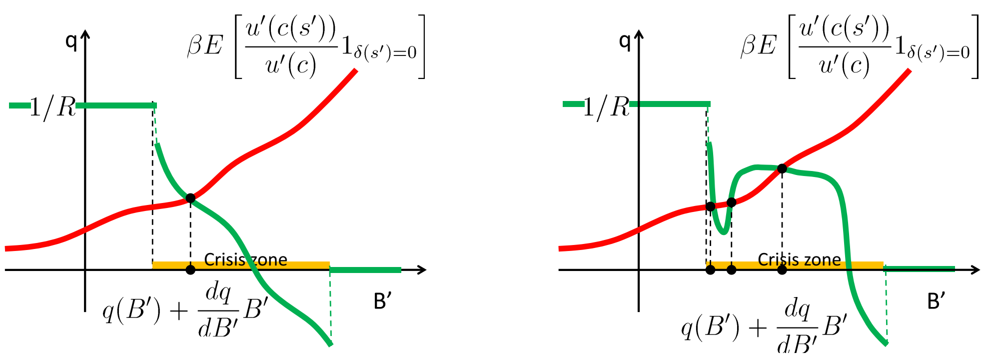


Figure 21: *Examining the first order condition (27)*

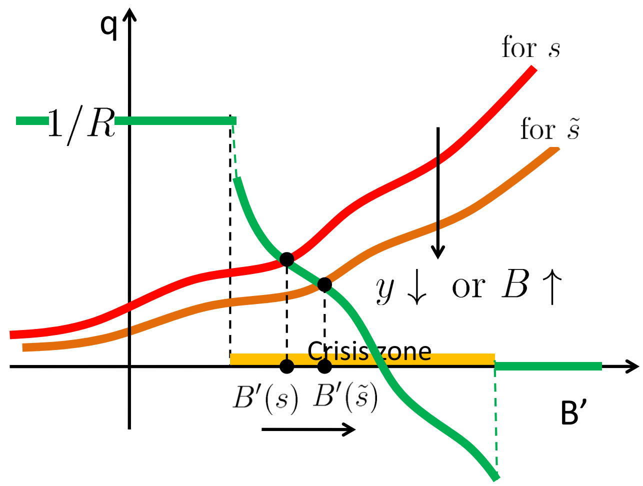


Figure 22: *The first-order condition (27) versus variations in the state s: implications for the new debt level B'.*

income are very small[5]. In that case, the price is nearly flat at $q = (1-\pi)/R$ in the crisis zone, $\min \underline{B}(z) \leq B' \max \bar{B}(z)$. Figure 23 shows the resulting version of (27), corresponding essentially to the situation described in Cole and Kehoe (2000). The question is now, how large B' is, compared to the debt level B leading into this scenario. Consider the case where $\beta R = 1$. If income is literally constant, then consumption should be constant and the debt level should likewise remain constant, **except** that the country can also avoid the cost of default altogether[6] by "saving itself" out of the crisis zone, as shown in Cole and Kehoe (2000). The version of (27) for an initial debt level $B = 0$ is shown in figure 24: at constant income and $\beta R = 1$, the

[5]This analysis is preliminary and rather speculative. Hopefully, we will succeed with a clean-up in a future version of this paper

[6]This appears to clash with the first-order condition derived above. The issue will be cleared up in a future version of this paper.

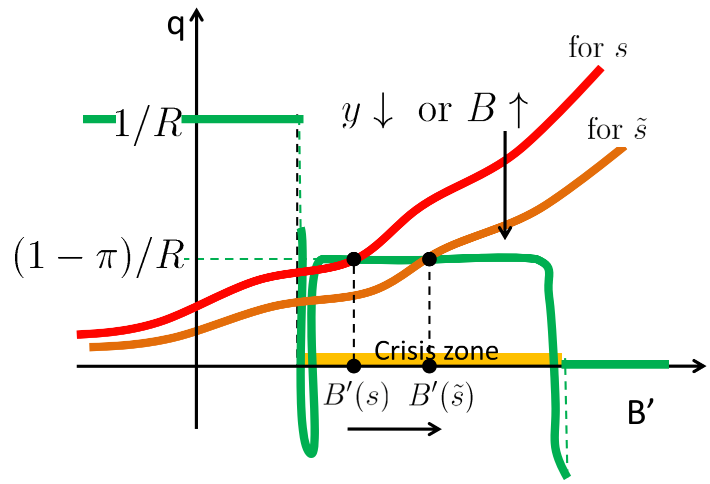


Figure 23: *The first-order condition (27) when income fluctuations are small.*

country will simply maintain that debt level rather than increase it.

Indeed, with a modest degree of income variation and for $\beta R = 1$, the country will choose to distance itself over time from the default zone as far as possible, saving for precautionary motives. The ensuing dynamics is shown in figure 25. If $\beta R < 1$, but close to 1, then the asset accumulation will not "run away", but still, the country will choose to accumulate large amounts of assets, as shown in figure 26. As a result, a sovereign debt crisis is highly unlikely. Here, it is therefore important to appeal to the political economy literature on sovereign debt accumulation, as in the literature cited in the introduction. If the government discounts the future sufficiently highly, i.e. if βR is considerably smaller than unity, then the country will possibly perch itself at a precarious point with an amount of debt in the crisis zone, as shown in figure 27. Indeed, reintroducing the income fluctuations in this picture results in a stationary distribution for the debt level, under suitable assumptions, as shown in figure 28.

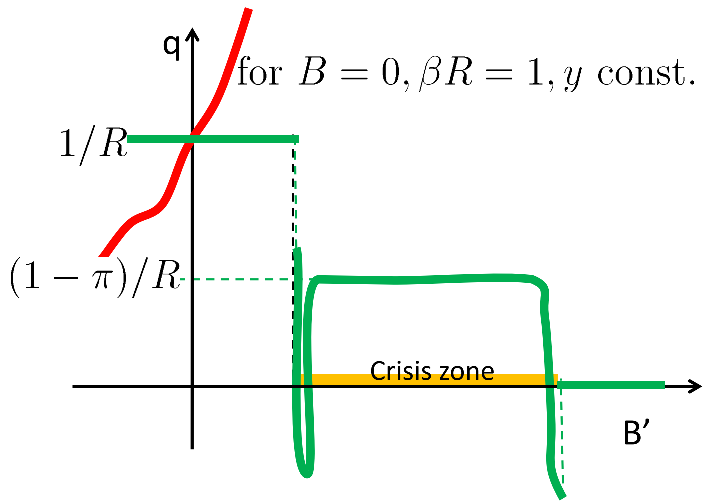


Figure 24: *The first-order condition (27) when income fluctuations are negligible and initial debt is zero.*

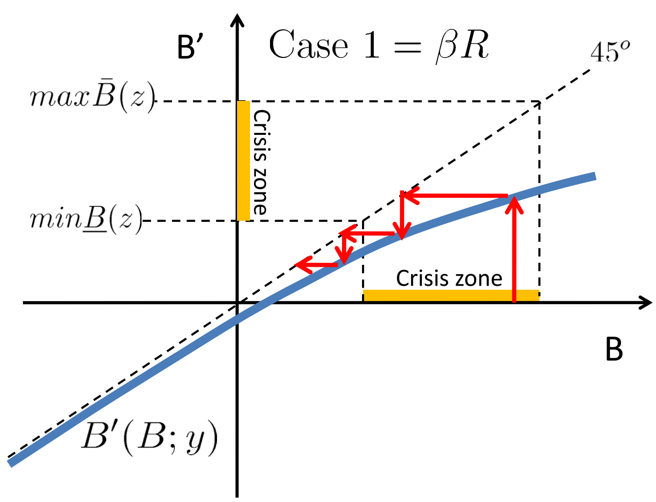


Figure 25: *The debt dynamics for small income fluctuations and $\beta R = 1$.*

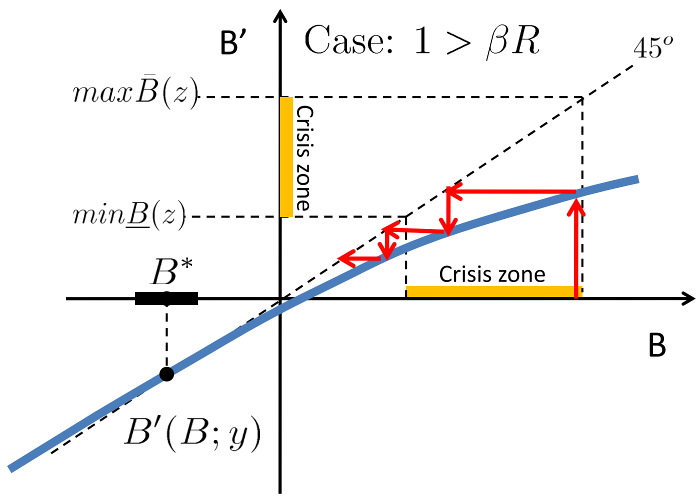


Figure 26: *The debt dynamics for small income fluctuations and βR below, but near* 1.

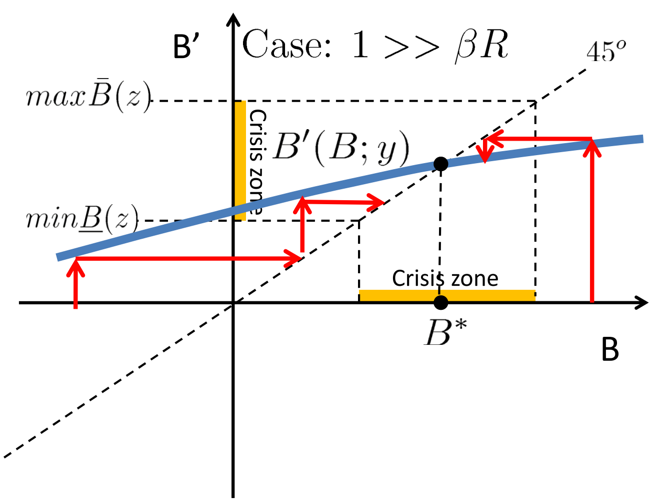


Figure 27: *The debt dynamics for small income fluctuations and βR far below* 1.

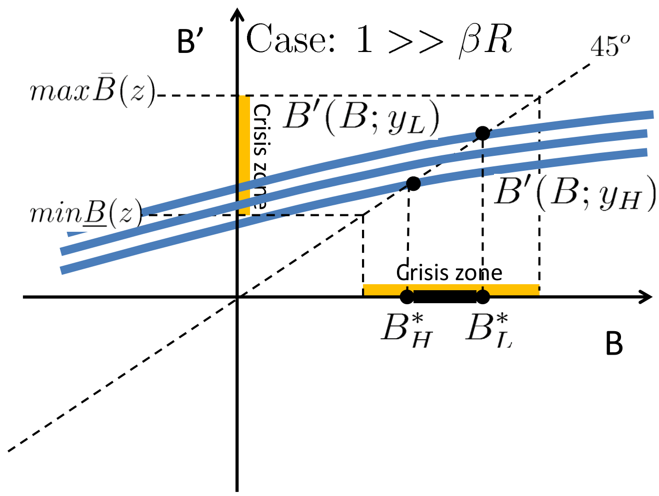


Figure 28: *The stationary debt dynamics for small income fluctuations and* βR *far below* 1*.*

C Other bailout mechanisms

Let us now consider the possibility for a bailouts, which may not necessarily be actuarily fair, as an extension of the discussion in the main body of the paper, and as these may be important for certain policy discussions. We shall focus on a few benchmark cases and explore their implications. First, suppose that, for a single period, debt can be sold at some fixed "assisted" price $0 < q_a < 1/R$ to some outside facility, provided the total amount B' of debt does not exceed some upper limit $\bar{B}_a$. This is a bailout and a stylized version of the one-time rescue for Greece or a one-time intervention by the European Financial Stability Facility. The resulting situation is shown in figure 29. The green line denotes the market price for existing debt sold to private lenders, while the blue line denotes the line, at which debt can be sold to the outside facility. The new debt level $B'_a(s)$ now exceeds the

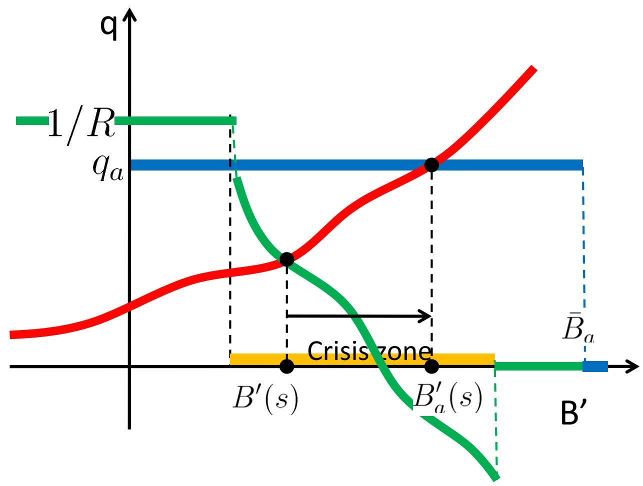


Figure 29: *The choice of the debt level in case of a one-time assistance or bailout.*

old debt level. Essentially, given the bailout, there is no longer quite the same pressure for the government of the country to cut back on government spending, due to the impending financial crisis. Indeed, we have seen how the attempts of government cut backs in Greece and Portugal have run into fierce local resistance: a luxury, that certainly would not have been there, if these countries needed to keep borrowing on private markets only and wished to avoid a default. As this is a one-time bailout, the resulting debt dynamics is given by figure 27, starting towards the right end, and indicated with the red arrow there (indeed, that arrow only applies in this situation: without the bailout, there would have been an assured default at that debt level outside the crisis zone).

It may be more interesting to consider a permanent version of this facility: all future borrowing by the country at hand can be done at some fixed price $0 < q_a < 1/R$, provided the total amount B' of debt does not

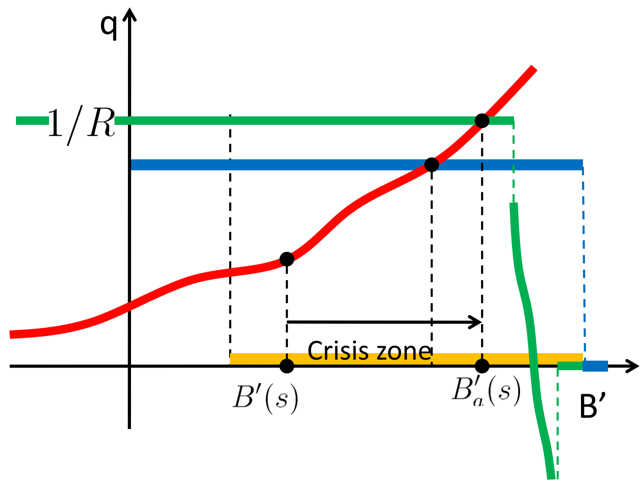


Figure 30: *The choice of the debt level in case of a permanent assistance or bailout.*

exceed some upper limit $\bar{B}_a$. In that case, the pricing is given by figure 30. The existence of the borrowing guarantee now removes the doubt of private lenders that the country will be able to borrow tomorrow. As a result, the country debt becomes safe and will be discounted at the usual safe rate R. The mere promise of the permanent facility results in a markedly reduced market interest on the country debt, provided the promised facility is fully credible.

This may appear to be a wonderful solution. This is so only at first blush, however. Note that the borrowing increases from $B'(s)$ to $B'_a(s)$. Indeed, the country will once again find its perch in the crisis zone of probabilistic default: this time, however, triggered by the debt limit imposed by the facility[7]. The country will borrow privately at the safe return R, until it gets near the

[7]Without a debt limit, the country will choose to run a Ponzi scheme, borrowing forever more without ever repaying.

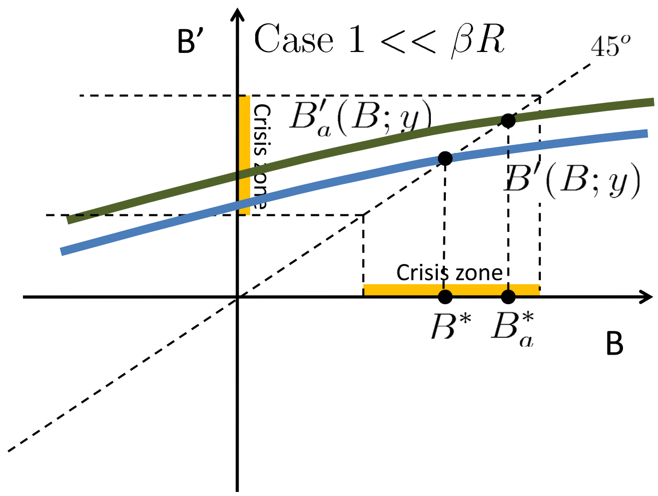


Figure 31: *The stationary debt dynamics for small income fluctuations and a permanent bailout facility.*

imposed debt limit. At that point, credibility on private credit markets collapses as a default is now viewed as likely, the country will borrow one last time, but this time from the facility at the reduced price, and will default in the next period. The proof is by contradiction: if it would not default in the next period (or if such a default would be very unlikely), then it would borrow privately, rather than at the "penalty rate" from the facility. The ensuing debt dynamics is shown in figure 31.

Both scenarios are in conflict with the observation, however, that yields on, say, Greece, Portugese and Irish debt are high and continue to be high, i.e. that there continue to be default fears by private markets. While it is conceivable, that we are simply in that "terminal" period described in the previous scenario, an alternative view here is that the bailout is probabilistic. This can be modelled in analogy to the default sunspot above. I.e., assume some bailout probability $0 < \omega < 1$. If the "bailout sunspot" ψ is below

ω, $\psi < \omega$, then the country can borrow at the price $0 < q_a < 1/R$ from the outside facility, provided the total amount B' of debt does not exceed some upper limit $\bar{B}_a$. If the "bailout sunspot" ψ exceeds ω, $\psi \geq \omega$, then the country must rely on private markets alone.

This will have the effect shown in figure 32. The level of debt at which a country will now prefer a default in those periods when no borrowing from the facility is possible, has increased compared to the "no bailout ever" scenario, as the country can hope for the option of borrowing from that facility in the future. Therefore, the crisis zone shifts to the right. The debt dynamics is shown in figure 33. Essentially, this is now a shifted version of the debt dynamics without that facility: rather than repaying the debt, the country shifts to higher debt levels, and the probability of a default is essentially the same as it was before. This takes a bit of time, of course. The facility therefore provides a temporary, but not a permanent resolution of the fiscal crisis. The debt is once again traded at a premium, as before, except that the probabilistic bailout means that these higher premium will be afforded at a higher debt level, than without that facility, while avoiding the default.

In essence, these scenarios show that the bailout facility only postpones the day of reckoning. It provides temporary relieve to the country in its desire to maintain a high level of government consumption, but leaves the default situation in a very similar and precarious situation as before, once the initial relief is "used up".

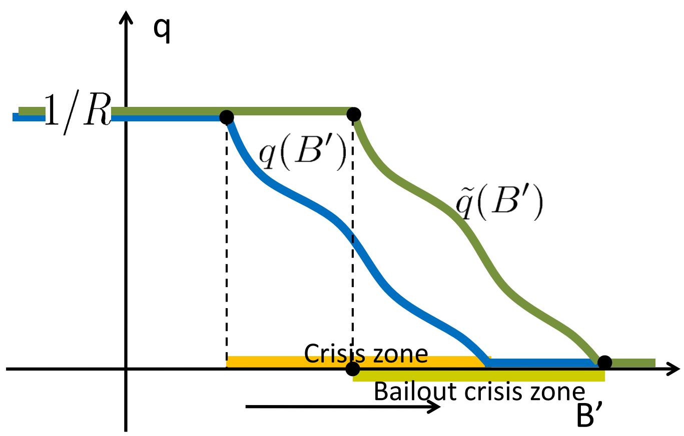


Figure 32: *Comparing the no-bailout private market pricing function $q(B')$ with the pricing function $\tilde{q}(B')$ in case of probabilistic bailouts.*

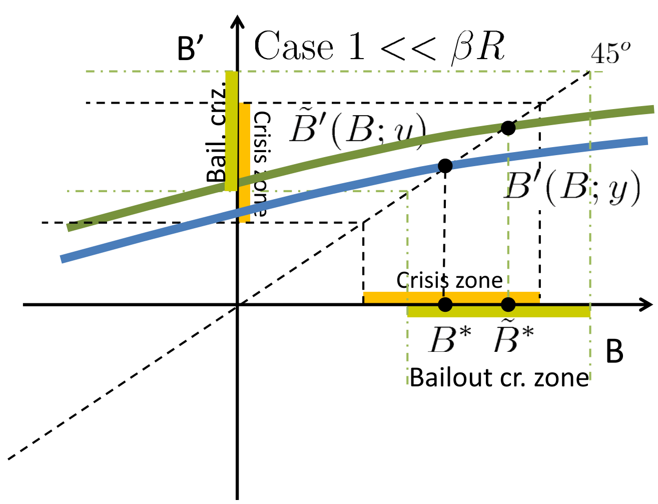


Figure 33: *The stationary debt dynamics for small income fluctuations and probabilistic bailout facility.*