

Inventories and the Role of Goods-Market Frictions for Business Cycles

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Abstract

Changes in the stock of inventories are important for fluctuations in aggregate output. However, the possibility that firms may fail to sell all produced goods and inventory accumulation are typically ignored in business cycle models. Using US data, the "ability to sell" is shown to be strongly procyclical. By including both a goods-market friction and a standard labor-market search friction, the model developed can substantially magnify and propagate shocks. Despite its simplicity, the model can also replicate key inventory facts. However, when these inventory facts are used to discipline parameter values, then goods-market frictions are quantitatively not very important.

Key Words: Matching models, search frictions, magnification, propagation.

JEL Classification: E12, E24, E32.

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1 Introduction

Firms are likely to hold back on hiring workers when demand for their products is low and consumers may very well postpone purchases when they worry about becoming unemployed. Such interaction between goods-market and labor-market frictions could deepen economic downturns. In modern business cycle models, such "Keynesian" interaction is typically due to nominal frictions, that is, due to the presence of sticky prices and wages: When prices are sticky, changes in demand have a stronger impact on production and changes in production have a stronger impact on employment when wages are sticky. This paper develops a business cycle model in which such Keynesian interaction is due to the presence of *real* frictions in both the labor market and the goods market. With frictions in both markets, there is a potentially powerful interaction between the goods market and the labor market, *even* when prices and wages are flexible. This paper is related to the coordination failure literature, but does not rely on self-fulfilling expectations nor on multiple equilibria.¹

It is common to incorporate labor-market search frictions in business cycle models and this approach is adopted here as well. Recently, several papers have incorporated goods-market search frictions into business cycle models.² Several of these papers assume that prices are flexible and by doing so make clear that Keynesian interaction between goods and labor markets is possible without relying on price rigidities. This paper shares with the recent literature the assumptions that (i) firms face frictions in finding buyers for their products and (ii) the severity of this friction varies over the business cycle. In contrast to the literature, the goods-market friction is not symmetric. The underlying idea is that firms may not sell all their products, for example, because they produce what consumers do not want, but consumers consider what is available and the only cost in acquiring the good is the purchase price. This is a minor difference. But the advantage is that Keynesian results in this paper do not rely on the cyclical nature of consumers' effort to acquire goods.³

A more essential aspect in which this paper differs from the literature is that the model includes

¹See Cooper (1999) for an overview of coordination failure models.

²For example, Arsenau (2007), Gourio and Rudanko (2011), Mathä and Pierrard (2011), Petrosky-Nadeau and Wasmer (2011), Bai, Ríos-Rull, and Storesletten (2012), Kaplan and Menzio (2013), and Michaillat and Saez (2013).

³It is not clear whether consumers' effort to acquire goods relative to the value of purchases is procyclical or countercyclical. In the models of Petrosky-Nadeau and Wasmer (2011) and Bai, Ríos-Rull, and Storesletten (2012), consumers put in *less* effort trying to acquire goods during recessions, which is bad for firms. In the model of Kaplan and Menzio (2013), unemployed consumers have more time to allocate to activities unrelated to working. Consequently, on average consumers put in *more* effort to acquire goods during recessions, since there are more unemployed during recessions. In the model of Kaplan and Menzio (2013), it is bad for firms if consumers put in more effort, since this means that consumers can visit more stores and bargain for lower prices.

inventories. There are several reasons to include inventories. As documented in this paper, the observed behavior of inventories is very informative about the characteristics of frictions in the goods market and the quantitative importance of such frictions for business cycles. This is not surprising. When there are cyclical changes in the frictions that firms face in selling products, then this is likely to affect the accumulation of inventories. Another important reason to include inventories in business cycle models is that changes in the investment in inventories are a quantitatively important aspect of cyclical changes in GDP. Blinder and Maccini (1991) document that the drop in inventory investment accounted on average for 87 percent of the drop in GNP in the postwar US recessions they considered. This paper confirms the empirical relevance of changes in investment in inventories for cyclical fluctuations in GDP, although the estimates are not as high as the one reported in Blinder and Maccini (1991).

This paper makes four contributions. First, the paper constructs a measure of "goods-market efficiency" and documents its properties. Second, the paper develops a business cycle model with inventories that is characterized by frictions in the labor *and* the goods market. Third, the paper documents that the model can match key aspects of US business cycles and in particular the cyclical behavior of inventories. Fourth, the paper documents the importance of goods market frictions when the model is consistent with the cyclical behavior of inventories. These contributions are discussed in more detail in the remainder of this section.

The measure of goods-market efficiency used is the amount of goods sold relative to the sum of newly produced goods and beginning-of-period inventories. A higher value means that firms sell a higher fraction of available products. This efficiency measure is a simple transformation of the inventory-sales ratio; if the inventory-sales ratio decreases (increases), then the goods-market efficiency measure increases (decreases). Section 2 documents that this measure of goods-market efficiency is strongly procyclical. This is not surprising given that the inventory-sales ratio is known to be countercyclical and the two measures are inversely related.⁴ A novel empirical finding is that the goods-market efficiency measure is negatively related to the beginning-of-period stock of aggregate inventories. This last aspect of goods-market efficiency turns out to play a key role in matching the observed behavior of inventories with the theoretical model.

The empirical findings provide the motivation for the specification of the goods-market friction that firms face in the theoretical model developed. Consistent with the observed positive dependence of goods-market efficiency on aggregate real activity, the paper follows Diamond (1982) and lets

⁴Bils and Kahn (2000) document that the inventory-sales ratio is countercyclical.

goods-market efficiency vary with market size. The idea is that a firm is more likely to find a customer who wants the firm's products in larger markets.⁵ The model incorporates this externality, but the externality is not strong enough to generate multiple equilibria as in Diamond (1982). Additional empirical support for this externality is given in Gavazza (2011); using transactions data for commercial aircraft markets, Gavazza (2011) shows that trading frictions diminish with the thickness of the market. In addition, goods-market efficiency is assumed to decrease when aggregate inventories increase, as indicated by the empirical analysis. Except for the presence of inventories and a goods-market friction, the model is a standard business cycle model with a labor-market search friction.

The model can match key facts regarding the behavior of inventories. Important facts regarding the joint behavior of inventories, sales, and real activity are that sales are *less* volatile than production, investment in inventories is *procyclical*, and the investment in inventories is *positively* correlated with sales.⁶ These properties have surprised the profession because they are inconsistent with the view that firms smooth production and use inventories as a buffer against unforeseen shocks to sales. Building models that can match the facts turned out to be a challenging exercise. There are now several ingenious business cycle models that are consistent with observed behavior, but successful inventory models tend to be characterized by non-trivial features such as Ss bands.⁷ In contrast, the model in this paper is extremely simple and can also match the facts. In existing models, the accumulation of inventories is a non-trivial choice problem for the firm. In the model of this paper, firms always try to sell *all* available goods and goods end up in inventories only because firms are not successful in selling goods. Firms could in principle choose to accumulate *additional* inventories, but it is never optimal to do so. To match the inventory facts, the behavior of the goods-market efficiency measure has to be consistent with its observed properties. In particular, both the observed positive dependence on aggregate real activity *and* the observed negative dependence of the goods-market efficiency measure on aggregate inventories are necessary. The simplicity of this approach to model inventories would make it possible to incorporate it in a broad range

⁵The idea is that sellers offer different types of products and that the chance of producing goods that customers do not want is smaller in bigger markets. That is, as the market grows, the law of large numbers becomes more appropriate and uncertainty about the outcome and the chance of mismatch become smaller.

⁶See Blinder and Maccini (1991), Ramey and West (1999), Bils and Kahn (2000), and McMahon (2011) for a discussion.

⁷Exemplary papers on this road towards success are Eichenbaum (1989), Ramey (1991), Bils and Kahn (2000), Coen-Pirani (2004), and Khan and Thomas (2007).

of business cycle models and by doing so include an important factor behind cyclical changes in aggregate output into the analysis.

The model is used to assess the importance of the goods-market friction for magnifying and propagating shocks when prices and wages are flexible. The paper documents that the procyclical aspect of the goods-market efficiency measure can create a powerful mechanism to magnify and propagate shocks. This is not too surprising, since Diamond (1982) shows that multiple equilibria are possible if the dependence of the goods-market friction on aggregate activity is strong enough. A more interesting question is whether cyclical changes in goods-market efficiency are still important when the model is consistent with observed inventory facts. The answer is no for two reasons. The first reason is that the positive dependence of goods-market efficiency on aggregate activity cannot be too strong. Consider a shock that negatively affects real activity. If the goods-market efficiency, i.e., the ease with which firms can find customers, drops a lot during economic downturns, then inventories would increase during recessions, whereas they decrease in the data, and sales would drop by more than output, whereas they drop by less in the data. The second reason is that the negative dependence of the goods-market efficiency measure on aggregate inventories also plays an important role in matching key inventory facts. This negative dependence means that cyclical changes in goods-market efficiency are short-lived. That is, following a negative shock, goods-market efficiency deteriorates initially, but it recovers quickly as the stock of inventories is reduced. The last section of the paper discusses some reasons why cyclical changes in goods-market efficiency may still be important, but the conclusion of this paper is that the observed behavior of inventories suggests that interaction between goods-market frictions and labor-market frictions does not seem to be very important, at least not in the type of model considered here and when prices and wages are flexible.

The remainder of this paper is organized as follows. Section 2 describes the goods-market efficiency measure used, its relationship to the inventory-sales ratio, and describes key aspects of its observed cyclical behavior. Section 3 describes the model. Section 4 motivates the parameter choices. Section 5 discusses the results. The last section concludes.

2 Empirical motivation

This paper focuses on the role of cyclical fluctuations in the efficiency of the process to get produced products into the hands of buyers. This section documents the cyclical behavior of this "goods-

market efficiency" and links the results to known properties of the cyclical behavior of inventories.

2.1 Goods-market efficiency

Let Y_t be total production in period t and let X_{t-1} be the stock of inventories carried over from the last period after depreciation. The maximum that could be sold in period t is equal to $Y_t + X_{t-1}$. Actual sales, S_t , are typically less. One reason is that goods that are ready to be sold do not find a buyer in the current period. Another reason is that some finished goods have not ended up on store shelves yet and are not ready to be sold. Finally, sales will also be less than $Y_t + X_{t-1}$ if X_{t-1} includes unfinished goods.

Goods-market efficiency, $\pi_{y,t}$, is defined as

$$\pi_{y,t} = \frac{S_t}{Y_t + X_{t-1}}. \quad (1)$$

This measure describes how many goods are sold relative to the sum of newly produced goods and the amount of goods carried over as inventories from last period. The amount produced, Y_t , is equal to the amount sold, S_t , plus the investment in inventories. That is,

$$Y_t = S_t + (X_t^{\text{eop}} - X_{t-1}), \quad (2)$$

where X_t^{eop} is the level of inventories at the end of period t , before depreciation. Combining the last two equations gives

$$\pi_{y,t} = \frac{S_t}{S_t + X_t^{\text{eop}}} = \frac{1}{1 + X_t^{\text{eop}}/S_t}. \quad (3)$$

That is, goods-market efficiency is inversely related to the inventory-sales ratio and both measures can be interpreted as measures that describe the efficiency of getting products in the hands of the customer.⁸

2.2 Cyclical properties of goods-market efficiency

The analysis is based on quarterly private non-farm inventory data from 1967Q1 to 2012Q, published by the Bureau of Economic Analysis.⁹ Results reported here are based on *aggregate* data and sales data are *final* sales, either final total sales by domestic businesses or final sales of goods and structures. Appendix B.1 reports results based on *disaggregated* data for the following five

⁸If X_{t-1} includes unfinished goods, then the efficiency measure could capture more than just frictions in the goods market. In particular, it could also include efficiencies in the production process.

⁹Detailed information about data sources is given in appendix A.

sectors: durable goods manufacturing, non-durable goods manufacturing, durable goods wholesale, non-durable goods wholesale, and retail.

The data are detrended using the Hodrick-Prescott (HP) filter in order to characterize data properties at business cycle frequencies. To study the possibility that data properties are different at high frequencies, two band-pass filters are used to extract the fluctuations that are associated with cycles that have a period of less than one year and with cycles that have a period of less than two years.¹⁰

Table 1 provides summary statistics and confirms some well-known facts about inventory behavior. In particular, inventories and sales are positively correlated at business cycle frequencies. At higher frequencies, however, there is a negative correlation between sales and inventories.¹¹ Sales are also positively correlated with the investment in inventories.¹² That is, inventories tend to increase during periods when the cyclical component of sales is positive. This property is closely related to another well-known property, namely that output is more volatile than sales.¹³ For the series considered here, output is roughly ten percent more volatile. This well-known ordering of volatilities has challenged the literature to come up with innovative inventory theories, since the traditional assumption of increasing marginal costs implies that firms would like to smooth production by using inventories as a buffer to absorb sales shocks.

The mean values of the goods-market efficiency for the two measures are equal to 40% and 55%. That is, quite a large fraction of newly produced output and inventories does not reach consumers within the quarter. Figure 1 displays the cyclical behavior of the goods-market efficiency measure. The solid line corresponds to the cyclical component of GDP (top panel) and the goods-market efficiency based on final sales of goods and structures (bottom panel). To better understand the importance of the cyclical changes, the mean of the goods-market efficiency measure is added to its cyclical component. The figure documents that the efficiency measure is clearly procyclical.

¹⁰The detrended value of an observation is obtained using a band-pass filter that uses the observation itself and 12 lagging and 12 leading observations.

¹¹Similar results are reported in Wen (2005).

¹²Since inventory investment can take on negative values, it is not possible to take logarithms to obtain a scale-free variable. The following is done to construct the cyclical component of inventory investment. First, inventory investment is divided by the trend value of GDP. Second, the HP-filter is applied to this ratio.

¹³Since output equals sales plus investment in inventories, output is necessarily more volatile than sales if sales and investment in inventories are positively correlated. Here, statistics are calculated for the logarithms of the variables. Consequently, the simple additive relationship no longer holds as an identity, but the logic carries over to the analysis using logarithms.

Since goods-market efficiency is a monotone inverse function of the inventory-sales ratio, this is just another way to state the well-known fact that the inventory-sales ratio is countercyclical. The correlation between goods-market efficiency and GDP is equal to 0.61 for the measure based on final sales of goods and structures. The magnitudes of the cyclical fluctuations are nontrivial. The cyclical component of the goods-market efficiency varies from a minimum of 38.2% to a maximum of 41.8%. Relative to the inventory-sales ratio, an advantage of the goods-market efficiency measure is that it is easier to interpret the magnitude of its cyclical fluctuations and to understand how important observed cyclical fluctuations potentially are for, for example, firm profitability. In particular, the observed difference between the just reported minimum and maximum values would correspond to a 8.5% drop in the sales price if firms would not be able to sell unsold goods in subsequent periods.¹⁴ If one compares this with, for example, the usual magnitude of fluctuations in aggregate TFP, then these are numbers that cannot be ignored.¹⁵

2.3 Tracking goods-market efficiency over the business cycle

To shed more light on the cyclical properties of goods-market efficiency, the following projection is calculated

$$\tilde{\pi}_{y,t} = \zeta_y \tilde{Y}_t + \zeta_x \tilde{X}_{t-1} + u_t, \quad (4)$$

where the tilde indicates that the series have been detrended. As in Diamond (1982), a positive value for ζ_Y captures the idea that finding a suitable trading partner is easier in larger markets. Basic national income accounting tells us that goods produced this period will lead to income such as wages and profits. Therefore, \tilde{Y}_t will affect both the supply side and the demand side of the goods market. Beginning-of-period inventories, \tilde{X}_{t-1} has less of an effect on this period's demand side, since it was produced in the past and typically generated income for workers and others when it was produced. If existing inventories mainly affect the supply side, then ζ_x would be negative. Even if demand is elastic, then an increase in inventories could reduce goods-market efficiency if goods are competing for shelf space and/or sales staff.

Details of this empirical exercise are given in appendix B.2. For all cases considered, the estimate for ζ_y is positive and the estimate for ζ_x is negative. Moreover, the fit improves considerably if

¹⁴Consequences for firm profits are less dramatic if inventories can be carried into the next period. However, inventory carrying costs are non-trivial. Richardson (1995) argues that inventory carrying costs are between 25% and 55% of the stock of inventories.

¹⁵Recall that the standard deviation of aggregate TFP is typically assumed to be $\sqrt{0.007^2/(1-0.95^2)}$, which is equal to 2.2 per cent.

inventories are included in the basis of the projection.

The bottom panel of figure 1 plots the results for the goods-market efficiency measure based on final sales of goods and structures for the preferred detrending procedure. The dotted line indicates the projection of the goods-market efficiency measure on just the cyclical GDP component. The dashed line is the projection on both cyclical GDP and cyclical inventories. The cyclical component of GDP clearly tracks key changes in the goods-market efficiency measures. When inventories are added to the basis of the projection, the projected values capture the severity of the fall in the goods-market efficiency during downturns much better. This may be surprising, since inventories are procyclical and the projection coefficient for inventories is negative. This would suggest that the fitted value of $\tilde{\pi}_{y,t}$ should decrease by *less* when inventories are added to the projection. The reason this does not always happen is the following. Cyclical fluctuations in inventories are larger than cyclical fluctuations in GDP. Moreover, it takes time to build down the large increase in the cyclical component of inventories that is formed during a boom. Consequently, the cyclical component of inventories can still be positive when the cyclical component of GDP is already negative. During such episodes both the negative cyclical component of GDP and the (still) positive cyclical component of inventories push the value of the goods-market efficiency down. This is exactly what happened during some of the deep recessions in the sample and can explain the improved fit during severe downturns when lagged inventories are included in the projection equation.

The explanatory variables are endogenous variables. Thus, these are just projections and the coefficients do not necessarily capture the causal effect of a right-hand side variable on the dependent variable. Nevertheless, the results do hint at the possibility that the process of getting goods in the hands of the consumer becomes easier when aggregate real activity increases and becomes more difficult (per unit of available good for sale) when firms have more goods in inventories. Independent evidence for the estimates found here is given in section 5 in which it is shown that the theoretical model needs a positive value for ζ_y and a negative value for ζ_x to match observed inventory facts.

2.4 Inventory accumulation during the recent recession

Although, inventories are procyclical at business cycle frequencies, they are countercyclical at higher frequencies as pointed out by Wen (2005) and confirmed here. The latter result is consistent with an increase in inventories at the onset of a recession. However, aggregate inventories follow changes in GDP quite quickly; during the recent recession, aggregate inventories also lag GDP, but the lag

seems to be not more than one quarter.¹⁶

The behavior of the aggregate series hide quite divergent behavior for the components. For example, from 2007Q3 to 2008Q2 (2008Q3), inventories of the durables-goods wholesale-trade sector increased by 4.2% (3.2%) compared with a drop in GDP of 1.1% (3.3%). Even larger increases are observed when inventories of particular subsectors are considered. Inventories of the "motor vehicles parts and supplies merchant" wholesale industry increased by 8% (11%) from 2007Q3 to 2008Q2 (2008Q3). Interestingly, the inventories of this sector display massive drops in subsequent quarters.¹⁷ Inventories of the computers and software merchant wholesale industry increased by 10% (4%) from 2007Q3 to 2008Q2 (2008Q3). In contrast, these inventories did not display sharp drops in subsequent quarters. The largest increase in inventories is observed in the petroleum and coal product manufacturing industry. Inventories in this sector increased by 23% from 2007Q3 to 2008Q1.

3 Model

There are three types of agents in the economy. The first is a representative household that receives the earnings from its members and determines how much of aggregate income to consume and how much to invest in capital. This representative household consists of a continuum of entrepreneurs and a continuum of workers. This section describes the choice problems of the three different agents, the characteristics of the labor and the goods market, wage setting, and the equilibrium conditions.

Notation and reason for the endowment good. Aggregate variables, such as market prices and choices made by the representative household, are denoted by uppercase characters. Variables associated with choices of the individual firms are denoted by lowercase characters. Prices are expressed in terms of an endowment good. This good plays no role in the model at all, but is helpful to describe price and wage setting. In particular, it makes it clear that the price of the market-produced consumption good is fully flexible and adjusts to clear the goods market. By focusing on the case with flexible prices, it becomes clear that there is an interaction between frictions in the goods market and frictions in the labor market *even* when prices and wages are flexible. All variables that are expressed in units of the endowment good are denoted by a symbol

¹⁶See appendix B.3.

¹⁷The American Recovery and Reinvestment Act of 2009 is likely to have played a role, but inventories started to drop before the act was signed into law on February 17 2009.

with a circumflex. In appendix B.4, it is shown that the model equations can be rewritten to a system of equations in which the endowment good does not appear.

Household. A representative household chooses the consumption of the market-produced good, C_t , the consumption of the endowment good, $C_{e,t}$, and the amount of capital to carry over into the next period, K_t . For stock variables, such as K_t , the subscript t means that it is determined in period t , and available for production in period $t + 1$.

The household consists of a continuum of workers that supply labor inelastically. The total mass of workers is given by Υ_N and the mass of employed workers is equal to N_t . The representative household receives income from employment, $\widehat{W}_t N_{t-1}$, income from renting out capital, $\widehat{R}_t K_{t-1}$, and income from firm ownership, \widehat{D}_t .

The maximization problem of the representative household is given by

$$\mathbf{V}(\mathbf{S}_t) = \max_{C_t, C_{e,t}, I_t, K_t} \frac{C_t^{1-\nu} - 1}{1-\nu} + U(C_{e,t}) + \beta \mathbf{E}_t [\mathbf{V}(\mathbf{S}_{t+1})]$$

s.t.

$$\widehat{P}_t C_t + \widehat{P}_t I_t + C_{e,t} = \underline{C}_e + \widehat{R}_t K_{t-1} + \widehat{W}_t N_{t-1} + \widehat{D}_t, \quad (5)$$

$$I_t = K_t - (1 - \delta_k) K_{t-1}, \quad (6)$$

where I_t is investment, \underline{C}_e is the quantity of the endowment good received, and \mathbf{S}_t is the set of state variables.¹⁸

The first-order conditions are given by

$$\Lambda_{e,t} = \frac{\partial U(C_{e,t})}{\partial C_{e,t}}, \quad (7)$$

$$\widehat{P}_t \Lambda_{e,t} = C_t^{-\nu}, \quad (8)$$

$$\widehat{P}_t \Lambda_{e,t} = \beta \mathbf{E}_t \left[\Lambda_{e,t+1} \left(\widehat{R}_{t+1} + \widehat{P}_{t+1} (1 - \delta_k) \right) \right]. \quad (9)$$

As explained below, transactions in the goods market are characterized by a friction. However, the friction only affects the ability of the firm to find a trading partner; consumers can buy whatever they want without incurring any disutility or any other type of cost except having to pay for the goods acquired. Consequently, the household problem is characterized by the standard set of equations.¹⁹

¹⁸The (not frequently used) symbols for the value function and the set of state variables are in bold and should be distinguished from the symbols for sales, S_t , and vacancies, V_t , which are not bold characters.

¹⁹If the household chooses negative *gross* investment, then equation (5) implies that capital goods are transformed

Existing firms/jobs. A firm consists of one entrepreneur and one worker. The firm hires capital to produce output. The Bellman equation of the entrepreneur's problem is given by

$$\widehat{v}(x_{t-1}; \mathbf{S}_t) = \max_{y_t, k_t, x_t} \left(\begin{array}{l} \left(\pi_{y,t} (y_t + x_{t-1}) \widehat{P}_t - \widehat{R}_t k_t - \widehat{W}_t \right) \\ + \beta (1 - \delta_n) \mathbb{E}_t [\Omega_{t+1} \widehat{v}(x_t; \mathbf{S}_{t+1})] \end{array} \right)$$

s.t.

$$y_t = \alpha_0 \exp(Z_t) k_t^\alpha, \tag{10}$$

$$x_t = (1 - \delta_x) (1 - \pi_{y,t}) (y_t + x_{t-1}), \tag{11}$$

where Ω_{t+1} is the marginal rate of substitution between one unit of wealth this period and one unit of wealth the next period. That is,

$$\Omega_{e,t+1} = \frac{\Lambda_{e,t+1}}{\Lambda_{e,t}} = \left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \frac{\widehat{P}_t}{\widehat{P}_{t+1}}. \tag{12}$$

Moreover, δ_n denotes the probability of exogenous firm exit.²⁰ Z_t is an exogenous random variable affecting productivity and its law of motion is given by

$$Z_t = \rho Z_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim N(0, \sigma^2).$$

The amount of products available for sale consists of newly produced output, y_t , and inventories available at the beginning of the period t , x_{t-1} . The probability to sell a good is equal to $\pi_{y,t}$. Thus, the quantity of unsold products is equal to $(1 - \pi_{y,t}) (\alpha_0 \exp(Z_t) k_{t-1}^\alpha + x_{t-1})$ of which the firm carries a fraction $(1 - \delta_x)$ as inventories into the next period. The parameter δ_x captures both physical depreciation as well as loss in value for other reasons.

The following first-order conditions characterize the solution of the entrepreneur's choice problem:

$$\widehat{R}_t = \left(\pi_{y,t} \widehat{P}_t + (1 - \pi_{y,t}) (1 - \delta_x) \widehat{\lambda}_{x,t} \right) \alpha A \exp(Z_t) k_t^{\alpha-1}, \tag{13}$$

$$\widehat{\lambda}_{x,t} = (1 - \delta_n) \beta \mathbb{E}_t \left[\Omega_{e,t+1} \frac{\partial \widehat{v}(x_t; \mathbf{S}_{t+1})}{\partial x_t} \right]. \tag{14}$$

Here $\widehat{\lambda}_{x,t}$ is the value of relaxing the constraint given in equation (11). It represents the value of leaving period t with one more unit of inventories (after depreciation). The value of a unit of

into goods that are immediately available for consumption without any cost or friction. This is a bit strange, since firms do face frictions when selling goods to consumers. This is not an issue, however, since gross investment turns out to be always positive.

²⁰ δ_n is also the worker separation rate, since each firm consists of one worker.

inventories at the beginning of the period is given by

$$\widehat{\mathbf{v}}_{x,t} = \frac{\partial \widehat{\mathbf{v}}(x_{t-1}; \mathbf{S}_t)}{\partial x_{t-1}} = \begin{pmatrix} \pi_{y,t} \widehat{P}_t \\ + (1 - \pi_{y,t}) (1 - \delta_x) \widehat{\lambda}_{x,t} \end{pmatrix}. \quad (15)$$

Using this equation, first-order condition (14) can be written as

$$\widehat{\lambda}_{x,t} = (1 - \delta_n) \beta \mathbb{E}_t \left[\Omega_{e,t+1} \begin{pmatrix} \pi_{y,t+1} \widehat{P}_{t+1} \\ + (1 - \pi_{y,t+1}) (1 - \delta_x) \widehat{\lambda}_{x,t+1} \end{pmatrix} \right]. \quad (16)$$

Choosing to accumulate additional inventory. In this model, firms passively accumulate inventories. The question arises whether it could be optimal to accumulate *additional* inventories. That is, could it ever be optimal to keep some goods in storage instead of trying to sell them? The answer is no. If a firm puts a unit of goods on the market, then the expected payoff is equal to $\pi_{y,t} \widehat{P}_t + (1 - \pi_{y,t}) (1 - \delta_x) \widehat{\lambda}_{x,t}$. If it chooses to keep the unit in inventories, then the expected payoff is equal to $(1 - \delta_x) \widehat{\lambda}_{x,t}$. It would only do the latter if $\widehat{\lambda}_{x,t} > \widehat{P}_t / (1 - \delta_x)$. Thus, a firm would *choose* to put a good into inventories if the value of doing so is sufficiently above the market value of a market-produced good this period. This never happens.²¹

Firm heterogeneity and firm value. A newly created firm starts with zero inventories. As time goes by, the firm will accumulate inventories. Firms only differ in the amount of inventories they hold. Moreover, the only aspect of the distribution of inventories that is relevant for agents' decisions and the behavior of aggregate variables is the *aggregate* level of inventories. Although $\pi_{y,t}$ is allowed to depend on *aggregate* inventories, the assumption is made that $\pi_{y,t}$ does not depend on the *firm's* level of inventories. This assumption implies that $\mathbf{v}_{x,t}$ does not depend on the level of x_{t-1} . Consequently,

$$\widehat{\mathbf{v}}(x_{t-1}; \mathbf{S}_t) = \widehat{\mathbf{v}}(0; \mathbf{S}_t) + x_{t-1} \widehat{\mathbf{v}}_{x,t}. \quad (18)$$

That is, the value of each firm consists of two parts. The first part is the value of the firm *without* inventories, $\widehat{\mathbf{v}}(0; \mathbf{S}_t)$. The second part is the value of the stock of inventories, $x_{t-1} \widehat{\mathbf{v}}_{x,t}$. Reallocations

²¹To understand why this is the case, suppose that there is no uncertainty. If $\widehat{\lambda}_{x,t} / \widehat{P}_t > (1 - \delta_x)^{-1}$, then equations (9) and (16) imply that

$$\frac{(1 - \delta_n)}{\left(\widehat{R}_{t+1} / \widehat{P}_{t+1} + (1 - \delta_k) \right)} \left[\pi_{y,t+1} + (1 - \pi_{y,t+1}) (1 - \delta_x) \frac{\widehat{\lambda}_{x,t+1}}{\widehat{P}_{t+1}} \right] = \frac{\widehat{\lambda}_{x,t}}{\widehat{P}_t} > \frac{1}{1 - \delta_x}, \quad (17)$$

which implies that $\lambda_{x,t+1} / \widehat{P}_{t+1}$ is also bigger than $(1 - \delta_x)^{-1}$ unless the net return on capital $\widehat{R}_{K,t+1} / \widehat{P}_{t+1} - \delta_K$ is sufficiently negative. Such speculative events do not occur in this model.

of inventories across firms have no aggregate consequences, since $\widehat{\mathbf{v}}_{x,t}$ does not depend on the level of x_t .

The value of a firm with no inventories is given by

$$\begin{aligned}\widehat{\mathbf{v}}(0; \mathbf{S}_t) &= \left(\begin{array}{c} \pi_{y,t} \widehat{P}_t \alpha_0 \exp(Z_t) k_t^\alpha - \widehat{R}_t k_t - \widehat{W}_t \\ + (1 - \delta_n) \beta \mathbf{E}_t [\Omega_{t+1} \widehat{\mathbf{v}}((1 - \pi_{y,t}) (1 - \delta_x) \alpha_0 \exp(Z_t) k_t^\alpha; \mathbf{S}_{t+1})] \end{array} \right) \\ &= \left(\begin{array}{c} \pi_{y,t} P_t \alpha_0 \exp(Z_t) k_t^\alpha - \widehat{R}_t k_t - \widehat{W}_t \\ + (1 - \delta_n) \beta \mathbf{E}_t \left[\Omega_{t+1} \left(\begin{array}{c} \widehat{\mathbf{v}}(0; \mathbf{S}_{t+1}) \\ + (1 - \pi_{y,t}) (1 - \delta_x) \alpha_0 \exp(Z_t) k_t^\alpha \widehat{\mathbf{v}}_{x,t+1} \end{array} \right) \right] \end{array} \right).\end{aligned}\quad (19)$$

where k_t is the optimal choice for capital.

Using equation (15), the last equation can be written as

$$\widehat{\mathbf{v}}(0; \mathbf{S}_t) = \left(\begin{array}{c} \left(\begin{array}{c} (\pi_{y,t} \widehat{P}_t + (1 - \pi_{y,t}) (1 - \delta_x) \widehat{\lambda}_{x,t}) \alpha_0 \exp(Z_t) k_t^\alpha \\ - \widehat{R}_t k_t - \widehat{W}_t \end{array} \right) \\ + (1 - \delta_n) \beta \mathbf{E}_t [\Omega_{t+1} \widehat{\mathbf{v}}(0; \mathbf{S}_{t+1})] \end{array} \right).\quad (20)$$

Labor market and labor market friction. Job creation requires an entrepreneur starting a project and finding a worker. The per-period cost of this joint activity is equal to ψ units of the market good. The assumption of free entry implies that in equilibrium the cost of creating a job equals the expected benefit. This means that

$$\psi \widehat{P}_t \Lambda_{e,t} = \pi_{f,t} \beta \mathbf{E}_t [\Lambda_{e,t+1} \widehat{\mathbf{v}}(0; \mathbf{S}_{t+1})],\quad (21)$$

where $\pi_{f,t}$ is the number of matches per vacancy.

The total number of jobs created, N_t^{new} , depends on the number of vacancies posted, V_t , and the number of unemployed workers ($\Upsilon_N - N_{t-1}$). The matching technology is characterized by a Cobb-Douglas production function, thus²²

$$N_t = (1 - \delta_n) N_{t-1} + \phi_0 V_t^{\phi_1} (\Upsilon_N - N_{t-1})^{1-\phi_1}, \text{ and}\quad (22)$$

$$\pi_{f,t} = \phi_0 \left(\frac{\Upsilon_N - N_{t-1}}{V_t} \right)^{1-\phi_1}.\quad (23)$$

Total investment in job creation is equal to ψV_t .

²²We allow for the possibility that $N_t^{\text{new}} > V_t$, that is, the number of matches could exceed the number of vacancies. In simulated data this does happen, but not very often. If it happens, then firms end up with more than one worker per vacancy. This is not problematic as long as $\pi_{f,t}$ is not interpreted as a probability. Imposing that $N_t^{\text{new}} \leq V_t$ makes it more difficult to solve the model accurately. The case in which $N_t^{\text{new}} > (\Upsilon_N - N_{t-1})$ did not occur.

Goods market and the goods-market friction. In the description above, firms do not always sell their products. This is motivated with a very simple matching friction according to which the firm does not find a buyer for every product it puts up for sale. If the standard approach would be used, then the amount of goods available *as well as* the search effort by consumers would affect total sales. It obviously makes sense to assume that consumers have to put in some effort to buy products, which for some consumers is an enjoyable activity and for some it is not. It is less clear, however, whether *changes* in the amount of effort that consumers put into the activity of acquiring goods are important for cyclical fluctuations in the number of goods firms sell when one controls for changes in demand for the good itself. Such changes do play a role in Petrosky-Nadeau and Wasmer (2011), Bai, Ríos-Rull, and Storesletten (2012), and Michaillat and Saez (2013). In the models of these papers, recessions are deeper because shopping itself requires effort?²³ That may be the case, but the search friction adopted here does not rely on changes in the search effort of consumers. Here it is assumed that variations in search effort over and above a minimum level are not important for the actual number of transactions and the following formulation is used:²⁴

$$S_t = \pi_{y,t} (N_{t-1}y_t + X_{t-1}), \quad (24)$$

and $\pi_{y,t}$ is given by

$$\pi_{y,t} = \bar{\pi}_y + \zeta_y (Y_t - \bar{Y}) + \zeta_x (X_{t-1} - \bar{X}), \quad (25)$$

where $\zeta_y \geq 0$, $\zeta_x \leq 0$, and a bar under a symbol indicates that it is the variable's steady state value. A positive dependence of $\pi_{y,t}$ on the size of the market, Y_t , is similar to the search externality in

²³In fact, one could argue that unemployed workers looking to buy something can devote more time searching for the best "match", which could imply that search frictions in the goods market are *less* severe during recessions, since more consumers are unemployed during recessions.

²⁴This formulation implicitly imposes that customers do put in the minimum level required so that sales are not zero. A more complete specification would be the following:

$$S_t = \begin{cases} \pi_{y,t} (N_{t-1}y_t + X_{t-1})^{\nu_1} \underline{E}^{\nu_2} & \text{if } E_t \geq \underline{E} \\ 0 & \text{if } E_t < \underline{E} \end{cases} \quad 0 < \nu_1, \nu_2 \leq 1,$$

where E_t denotes the effort level and \underline{E} denotes the minimum effort level, e.g., the cost of going to the shopping mall. If an increase in E_t reduces utility, then $E_t = \underline{E}$. The assumption is made that the disutility of putting in \underline{E} is low enough, so that E_t is always equal to \underline{E} . We also assume that $\nu_1 = 1$. For the results in this paper, the value of ν_1 does not matter, since a process for $\pi_{y,t}$ is chosen such that goods-market efficiency, i.e., the level of sales, S_t , relative to the amount of available goods, $N_{t-1}y_t + X_{t-1}$, mimics the cyclical nature of its empirical counterpart. The lower ν_1 , the more procyclical $\pi_{y,t}$ has to be to make goods-market efficiency procyclical, that is, the calibrated value of ζ_y would be higher.

the pathbreaking model in Diamond (1982). Moreover, a positive value for ζ_y is consistent with the empirical findings based on aggregate data of section 2.3 and the empirical findings based on commercial aircraft markets of Gavazza (2011). The empirical analysis of this paper indicates a negative dependence of $\pi_{y,t}$ on beginning-of-period aggregate inventories. It does not seem unreasonable, that a higher stock of inventories reduces the chance of selling a given good. This raises the question why it also would not be more difficult to sell goods when the amount of *newly* produced goods, Y_t , increases. However, there is an important difference between a higher GDP, Y_t , and a higher level of aggregate inventories, X_{t-1} . A higher level of GDP not only means that the supply of goods increases, it also means that demand increases, since income earned is higher. In contrast, a higher level of beginning-of-period aggregate inventories definitely means that the supply of goods is higher, but will in general not lead to an equal increase in income.²⁵

Wages. Instead of relying on a theory such as Nash bargaining to describe wage setting, I adopt a flexible approach to model the behavior of the real wage rate. In particular, the wage rate rule is given by

$$\frac{\widehat{W}_t}{\widehat{P}_t} = \omega_0 \left(\begin{array}{l} \omega_1 \frac{((\pi_{y,t}\widehat{P}_t + (1-\pi_{y,t})(1-\delta_x)\widehat{\lambda}_{x,t})\alpha_0 k_t^\alpha - \widehat{R}_t k_t)}{\widehat{P}_t} \\ + (1 - \omega_1) \frac{((\pi_y\widehat{P} + (1-\pi_y)(1-\delta_x)\widehat{\lambda}_x)\alpha_0 k^\alpha - \widehat{R} k)}{\widehat{P}} \end{array} \right), \quad (26)$$

where a lower bar indicates the steady state value, $0 \leq \omega_1 \leq 1$, $0 < \omega_0 < 1$, and all variables are expressed in units of the market-produced good. The two terms on the right-hand side are the level of current-period revenues net of rental costs with unsold goods valued at $(1 - \delta_x)\widehat{\lambda}_{x,t}$ and its steady state equivalent. If $\omega_1 = 0$, then the real wage rate is fixed. If $\omega_1 > 0$, then wages increase with the firm's net revenues. ω_0 indicates the average share of revenues net of rental costs that goes to the worker. The other fraction goes to the entrepreneur as compensation for creating the job.

Inventories when prices are flexible. It is assumed that prices in the goods market are such that the consumer is indifferent between buying and not buying an additional unit. That is,

$$\widehat{P}_t \frac{\partial U(C_{e,t})}{\partial C_{e,t}} = C_t^{-\nu}. \quad (27)$$

²⁵Inventories are produced in the past. Workers that produced these inventories were paid in the past. Depending on how inventories are valued, the production of inventories may even have generated income through profits. The actual sale of inventories may generate *additional* income in the current period when the sale price exceeds the accounting price used to value inventories, but the value of this additional income is likely to be less than the total value of the inventories available for sale.

The price level is flexible. This raises the question why the price level does not adjust to ensure that all products are sold. The reason is the following. Ex ante, i.e., before trading starts, all firms are the same. Thus, it makes sense to focus on the case in which all firms charge the same price. In equilibrium, prices are such that the implied amount that customers demand, S_t , and the implied amount that firms supply, Q_t , is such that

$$S_t = \pi_{y,t} Q_t, \text{ where} \quad (28)$$

$$Q_t = N_{t-1} \alpha_0 \exp(Z_t) k_t^\alpha + X_{t-1}. \quad (29)$$

When choosing the amount of goods supplied, Q_t , firms take into account that only a fraction $\pi_{y,t}$ is sold. Ex post, some goods are sold and some are not. If a firm did not sell some products, then it has an incentive to lower the price of these unsold goods *if* the goods can still be sold within the same period. This possibility is ruled out by assumption. That is, firms only find out at the end of the period whether a good is sold or not. At that point, the next period starts. At the beginning of this next period, a good that is newly produced is not distinguishable from a good that was produced in the past and did not sell (adjusted for any possible depreciation). Consequently there is no reason why the firm offering goods out of inventories should charge lower prices.

Aggregation and equilibrium. Individual firms have different levels of inventories. For example, newly created firms have no inventories at all. But it is easy to obtain an expression for aggregate inventories. All firms face the same value for $\pi_{y,t}$, which implies that all firms choose the same level for capital, i.e., $k_{i,t} = k_t$. The law of motion for aggregate inventories, X_t , is thus equal to

$$\begin{aligned} X_t &= (1 - \delta_n)(1 - \delta_x) \sum_i [(1 - \pi_{y,t})(\alpha_0 \exp(Z_t) k_t^\alpha) + x_{i,t-1}] \\ &= (1 - \delta_n)(1 - \delta_x) [N_{t-1}(1 - \pi_{y,t}) \alpha_0 \exp(Z_t) k_t^\alpha] + (1 - \pi_{y,t}) X_{t-1} \\ &= (1 - \delta_n)(1 - \delta_x)(1 - \pi_{y,t}) (\alpha_0 \exp(Z_t) K_{t-1}^\alpha N_{t-1}^{1-\alpha} + X_{t-1}). \end{aligned}$$

Equilibrium in the rental market for capital goods requires that

$$N_{t-1} k_t = K_{t-1}, \quad (30)$$

that is, the amount of capital firms choose in period t , k_t , is equal to the available amount of capital per firm. Total amount of cash flows generated in the corporate sector, \widehat{D}_t , is given by

$$\widehat{D}_t = \pi_{y,t} \widehat{P}_t (N_{t-1} \alpha_0 \exp(Z_t) k_t^\alpha + X_{t-1}) - \widehat{W}_t N_{t-1} - \widehat{R}_t K_{t-1} - \psi V_t. \quad (31)$$

An equilibrium is a set of functions $\pi_y(\mathbf{S}_t)$, $\pi_f(\mathbf{S}_t)$, $\widehat{P}(\mathbf{S}_t)$, and $\widehat{W}(\mathbf{S}_t)$ and a set of policy functions for the agents' choices such that (i) the policy functions solve the corresponding optimization problems taking probabilities and prices as given and (ii) and the policy functions imply $\pi_y(\mathbf{S}_t)$, $\pi_f(\mathbf{S}_t)$, $\widehat{P}(\mathbf{S}_t)$, and $\widehat{W}(\mathbf{S}_t)$.

Walras law. Goods market equilibrium requires that

$$C_t + I_t + \psi V_t = \pi_{y,t} (\alpha_0 \exp(Z_t) K_t^\alpha N_t^{1-\alpha} + X_{t-1}).$$

This equation is implied by the budget constraint of the household and the definition of \widehat{D}_t .

4 Calibration

The parameters β , α , δ_k , and ν are set to standard values. In particular, $\beta = 0.99$, $\alpha = 0.3$, $\delta_k = 0.025$, and $\nu = 1$. Typical values for the parameters of the law of motion for productivity, ρ and σ , are 0.95 and 0.007. In addition, the results are given for a process with a value for ρ equal to 0.7 and a value for σ such that the volatility of Z_t is the same for the two processes. By considering a less persistent process for the stochastic driving variable, it becomes clear that the model can generate very persistent behavior even when Z_t itself is not that persistent. The depreciation rate of inventories, δ_x , is set equal to 0.10. This captures physical depreciation but also other possible reasons for value reduction and storage costs.²⁶

The wage process is characterized by two parameters, ω_0 and ω_1 . The value of ω_0 is chosen to match a measure of observed employment volatility, namely $\sigma(\ln N) / \sigma(\ln Y)$. The value of the target is equal to 0.466 which is also used in Den Haan and Kaltenbrunner (2009). Several empirical studies suggest that wages are not that responsive.²⁷ Following Den Haan and Kaltenbrunner (2009), ω_1 is set equal to 3/4, that is, wages respond quite strongly to current-period profits. Thus, the results here do not rely on having sticky wages.²⁸

²⁶The value of this parameter is conservative. It is slightly lower than the value used by Khan and Thomas (2007), who calculate the cost of inventory storage cost to be equal to 12% of the value of inventories held. Their calculations are based on data provided by Stock and Lambert (1987) and Richardson (1995). The estimates of the latter are substantially higher, because they include the cost of money, insurance, and taxes, which should not be part of δ_x in this model.

²⁷This is true for results based on estimated DSGE models and for results based on micro-level wage data. See Barattieri, Basu, and Gottschalk (2010).

²⁸In fact, the value of ω_1 is not important in this paper. The reason is that ω_0 is set to match observed employment volatility. If ω_1 is lowered, then a higher value of ω_0 would ensure that employment volatility would not be affected.

The specification for goods-market efficiency depends on three parameters, $\underline{\pi}_y$, ζ_y , and ζ_x . The value of $\underline{\pi}_y$ is the steady state value of $\pi_{y,t}$ and is set equal 0.4, which is the average of the observed measure for goods-market efficiency for final sales of domestic businesses, as documented in table 1. As discussed below, the values of ζ_y and ζ_x are chosen to match a measure of the volatility of $\pi_{y,t}$, namely $\sigma(\pi_y)/\sigma(Y)$, and a measure of the volatility of sales, namely $\sigma(S)/\sigma(Y)$.

The remaining parameters are related to employment determination. Following the literature, ϕ_1 is set equal to 0.5, which means that the elasticity of $\pi_{f,t}$ with respect to labor market tightness is equal to one half.²⁹ Based on results in Den Haan, Ramey, and Watson (2000), the job destruction rate, δ_n , is set equal to 0.052 and the values for the scaling coefficient in the matching function, ϕ_0 , and the cost of starting a project, ψ , are such that the steady state unemployment rate is equal to 12% and the steady state value for the number of matches per vacancy is equal to 0.71.³⁰ This measure for the unemployment rate takes into account those workers that indicate that they would like to work but are not counted in the formal unemployment definition.

5 Results

Two experiments are discussed to bring to light key properties of the model. In the first experiment, the ability to sell, $\pi_{y,t}$ only depends on aggregate output and not on beginning-of-period aggregate inventories. The parameter affecting the dependence of $\pi_{y,t}$ on aggregate output, ζ_y , is chosen such that the volatility and the procyclical behavior of goods-market efficiency, $\pi_{y,t}$, match their empirical counterparts. Another key parameter in this experiment is ω_0 , the share of revenues that accrues to the workers. Hagedorn and Manovskii (2008) point out that the response of employment to changes in firm revenues is larger when ω_0 is higher and the profit margin is, thus, lower. Therefore, changes in $\pi_{y,t}$ will have a larger impact on the economy if ω_0 is closer to 1. To discipline the model's response to changes in $\pi_{y,t}$, the value of ω_0 is chosen such that the model generates a realistic amount of employment volatility.³¹

In the second experiment, $\pi_{y,t}$ depends on aggregate activity, and—motivated by this paper's empirical findings—depends negatively on the beginning-of-period aggregate level of inventories.

²⁹Empirical support for this value is given in Petrongolo and Pissarides (2001).

³⁰The latter is based on van Ours and Ridder (1992).

³¹It is not straightforward to calibrate ω_0 using direct measures of entrepreneurial compensation. Observed profit shares include compensation for equity financing, while in the model $1 - \omega_0$ is only the compensation for the entrepreneurial activity of creating a job.

Finding the parameter values at which the model exactly hits the targets entails a non-trivial search in the parameter space. Moreover, the calibration procedure relies on second-order moments, the calculation of which requires a numerical solution of the policy functions. Consequently, a fast solution method is needed. The results reported are based on first-order perturbation. At the calibrated parameter values, the model is also solved with a global solution method and the results reported are very similar for the two solution methods.

5.1 The role of a procyclical goods-market friction for business cycles

As documented in section 2, goods-market efficiency, $\pi_{y,t}$, is procyclical and quite volatile. The ability to sell, $\pi_{y,t}$, affects firm profitability and, thus, aggregate activity and it is in turn affected by the level of aggregate activity. Consequently, variation in $\pi_{y,t}$ could be an important channel through which shocks are magnified and propagated. In this subsection, the specification for $\pi_{y,t}$ is given by

$$\pi_{y,t} = \underline{\pi}_y + \zeta_y (Y_t - \underline{Y}). \quad (32)$$

That is, $\pi_{y,t}$ is allowed to depend on aggregate real activity, but not on aggregate inventories. Model properties are presented in table 2, which reports unconditional business cycle moments, and in figure 2, which displays the impulse response functions (IRFs).

The role of inventories for GDP fluctuations. First consider the benchmark results when ζ_y and ω_0 are chosen such that the model exactly matches the observed cyclical behavior of goods market efficiency and employment. Since goods-market efficiency is a simple transformation of the inventory-sales ratio, the calibration automatically ensures that the model also matches the cyclical behavior of the inventory-sales ratio. Table 2 documents that the model predicts the typical ordering of the volatility of consumption, investment, and output. The calculated shares of investment in inventories for GDP fluctuations are equal to 0.149 and 0.094 when ρ is equal to 0.7 and 0.95, respectively.³² The empirical counterpart is equal to 0.193. Thus, a non-trivial part of GDP fluctuations is attributable to investment in inventories, although this version of the model somewhat underpredicts the importance of inventories for business cycle fluctuations of GDP.

³²Following Fujita and Ramey (2009), this contribution is calculated as follows. Here, \tilde{Y}_t is the cyclical component of GDP divided by its trend value, \tilde{X}_t is the cyclical component of investment in inventories divided by the trend value of GDP, and \tilde{Y}^* is defined by $\tilde{Y}_t = \tilde{Y}^* + \tilde{X}_t$. The latter identity implies that $\text{variance}(\tilde{Y}_t) = \text{covariance}(\tilde{Y}_t, \tilde{Y}_t^*) + \text{covariance}(\tilde{Y}_t, \tilde{X}_t)$. The fraction of GDP fluctuations that is attributed to investment in inventories is, thus, given by $\text{covariance}(\tilde{Y}_t, \tilde{X}_t) / \text{variance}(\tilde{Y}_t)$.

At the calibrated parameter values, the IRFs of inventories and sales associated with a positive shock to Z_t are positive at all time horizons. With both responses being positive, it is not surprising that the model correctly predicts that inventories are positively correlated at business cycle frequencies. The model also correctly predicts that inventories and sales are *negatively* correlated at higher frequencies. This is more surprising given that the IRFs of both variables are positive. The reason is that the response of inventories is a bit delayed. This means that the high-frequency component of the inventories response is initially negative, whereas the high-frequency component of the real activity response is initially positive.

Magnification and persistence. The autocorrelation coefficients for employment and output indicate that the model is capable of adding quite a bit of persistence. For example, when $\rho = 0.7$, the autocorrelation coefficients are equal to 0.982 and 0.997 for employment and output, respectively.³³

Figure 2 displays the IRFs of employment, output, and goods-market efficiency. To facilitate comparison, the IRF of productivity is also shown in the panels for the employment IRF and the output IRF. The variance of the innovation is chosen such that the unconditional variance of Z_t is the same for the two values of ρ . Consequently, the process with the higher value for ρ has a smaller innovation variance and, thus, smaller initial responses. The IRFs are given for three different values of ζ_y . The first value is the one for which model predictions for $\pi_{y,t}$ match the observed cyclical behavior of its empirical counterpart. The second value of ζ_y considered is 0. A comparison of the responses when $\zeta_y = 0$ with the responses for the calibrated value of ζ_y reveals the role of goods-market friction in magnifying and propagating shocks. The third value of ζ_y is such that the model responses to the *non-permanent* shock considered here are close to being permanent. By considering higher values of ζ_y one learns what the role of goods-market frictions can be if parameter choice is not constrained by the observed volatility of $\pi_{y,t}$.

The graphs show that the employment and output responses to a shock to Z_t are substantially more persistent than the responses of Z_t itself. This is also true when $\zeta_y = 0$ and $\pi_{y,t}$ is, thus, constant. When ζ_y equals zero, shocks are propagated because of the matching friction and the desire to smooth consumption. The responses are more persistent, however, when ζ_y is equal to its calibrated value and substantially so when $\rho = 0.7$. The same is true for the magnification of the shock. When $\rho = 0.95$, that is, when the underlying shock is already quite persistent, then

³³Since filtering also affects the autocorrelation, the unfiltered series are used to calculate these statistics.

the model does not add a lot of magnification and additional persistence when ζ_y is equal to its calibrated value. When ζ_y is increased above its calibrated value, however, then the goods-market friction also generates remarkable magnification and propagation when $\rho = 0.95$.

Why this version cannot match all inventory and sales facts. At the calibrated parameter values, the model predicts that output and sales have roughly the same volatility. In the data, however, sales are *less* volatile than output. This somewhat surprising empirical finding has triggered an extensive literature with ingenious attempts to build models to get this right. The model developed here could generate the right ordering for the volatility of sales and output quite easily. As indicated in the " $\zeta_y = 0$ " column in table 2, sales are substantially less volatile than output when goods-market efficiency is constant, especially when $\rho = 0.7$. When $\pi_{y,t}$ is constant, then sales are simply a fraction of the amount of available goods for sale, that is, newly produced goods *plus* the stock of inventories. The latter is a stock variable and less volatile than output. Consequently, when sales are a constant fraction of the sum of output and inventories, then sales will be less volatile than output.

The problem with setting ζ_y equal to zero and keeping $\pi_{y,t}$ constant, however, is that the model would no longer generate the right cyclical behavior for the goods-market efficiency measure, $\pi_{y,t}$, and, thus, would not generate the right cyclical behavior of the inventory-sales ratio. Starting at zero, an increase in ζ_y induces volatility in the goods-market efficiency measure, which is consistent with the data. As long as ζ_y is low enough, the model also correctly predicts that sales are less volatile than output. However, when ζ_y is such that the model matches the volatility of $\pi_{y,t}$, the volatility of sales exceeds the volatility of output. Consequently, the model cannot match both the correct procyclical behavior of $\pi_{y,t}$ and the right relative volatility of sales and output by only changing ζ_y . In the next subsection, it will be shown that the model can match both properties by allowing $\pi_{y,t}$ to also depend on aggregate inventories.

The role of the goods-market friction when $\zeta_x = 0$. The finding that the model's implications become increasingly at odds with well-known facts from the inventory literature as ζ_y takes on higher values also means that the role of the goods-market friction for magnification and propagation is limited. This is most clear when $\rho = 0.95$. In this case, the value of ζ_y , which directly affects the magnitude of cyclical fluctuations in the goods-market friction, can be increased a lot before the model's solution becomes explosive. As documented in figure 2, the model generates stunning magnification and propagation at high values for ζ_y . Moreover, figure 2 also documents that $\pi_{y,t}$

drops just a few percentage points at the highest value for ζ_y considered. Although the implied volatility for $\pi_{y,t}$ is higher than what is observed in the data, the generated changes in $\pi_{y,t}$ do not seem outlandish. However, some implications for the model's properties regarding inventories are clearly inconsistent with the data when ζ_y takes on high values.

It is quite intuitive that making goods-market frictions more important will at some point imply that the model's predictions for sales and inventories deteriorates. Consider a negative TFP shock. The reduction in economic activity induces a reduction in $\pi_{y,t}$. The larger the value of ζ_y , the larger the reduction in $\pi_{y,t}$, which in turn implies stronger magnification and more persistence. But the reduction in $\pi_{y,t}$ also implies that less is sold relative to what is produced. As the reduction in $\pi_{y,t}$ becomes larger, then at some point sales will drop by *more* than output and inventories will *increase*. Both properties are inconsistent with observed facts. Again, consider the case when $\rho = 0.95$ and ζ_y is set equal to its highest possible value. In this case, the standard deviation of sales is 1.823 times the standard deviation of output, whereas the empirical ratio is only 0.901. Similarly, the correlation between inventories and sales is *negative* whereas it is positive in the data.

5.2 Results when goods market friction also depends on inventories

The results discussed so far show that the model cannot simultaneously match the correct cyclical behavior of goods-market efficiency and predict that sales are less volatile than output when $\pi_{y,t}$ *only* varies with aggregate output. The empirical results in section 2 indicate, however, that $\pi_{y,t}$ not only depends on output, but also depends (negatively) on beginning-of-period aggregate inventories. To capture both aspects the following specification for $\pi_{y,t}$ is considered:

$$\pi_{y,t} = \underline{\pi}_y + \zeta_y (Y_t - \underline{Y}) + \zeta_x (X_{t-1} - \underline{X}) \text{ with } \zeta_y > 0, \zeta_x < 0. \quad (33)$$

The values of ζ_y , ζ_x , and ω_0 are chosen to match the observed volatility of employment, the observed cyclical behavior of $\pi_{y,t}$, *and* the observed value for the volatility of sales relative to the volatility of output. Table 3 reports unconditional business cycle moments and figure 3 displays the impulse response functions (IRFs).

The role of inventories for GDP fluctuations. As documented in table 3, this version of the model also generates the right ordering for the volatility of consumption, investment, and output. At the calibrated parameter values, the share of investment in inventories for cyclical fluctuations in GDP is equal to 0.240 when ρ equals 0.7 and 0.259 when $\rho = 0.95$. Both are fairly close

to the observed share which is equal to 0.193. Moreover, at the calibrated parameter values the model predicts correctly (again) that inventories and sales are positively correlated at business cycle frequencies and negatively correlated at high frequencies.

Why this version can match the inventory and sales facts. As pointed out in the previous subsection, $\pi_{y,t}$ cannot respond too strongly to changes in real activity, because sales would be more volatile than output if the response is large enough. On the other hand, the response has to be sufficiently strong to ensure that $\pi_{y,t}$ is sufficiently volatile. The dilemma of matching both properties can be solved by letting $\pi_{y,t}$ depend positively on real activity (that is, $\zeta_y > 0$) and—as indicated by the empirical findings discussed in section 2—negatively on beginning-of-period aggregate inventories (that is, $\zeta_x < 0$). In fact, with the appropriate choice of ζ_y and ζ_x the model can exactly match the observed volatility and procyclical behavior of $\pi_{y,t}$ (and, thus, match the observed cyclical behavior of the inventory-sales ratio) as well as exactly match the observed volatility of sales relative to the volatility of output. Additional support for the specification used can be found in the fact that the calibrated values for ζ_y and ζ_x are not that different from the empirical estimates discussed in section 2. For example, when $\rho = 0.95$, then the calibrated values are 0.161 and -0.191 for ζ_y and ζ_x , respectively. The empirical estimates for these two parameters are equal to 0.25 and -0.14 .³⁴

What does the calibrated specification for $\pi_{y,t}$ imply for the behavior of $\pi_{y,t}$ following a shock to Z_t . The results are given in the two panels of the bottom row of figure 3. Similar to the results with $\zeta_x = 0$, $\pi_{y,t}$ displays a sharp drop when Z_t is hit by a negative shock. In contrast to the results with $\zeta_x = 0$, $\pi_{y,t}$ recovers rapidly and goes *above* its pre-shock value as the reduction in aggregate inventories puts upward pressure on $\pi_{y,t}$. The result that the response of $\pi_{y,t}$ switches signs makes it possible to have a sufficiently volatile $\pi_{y,t}$ without making sales too volatile.

The role of the goods-market friction when $\zeta_x < 0$. Compared with other models in the literature that incorporate inventories into business cycle models, the model developed here is remarkably simple. Despite its simplicity, it can generate key facts about inventories and it captures the observed importance of investment in inventories for fluctuations in aggregate output. The question arises whether goods-market frictions are an important channel through which shocks get magnified and propagated when the model matches all key facts regarding the joint behavior of

³⁴Since the regression is affected by endogeneity issues, the estimates of ζ_y and ζ_x should be interpreted with care, but these theoretical results suggest that a more causal interpreted may not be that unreasonable.

inventories, sales, and output.

Figure 3 plots the employment and output IRFs at the calibrated values for ζ_y and ζ_x and when ζ_y is set as high as possible without having explosive responses, keeping ζ_x fixed. Resembling the results in section 5.1, employment and output responses are larger and more persistent at higher values of ζ_y . Thus, by increasing ζ_y the model can magnify and propagate shocks, but an increase in ζ_y above its calibrated value comes at the cost of doing worse in terms of matching the observed behavior of inventories. In particular, sales become too volatile relative to output.

The question arises how the model in which $\pi_{y,t}$ is constant compares to the model in which $\pi_{y,t}$ responds to real activity and accumulated inventories as indicated by the calibrated values for ζ_y and ζ_x . That is, how important are changes in goods-market efficiency when the model is calibrated to be consistent with the joint behavior of inventories, sales, and output. The IRFs for the case when both ζ_y and ζ_x are equal to zero are also plotted in figure 3. The figure shows that eliminating the calibrated fluctuations in $\pi_{y,t}$ results in *more* magnification and *more* persistence, whereas the opposite was found in the previous subsection. The reason is the following. Consistent with the results in the previous subsection, eliminating the positive dependence of $\pi_{y,t}$ on real activity leads to less magnification and less persistence. Eliminating the negative dependence of $\pi_{y,t}$ on aggregate inventories, however, leads to *more* magnification and *more* persistence and this effect turns out to be stronger. The latter effect is only slightly stronger and the employment and output IRFs based on the calibrated specification for $\pi_{y,t}$ are quite similar to the IRFs based on a constant value for $\pi_{y,t}$. Although the richer specification for $\pi_{y,t}$ makes it possible to match the key facts regarding the behavior of inventories, sales, and output, it also means that variation in the goods-market friction no longer works as a mechanism to magnify and propagate shocks. The concluding section points out that this does not necessarily mean that goods-market frictions do not play an important role in the transmission of shocks, but this role does seem to be restricted by the observed behavior of inventories. At least in this type of model without any other type of friction such as sticky prices.

6 Goods-market frictions, the verdict

The presumption that frictions in goods markets and frictions in labor markets, and especially their interaction, are important for business cycles seems reasonable. If frictions prevent goods market from working efficiently, then this is likely to affect firms' sales and firms' hiring decisions. Similarly, if labor markets do not work efficiently, then this will affect the job-finding rate, which in turn will

affect goods-market activity. This paper formalizes this idea and shows that a model with goods and labor-market frictions can quite easily magnify and propagate shocks. Moreover, the model can also replicate key aspects of the behavior of inventories, sales, and output. The problem is that it cannot do both at the same time. Does this mean that realistic goods-market frictions do not change the dynamics of business cycles very much and that there is, thus, not much point in incorporating a goods-market friction in business cycle models?

Before addressing these questions, the key aspects of the restrictions that observed inventories, sales, and output data impose on cyclical changes in the goods-market friction are highlighted. Suppose that a negative shock hits the economy. If goods-market frictions are procyclical, then this would mean that such a negative shock would impede sales. The data imply, however, that firms manage to let output drop by *more* than sales. This *seems* to indicate that firms are quite efficient in scaling down the size of operations during downturns. Moreover, if output drops by more than sales, then the probability to sell, i.e., the severity of the goods-market friction, cannot have worsened too much. That is, the level of sales are not that bad *relative to* the level of output. If the goods-market friction would worsen too much, then a negative shock would lead to an *increase* in inventories and the drop in sales would *exceed* the drop in output.

Nevertheless, it is still a good idea to incorporate goods-market frictions, since—as documented in this paper—a simple goods-market friction can match key facts about inventories. Given that changes in the investment in inventories are known to be important for GDP fluctuations, it makes sense to include inventories in business cycle models.

Now consider the question whether the results in this paper indicate that cyclical changes in goods-market frictions are unlikely to be quantitatively important for aggregate fluctuations. The provision of many types of services does not allow for inventories. If a hairdresser has no customers, then this does not lead to an increase in inventories. If there are no inventories, then the observed behavior of inventories cannot impose restrictions on the properties of the goods-market friction like they do in this paper. But the question arises whether the behavior of goods-market frictions would be very different for services than for manufacturing and wholesale.

Another reason why goods market frictions could be more important than the results in this paper indicate is that the cyclicity of the goods-market friction measure used in this paper understates the procyclical behavior of the true goods-market friction, because this paper's measure is based on *actual* output instead of *potential* output. To explore this possibility, consider the following example. During normal times, firms produce 100 goods, start the period with 100 goods in

inventories, and sell 100 goods. Thus, the sell probability is equal to one-half. In addition, suppose that firms would like to reduce output to 80 goods when the economy is hit by a negative shock *and* the sell probability would remain equal to one-half. If the sell probability would indeed remain constant, then sales would drop by 10 to 90, which is less than the drop in output, and inventories would drop to 90. Both responses are consistent with the data. Now suppose that the sell probability does not remain equal to one-half, but drops to one third during an economic downturn. If the firms would still produce 80, then sales would drop to 60, i.e., one third of 180 (80 produced goods and 100 from inventories). Inventories would increase and the drop in sales is bigger than the drop in output. Both responses are inconsistent with the data, which is the reasons why the calibrated models did not consider such large changes in the goods-market friction. But now suppose that firms can choose to keep labor idle and that there is some benefit of doing so.³⁵ Faced with a sharp drop in sales, one could argue that the firm should lower output further, say to 20 units and enter the market this period with 20 newly produced goods and 100 goods in inventories. If the firm still sells 60, then the *observed* value for the sell probability would be equal to one-half, that is, the observed goods-market friction would show no change even though the firm faces a sharp reduction in the sell probability if one considers actual sales relative to what the firm could produce given the size of its workforce.

Unfortunately, there are several problems with this reasoning. First, in this numerical example the amount firms can sell does not depend on the amount of goods that are available. That is, sales are kept constant at 60 when production is reduced. But the idea of the goods-market friction is that mismatch between what producers produce and what consumers want is smaller when markets are bigger. More importantly, if firms can lower actual production during recessions without negatively affecting the amount they sell, then the question arises why they would not do so during normal times? If output can be reduced without negatively affecting sales, then firms could lower production during normal times as well, for example, to a level of 50 units, which—if sales remain fixed at 100—would imply that the probability to sell increases from one-half to two-thirds. One would have to argue that this increase in efficiency only happens during downturns, perhaps because operating efficiently is only essential during downturns or the chance of stockouts are less problematic during downturns.

Finally, consider the possibility that inventories do not increase during economic downturns and the sell probability does not drop by that much exactly because the supply of goods falls sharply

³⁵The benefit could be a reduction in material costs or a direct utility benefit of working less.

during down turns. This may very well be the case, but if—in the end—the sell probability does not drop by that much, then why would output drop by so much? If the entrepreneur’s share of the surplus, $1 - \omega_0$, is small on average, then small changes in $\pi_{y,t}$ can induce large proportional changes in the entrepreneur’s revenue, which in turn leads to large fluctuations in job creation and aggregate output. To generate sufficiently volatile employment and aggregate output, this paper follows Hagedorn and Manovskii (2008) and adopts values for ω_0 that are already quite high.³⁶ But choosing an even higher value for ω_0 would not change the conclusion. For example, consider the case when ρ is equal to 0.95, $\zeta_y = 0.161$, and $\zeta_x = 0$. As documented in figure 2 and table 2, in this case the goods-market friction magnifies shocks somewhat, but not by much. If ω_0 is increased with one percentage point, then shocks have a substantially larger impact on the economy. For example, the maximum drop in employment increases considerably, namely from 4.8% to 7.0%. But a similar increase is observed for the responses when $\zeta_y = 0$. And relative to the " $\zeta_y = 0$ " responses, the proportional increase in responses when ζ_y is increased to 0.161 is very similar for the two values of ω_0 considered.

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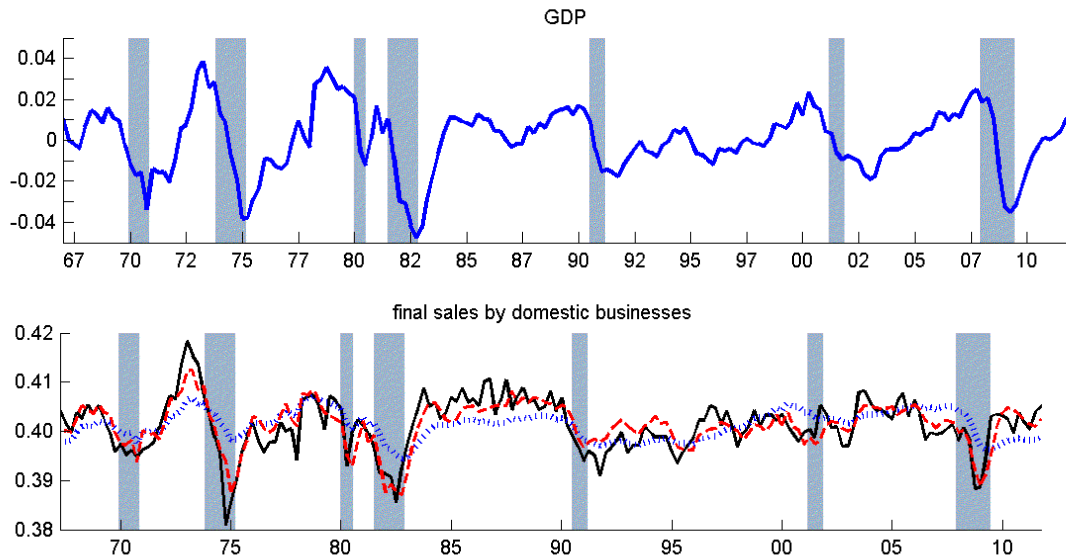
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³⁶See tables 2 and 3.

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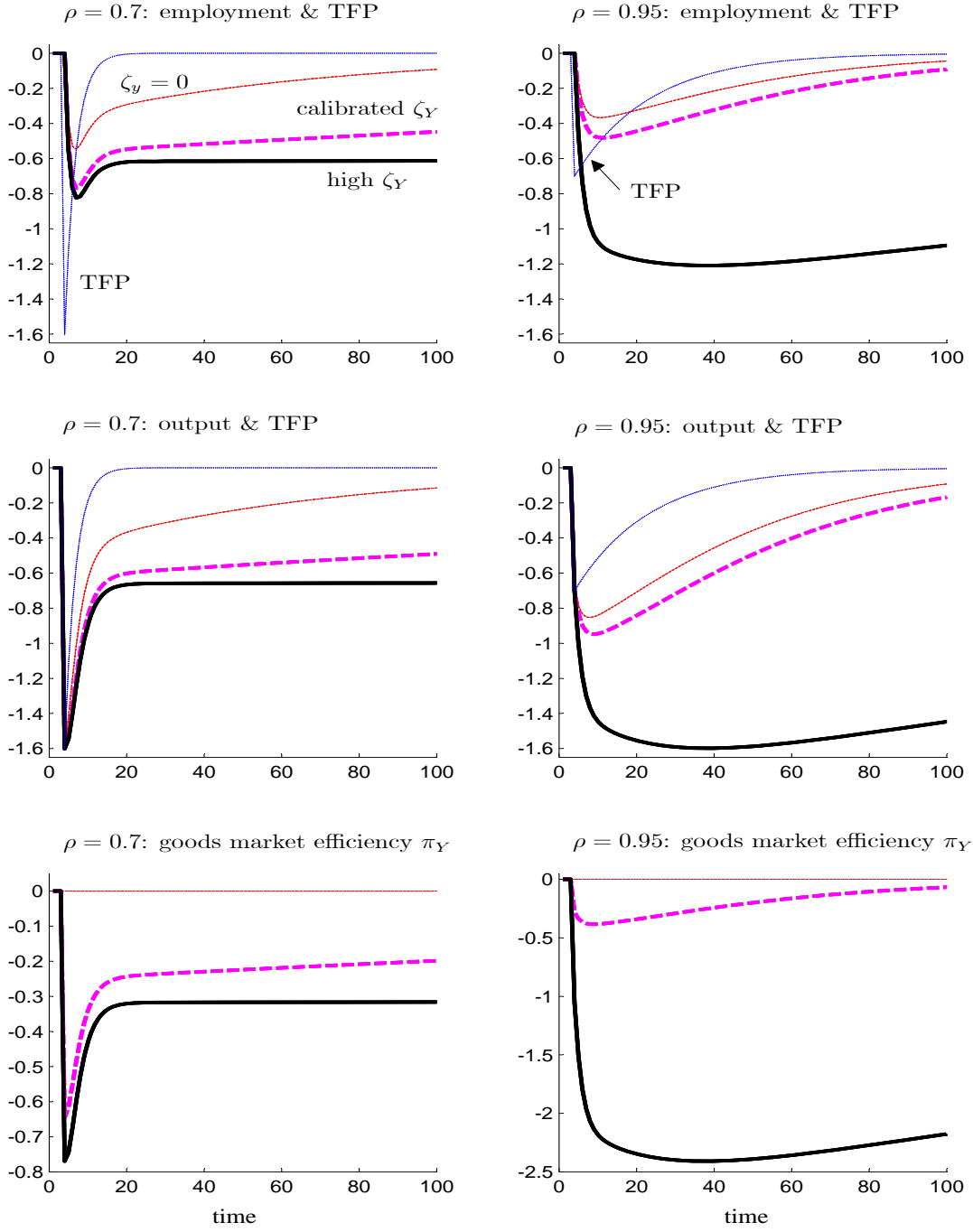
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Figure 1: Cyclical behavior of goods-market efficiency



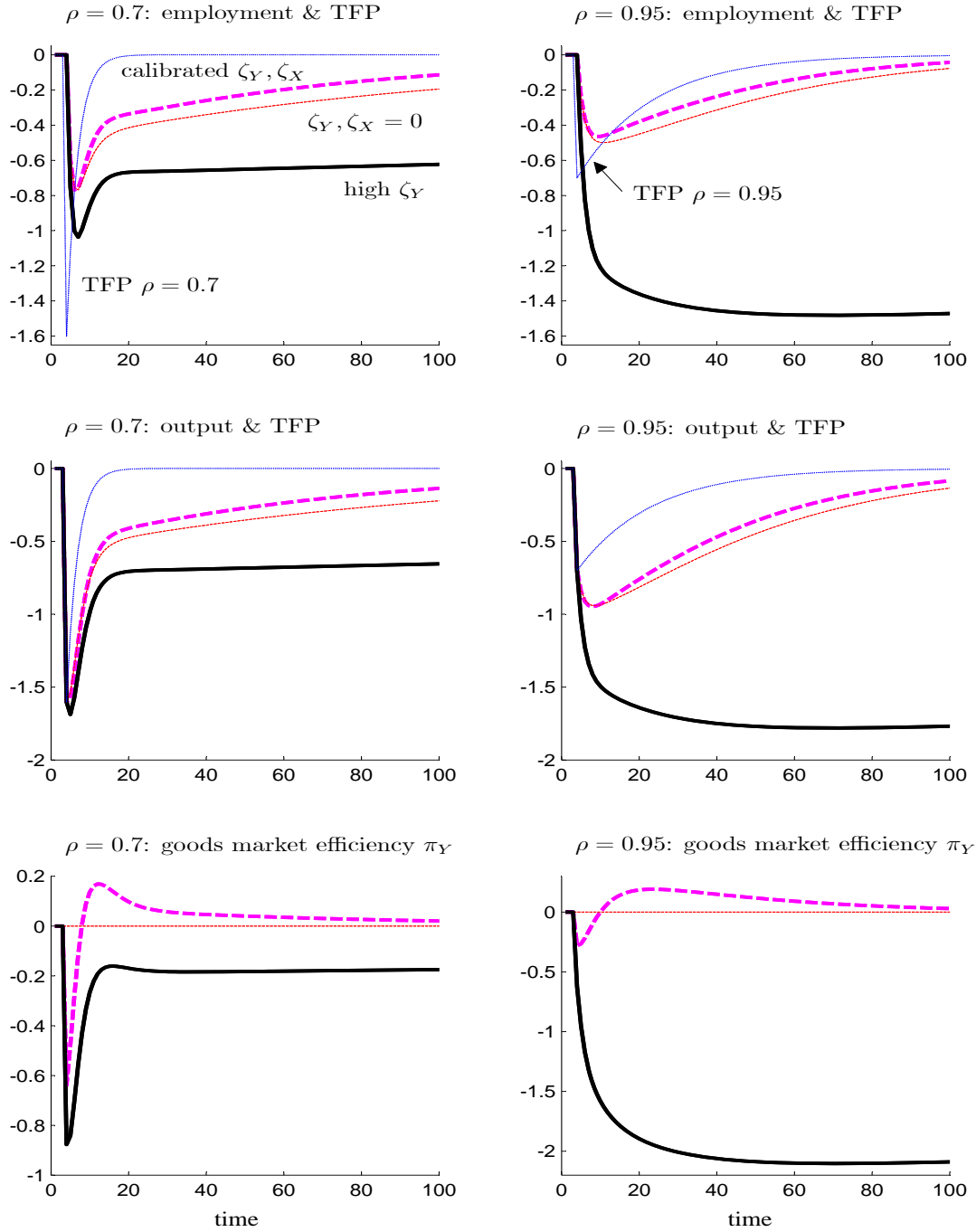
Notes: The top panel plots the cyclical component of GDP. The solid line in the bottom panel is the cyclical component of goods-market efficiency (plus the mean). If the efficiency measure is equal to 0.5, then sales are 50% of newly produced output plus inventories. The bottom panel also plots the fitted values from a projection using cyclical GDP (dotted line) and the fitted values from a projection using cyclical GDP and lagged cyclical inventories (dashed line).

Figure 2: IRFs when not all key inventory facts are matched



Notes: Each panel plots the responses to a productivity shock. The IRF labeled "calibrated ζ_y " corresponds to the case when ζ_y and ω_0 are chosen to match σ_N/σ_Y and σ_{π_y}/σ_Y . This version of the model does not match the observed value of σ_S/σ_Y .

Figure 3: IRFs when key inventory facts are matched



Notes: Each panel plots the responses to a productivity shock. The IRF labeled "calibrated ζ_y, ζ_x " corresponds to the case when ζ_y, ζ_x , and ω_0 are chosen to match $\sigma_N/\sigma_Y, \sigma_{\pi_y}/\sigma_Y$, and σ_S/σ_Y .

Table 1: Summary statistics - Private non-farm inventories and final sales

	total	goods + structures
$\rho_{X,S}$	0.632	0.648
$\rho_{X,S}, BP_{\leq 4Q}$	-0.364	-0.358
$\rho_{X,S}, BP_{\leq 8Q}$	-0.269	-0.270
$\rho_{\Delta X,S}$	0.356	0.361
σ_S/σ_Y	0.909	0.902
$\rho_{X,S}, BP_{\leq 4Q}$	1.033	0.970
$\rho_{X,S}, BP_{\leq 8Q}$	1.006	0.972
mean $\pi_{y,t}$	0.550	0.401
mean X_t/S_t	0.821	1.498
σ_{π_y}	0.0041	0.0047
σ_{π_y}/σ_S	0.215	0.184
$\rho_{\pi_y,Y}$	0.362	0.607
$\rho_{\pi_y,X_{-1}}$	-0.508	-0.251

Notes: $BP_{\leq NQ}$ indicates that the band-pass filter is used to extract that part of the series that is associated with fluctuations with a period less than N quarters. All other second-order moments are for HP-detrended data. σ_i is the standard deviation of variable i ; $\rho_{i,j}$ is the correlation coefficient of variables i and j ; S stands for sales, X stands for inventories, Y stands for GDP, and $\pi_y = S/(Y + X_{-1})$ is the measure of goods-market efficiency.

Table 2: Results when not all key inventory facts are matched

	<i>data</i>	<i>model with $\rho = 0.7$</i>			<i>model with $\rho = 0.95$</i>		
		ζ_y, ω_0 calibrated	$\zeta_y = 0$	high ζ_Y	ζ_y, ω_0 calibrated	$\zeta_y = 0$	high ζ_y
<i>parameter values</i>							
ζ_y		0.162	0	0.193	0.162	0	0.600
ζ_x		0	0	0	0	0	0
ω_0		0.993	0.993	0.993	0.970	0.970	0.970
<i>calibrated moments</i>							
σ_N/σ_Y	0.466	=	0.357	0.499	=	0.386	0.732
σ_{π_Y}/σ_Y	0.162	=	0	0.189	=	0	0.566
σ_S/σ_Y	0.901	1.006	0.775	1.064	1.045	0.853	1.823
<i>inventory properties</i>							
$\rho_{X,S}$	0.648	0.674	0.845	0.644	0.803	0.913	-0.509
$\rho_{X,S}, BP_{\leq 4Q}$	-0.358	-0.690	-0.485	-0.700	-0.602	-0.364	0.485
$\rho_{X,S}, BP_{\leq 8Q}$	-0.270	-0.086	0.286	-0.118	0.010	0.369	0.123
<i>standard business cycle statistics</i>							
σ_C/σ_Y	0.535	0.338	0.238	0.367	0.447	0.354	0.896
σ_I/σ_Y	3.554	3.710	2.906	3.935	3.199	2.676	12.431
<i>role of investment in inventories for GDP fluctuations</i>							
ρ_{XY}/σ_Y^2	0.193	0.149	0.384	0.108	0.094	0.316	-0.461
<i>autocorrelation unfiltered series</i>							
$\rho_{N,N(-1)}$		0.982	0.9153	0.994	0.993	0.989	0.999
$\rho_{Y,Y(-1)}$		0.997	0.984	0.999	0.996	0.995	1.000

Notes: This table reports summary statistics of model-generated data and the empirical counterparts. $BP_{\leq NQ}$ indicates that the band-pass filter is used to extract that part of the series that is associated with fluctuations with a period less than N quarters. All other second-order moments are for HP-detrended data. σ_i is the standard deviation of variable i ; $\rho_{i,j}$ is the correlation coefficient of variables i and j ; S stands for sales, X stands for inventories, Y stands for GDP, Y^* is the output measure for these firms data (constructed using the sales and inventory data), and $\pi_y = S/(Y + X_{-1})$ is the measure of goods-market efficiency. ρ is the autoregressive coefficient in the law of motion for productivity, Z_t . For both values of ρ , the table has three columns. The first column gives the results when ζ_y and ω_0 are chosen to match σ_N/σ_Y and σ_{π_y}/σ_Y . Not matched is the value of σ_S/σ_Y . The second column gives the results when ζ_y is set equal to 0. The third column gives the results when ζ_y is set to the highest possible value for which model data are non-explosive. "=" indicates that this model characteristic matches its empirical counterpart by construction.

Table 3: Results when all key inventory facts are matched

	<i>data</i>	<i>model $\rho = 0.7$</i>			<i>model $\rho = 0.95$</i>		
		$\zeta_y, \zeta_x, \omega_0$ calibrated	$\zeta_y = 0$ $\zeta_x = 0$	high ζ_y	$\zeta_y, \zeta_x, \omega_0$ calibrated	$\zeta_y = 0$ $\zeta_x = 0$	high ζ_y
<i>parameter values</i>							
ζ_y		0.161	0	0.220	0.161	0	0.355
ζ_x		-0.178	0	-0.178	-0.191	0	-0.191
ω_0		0.993	0.993	0.993	0.980	0.980	0.980
<i>calibrated moments</i>							
σ_N/σ_Y	0.466	=	0.474	0.582	=	0.488	0.757
σ_{π_y}/σ_Y	0.162	=	0	0.211	=	0	0.314
σ_S/σ_Y	0.901	=	0.788	1.046	=	0.861	2.521
<i>inventory properties</i>							
$\rho_{X,S}$	0.648	0.327	0.858	0.326	0.433	0.919	-0.228
$\rho_{X,S}, BP_{\leq 4Q}$	-0.358	-0.741	-0.456	-0.721	-0.519	-0.323	0.128
$\rho_{X,S}, BP_{\leq 8Q}$	-0.270	-0.362	0.903	-0.351	-0.215	0.340	0.050
<i>standard business cycle statistics</i>							
σ_C/σ_Y	0.535	0.252	0.262	0.353	0.359	0.378	1.008
σ_I/σ_Y	3.554	3.571	2.910	3.92	2.903	2.724	16.900
<i>role of investment in inventories for GDP fluctuations</i>							
ρ_{XY}/σ_Y^2	0.193	0.240	0.372	0.118	0.259	0.308	-0.265
<i>autocorrelation unfiltered series</i>							
$\rho_{N,N(-1)}$		0.933	0.949	0.991	0.990	0.992	1.000
$\rho_{Y,Y(-1)}$		0.977	0.987	0.998	0.993	0.995	1.000

Notes: This table reports summary statistics of model-generated data and the empirical counterparts. $BP_{\leq NQ}$ indicates that the band-pass filter is used to extract that part of the series that is associated with fluctuations with a period less than N quarters. All other second-order moments are for HP-detrended data. σ_i is the standard deviation of variable i ; $\rho_{i,j}$ is the correlation coefficient of variables i and j ; S stands for sales, X stands for inventories, Y stands for GDP, Y^* is the output measure for these firms data (constructed using the sales and inventory data), and $\pi_y = S/(Y + X_{-1})$ is the measure of goods-market efficiency. ρ is the autoregressive coefficient in the law of motion for productivity, Z_t . For both values of ρ , the table has three columns. The first column gives the results when ζ_y , ζ_x , and ω_0 are chosen to match σ_N/σ_Y , σ_S/σ_Y , and σ_{π_y}/σ_Y . The second column gives the results when ζ_y and ζ_x are set equal to 0. The third column gives the results when ζ_y is set to the highest possible value for which model data are non-explosive. "=" indicates that this model characteristic matches its empirical counterpart by construction.