

Competitive Targeted Advertising with Price Discrimination*

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Abstract

This paper examines how should firms allocate their advertising budgets between consumers who have a high preference for their products (strong segment) and those who prefer competing products (weak segment). Targeted advertising transmits relevant information to otherwise uninformed consumers and it is used as a price discrimination device. With targeted advertising and price discrimination, we find that, when the attractiveness of the weak segment is low, each firm advertises more intensively in its strong segment than in its weak segment. The same result arises when the attractiveness of the weak segment is high and advertising is expensive enough. Interestingly, when the attractiveness of the weak segment is high but advertising costs are sufficiently low, it is optimal for each firm to advertise more intensively in its weak segment than in its strong segment. The paper also investigates how advertising strategies and equilibrium profits are affected by price discrimination. In comparison to uniform pricing, firms can increase or reduce the intensity of advertising targeted to each segment when price discrimination is allowed. Further, when the attractiveness of

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the weak market is high, price discrimination boosts firms' profits provided that advertising costs are sufficiently low. The reverse happens when advertising costs are high.

1 Introduction

In many markets firms need to invest in advertising to create awareness for new products, prices and special offers (informative view of advertising). Until recently, the firms' advertising strategies were mostly tailored to traditional media and mass audiences. Today, firms can exploit new possibilities to deliver ads targeted to specific segments within a market,¹ using for instance mobile coupons, sophisticated forms of location-based advertising, including geo-fencing and geo-conquesting,² and product search apps (see "The New Retail: from mobile aspirations to business results," *The Economist*, March 2014). The use of such advertising techniques drastically increases the scope for targeted advertising and price discrimination.³

In real markets, not all consumers are equally valuable to firms. While some consumers may have a relative preference for a firm's product (*strong* segment) the remaining ones may have a relative preference for the competitors' product (*weak* segment). Hence, firms in these markets need to choose the intensity of advertising and the price to be tailored to each market segment.

The central question of this paper is the following. Should a firm advertise more intensively in its strong segment or rather in its weak segment? Other relevant questions are: Which customer segment should be rewarded? Is price discrimination profitable? Further, what changes in terms of per-segment advertising spending and profits when we depart from a setting of targeted advertising and uniform pricing to one of targeted advertising and price discrimination?

We consider a model with two firms A and B launching a new product to consumers who may buy from a firm only if they receive an advertising message (henceforth ad) from it. Advertising creates awareness (and also informs about prices). The set of potential buyers is composed of two

¹eMarketer estimates that mobile ad spending will increase from \$8.4 billion in 2012 to \$37 billion in 2016. See <http://www.forbes.com/sites/chuckjones/2013/01/04/mobile-ad-spending-forecast-to-increase-4x-over-the-next-4-years/>.

²According to "The Location Terminology Guide – The Language of Location" developed by a working group of Mobile Marketing Association, geofencing identifies a point of interest on a map and establishes a radius around it for targeting purposes. Geo-conquesting is used when this point of interest is the competitor's location.

³For example, a Wall Street Journal investigation found that the Staples website displays different prices to people after tracking their locations. Staples appeared to consider the person's distance from a competitor's physical store. If a competitor had stores within 20 miles or so, Staples.com usually showed a discounted price. See <http://www.mmaglobal.com/location-terminology-guide>).

distinct segments of equal size, half of consumers have a relative preference for product A, while the remaining ones have a relative preference for product B. The desutility of not buying the most preferred brand is exogenously given by $\gamma > 0$. In a location interpretation, this means that consumers can purchase costlessly from the firm in their neighborhood but they incur a transport cost γ if they go to the more distant firm. This demand structure (à la Shilony, 1977) suggests that, although firms may have some advantage over their competitors, all (informed) consumers may, in the end, be induced to switch. With targeted advertising and price discrimination, each firm’s strategy consists in choosing an intensity of advertising and a different price to be tailored to the strong and to the weak segments of the market. By investing in advertising, firms endogenously segment the market into captive (partially informed), selective (fully informed) and uninformed customers.

To motivate our model consider the following example taken from location-based advertising via geo-fencing/conquesting, which has become a hot topic in the advertising and marketing world. Suppose two firms—e.g. McDonalds (Mc) and Burguer King (BK)—are running an awareness mobile advertising campaign to a new menu item. Both are perfectly informed about the consumers’ location (i.e., they know whether a consumer is near the Mc store or the BK store) and they have access to LBA tools that allow them to send ads with different offers (prices) to customers in different locations. For instance, consider a potential customer standing in front of BK’s door. BK can send this customer a relevant advertising offer. The consumer may also be tracked by Mc in the neighborhood, which may send him/her an ad with a special offer (discounts or other rewards). If the last ad is compelling enough, Mc can entice the consumer to travel to the more distant store (incurring the cost γ). The practice of targeting consumers located near the competitor has recently been labelled as a *geo-conquesting* strategy. Today, geo-conquesting ads are frequently used in markets in which there is a small window of thought before buying (e.g. retail, restaurants, hotels, travel,...) as well as in businesses that sell bigger items (e.g. automobiles).⁴

The model addressed in this paper fits well advertising and pricing policies that are nowadays

⁴Some well known geo-conquest campaigns have been used by Chrysler, Lexus, Outback Steakhouse or Best Western hotels. Tom MacIsaac, CEO of location-centric mobile ad network Verve Mobile says that “The company looked at click-through rates across 17 Mother’s Day campaigns by retailers using the tactic, as well as geo-fencing their own locations to promote cards, flowers, baked goods, and other gift items... The results showed that geo-conquesting led to a 30% higher click-through rate than standard geo-fencing.” See <http://www.mediapost.com/publications/article/200578/geo-conquesting-drives-higher-mobile-click-rates.html>

possible through the use of mobile devices, such as LBA and mobile coupons. This kind of advertising/pricing strategies have already been employed by brands like Starbucks, Burger King, Taco Bell, Tasti-D-Lite, Macy's and Pepsi. For example, the CEO of the New York City-based startup PlaceIQ said recently that PlaceIQ can be used to lure potential customers away from a competitor's location. Through the use of this technology, Lexus could potentially identify mobile phone users at an Audi dealership and serve them a mobile ad directing them to the nearest Lexus lot.

In the previous examples, consumers' physical location is a key determinant of firms' advertising strategies. However, our stylized model is also suitable to analyze other forms of targeting advertising strategies in which the geographical element is not intrinsically present. For example, in the case of contextual advertising through search engines, firms may engage on conquering ads by targeting them to consumers with an intrinsic preference for the rival firm.⁵

This paper offers important insights to the understanding of firms' advertising decisions to their strong and weak market segments. An important contribution is to clearly describe what market features are needed for the two advertising outcomes—more intensive advertising in a firm's strong market, or in its weak market—to arise in equilibrium when firms are able to engage in targeted informative advertising. We show that the relative attractiveness of the weak market and the level of advertising costs are key equilibrium determinants. When the attractiveness of the weak segment is low, regardless of advertising costs, the standard result in the literature prevails—it is always optimal for each firm to advertise more in its strong segment than in its weak segment. When the attractiveness of the weak segment is high enough, the two equilibrium outcomes are possible: each firm prefers to advertise more intensively in its weak segment when advertising costs are sufficiently low; the reverse happens when advertising costs are high (Proposition 3). This result allows us to provide a theoretical strategic rationale for the increasingly popular geo-conquering advertising strategies: by advertising less intensively in its strong market, each firm invites the rival to play less aggressively in that market.

The paper also investigates what changes in terms of targeted advertising decisions and equilibrium profits when firms move from a world of uniform pricing to one with price discrimination. In particular, the result of greater advertising to the weak market only arises in the equilibrium

⁵The Wall Street Journal referred that search engines allow firms “to bid on an adversary's trademarked search terms. A recent search for "Taco Bell" on Google, for example, revealed a sponsored link advertising Wendy's new steakhouse double melt sandwich.” See <http://online.wsj.com/news/articles/SB118118288230427401>

with price discrimination. With uniform pricing, each firm always prefers to advertise more intensively in its strong market. Finally, the paper also shows that price discrimination by means of targeted advertising can boost firms' profits and so it does not necessarily lead to the classic prisoner dilemma obtained in the competitive price discrimination literature.

This paper is mainly related to two strands of the literature, the literature on competition with informative targeted advertising and the literature on price discrimination based on customer recognition. We contribute to these strands of the literature by looking at the firms' targeted advertising and pricing decisions regarding their strong and weak segment of consumers. Galeotti and Moraga-Gonzalez (2008) and Brahim et al. (2011) have also looked at this issue in the world of targeted advertising and uniform pricing. In these papers, without advertising consumers stay out of the market. Galeotti and Moraga-Gonzalez (2008) look at the firms' advertising and pricing strategies in a homogeneous product market, where one of the market segments is exogenously more profitable than the other. Brahim et al. (2011) also investigate the profit effects of targeted advertising in a Hotelling competition model with no price discrimination. Both papers show that firms advertise more intensively to their strong markets than to their weak markets.

Our analysis is more closely related to Iyer et al. (2005). They characterize a differentiated market in a Varian (1980) type set-up: the market is exogenously segmented between captive consumers and comparison shoppers. When firms decide to advertise to a specific segment they inform the whole segment. They show that, provided all consumers remain uninformed without advertising, firms will always advertise more heavily to their high preference segment than to comparison shoppers.⁶ By doing so, firms strategically avoid Bertrand competition in this weak market. Finally, Iyer et al. (2005) also compare firms' targeted advertising decisions under uniform pricing versus price discrimination. In their setting price discrimination does not affect the advertising intensity targeted to each segment of the market neither the firm's profits.

Thus, the present paper complements Iyer et al. (2005) by studying targeted advertising and pricing with a different demand structure. In doing so, new insights can be obtained regarding

⁶It is important to stress that more advertising to comparison shoppers (weak segment) can also arise in Iyer et al. (2005) when a significant proportion of consumers in the strong segment (who are captive) are already informed about the firm's existence. The higher is the proportion of exogenous informed (strong) consumers the lower is the incentive of firms to employ costly advertising to their strong segment. However, in their set-up, as long as this proportion is equal to zero, i.e., without advertising consumers stay out of the market (like in our setting), the result of more advertising in a firm's weak segment never arises in equilibrium.

the firms' advertising strategies and the impact of price discrimination on firm's advertising strategies and profits. An important difference between our work and theirs lies on the behavior of loyal consumers (those with a strong preference for a brand). While Iyer et al. (2005) assume that loyal consumers always buy from their favorite brand (regardless of the price of the rival brand), we assume that consumers in a certain market segment prefer the corresponding brand by a certain amount but they are willing to consider buying the rival brand if the price difference is conducive. This assumption is consistent with empirical results showing that consumers may switch brands for pricing reasons (see, e.g., Keaveney, 1995, Bolton and Lemon, 1999). Krishnamurthi and Raj (1991) find that loyals are less price sensitive than nonloyals in the choice decision but they still react to price changes. Second, in our set-up, firms' advertising decisions endogenously segment the market into captive (partially informed) consumers and selective (fully informed) consumers.⁷ In light of this, we find that in markets where consumers are uninformed without advertising, the equilibrium outcome may produce more advertising to weak markets. Our explanation is a strategic one, given the demand formulation à la Shilony, the firms' ability to engage in price discrimination and the interplay between advertising costs and the attractiveness of the weak market segment.

Finally, this paper is also related to the literature on competitive price discrimination with customer recognition (e.g. Chen, 1997, Villas-Boas, 1999, Fudenberg and Tirole, 2000, and Esteves, 2010).⁸ In these models it is generally the case that the market exhibits best-response asymmetry (Corts, 1998): the strong market segment of one firm is the weak market segment of the competitor. A common finding in such models (with symmetric firms and full informed consumers) is that firms charge lower prices to customers in weak markets and, in comparison to uniform pricing, equilibrium profits fall with price discrimination.⁹

The rest of this paper is organized as follows. Section 2 describes the main ingredients of the model. Section 3 analyses the equilibrium advertising and pricing strategies when firms can

⁷Another difference between our demand structure and the one used in Iyer *et al.* (2005) lies on the existence of a group of pure switchers that have no loyalty or preferences for the brands in their model. We have analyzed how the existence of a segment of switchers would affect our model of target advertising with price discrimination and we find that this change would not qualitatively affect our result that firms may advertise more intensively to their weak market than to their strong market. Proofs can be obtained from the authors upon request.

⁸See Fudenberg and Villas-Boas (2007) for a comprehensive survey on this literature.

⁹More recently, Chen and Percy (2010) and Shin and Sudhir (2010) have shown that profits can increase with price discrimination, basically when consumer preferences are weakly correlated across time as well as in the case of high enough consumer heterogeneity.

price discriminate through the use of perfect targeted advertising. Section 4 derives the firms' equilibrium advertising and pricing strategies in the context of targeted advertising but uniform pricing and compares the firms' advertising strategies and overall equilibrium profits in the two price regimes. Section 5 concludes and an appendix collects the proofs that were omitted from the text.

2 The Model

Two firms A and B are each launching a new good, produced at a constant marginal cost, which is assumed to be zero without loss of generality. There is a large number of potential buyers, with mass normalized to one, each of whom wishes to buy a single unit of either good A or B . The set of potential buyers is composed of two distinct segments of equal size, half of consumers have a relative preference for product A , while the remaining ones have a relative preference for product B . The consumers have a common reservation price v for the goods but consumers in segment i prefer product i over product j by a degree equal to $\gamma > 0$. As in Shilony (1977), Raju et al. (1990) and Esteves (2010), γ can be defined as a measure of the degree of a consumer's preference towards his favorite product. In a location interpretation, this means that consumers can purchase costlessly from the firm in their neighborhood but they incur a transport cost γ if they go to the more distant firm. Thus, γ is the minimum difference between the prices of the two competing products that induces consumers to buy from the least preferred firm. Although firms may have some advantage over their competitors, due for instance to brand loyalty or transport costs, all consumers may, in the end, be induced to switch.¹⁰ In other words, each firm has a strong and a weak segment of consumers. For firm i , segment i is its strong segment, while segment j is its weak segment, $i, j = A, B$ and $i \neq j$.

Although consumers are endowed with preferences over the products, it is assumed that consumers are initially uninformed about the existence and the price of the goods. Like in Butters (1977) and Stahl (1994), a potential consumer cannot be an actual buyer unless firms invest in advertising. We assume that each firm can send two types of ads: ads targeted to the strong segment of consumers and ads targeted to the weak segment of consumers.¹¹ Within each

¹⁰For example, Brynjolfsson and Smith (2000) have found that Amazon customers are willing to pay up to 5-8 percent more before they consider switching to another seller.

¹¹Given our motivating example (in the Introduction), Burger King can assume that almost everyone standing at the entrance of a McDonald's store is a potential BK customer. With a geo-conquesting campaign BK can

segment, we assume that ads are randomly distributed. Moreover, all ads contain truthful and complete information about the existence and the price of the goods.

The game is static and proceeds as follows. Firms choose advertising intensities and prices simultaneously and non-cooperatively. Firm i 's strategy then consists of choosing an optimal advertising reach and an optimal pricing strategy to each segment of the market. The advertising intensities ϕ_i^o and ϕ_i^r respectively denote the advertising intensity of firm i targeted to its own (strong) market and to the rival's market (firm i 's weak market). Ads targeted to each segment announce different prices, respectively p_i^o and p_i^r . The firms' targeting ability is assumed to be perfect, i.e. $\Pr(\text{fall in } i \mid \text{targeted to } i) = 1$ while $\Pr(\text{fall in } i \mid \text{targeted to } j) = 0$. Perfect advertising targeting is also assumed in Brahim et al. (2011) or Galeotti and Moraga-González (2008) and it means that there is no leakage of ads between groups. Thus, consumers in segment i will only be aware (as long as informed) of p_i^o and p_i^r . The remaining prices, namely p_j^r and p_j^o , are quoted in ads targeted to consumers in segment j .

After firms have sent their ads independently, some consumers will be reached by one of the firms, by both firms or by none of them. In each segment of the market, consumers can then be divided into captive, selective and uninformed consumers. Specifically, in segment i , a proportion ϕ_i^o and ϕ_j^r of customers is reached, respectively, by firm i and j 's ads. Some consumers are captive to firm i , namely $\phi_i^o (1 - \phi_j^r)$, while others are captive to firm j , namely $\phi_j^r (1 - \phi_i^o)$. Captive consumers are only aware of one of the firms. They purchase the product they know as long as they obtain a non-negative surplus. To guarantee that firms can always serve consumers in both market segments, we need to impose that $v > \gamma$. The group of selective customers in segment i , $\phi_i^o \phi_j^r$, receive ads from both firms. Hence, consumers in this group buy from the firm offering them the highest surplus. Finally, in segment i , there is a group of $(1 - \phi_i^o) (1 - \phi_j^r)$ consumers who receive none of the ads and so they are excluded from the market. For $p_i^o \leq v$, the expected demand of firm i in its strong market, D_i^o , writes as follows:

$$D_i^o = \left[\frac{1}{2} \phi_i^o (1 - \phi_j^r) + \frac{1}{2} \phi_i^o \phi_j^r \Pr(p_i^o < p_j^r + \gamma) \right]. \quad (1)$$

Analogously, for $p_i^r \leq v - \gamma$, firm i 's expected demand in its weak market, D_i^r , is equal to:

$$D_i^r = \left[\frac{1}{2} \phi_i^r (1 - \phi_j^o) + \frac{1}{2} \phi_i^r \phi_j^o \Pr(p_i^r + \gamma < p_j^o) \right]. \quad (2)$$

deliver targeted ads with different content to mobile devices of consumers who are near the Mc's store (compared to ads sent to consumers who are near its store).

Advertising is a costly activity for firms. The advertising technology is exogenously given and it is the same for both firms. In line with Iyer et al. (2005), Galeotti and Moraga-Gonzalez (2008) and Brahim et al. (2011), we assume that the advertising cost function is additive separable in ϕ_i^o and ϕ_i^r . We denote by $A(\phi_i^k)$ firm i 's total cost of advertising in segment $k = \{o, r\}$. Following the standard literature, we impose $A_{\phi_i^k} > 0$ and $A_{\phi_i^k \phi_i^k} \geq 0$, where the subscripts stand for partial derivatives. In what follows, whenever a functional form is needed, we consider that the cost of reaching a fraction ϕ_i^k of consumers is given by the quadratic advertising function: $A(\phi_i^k) = \lambda (\phi_i^k)^2$. Since we assume that there is a large number of buyers, normalized to unit, λ can be identified with the cost per ad.

3 Targeted Advertising with Price Discrimination

Firms make their advertising and pricing choices simultaneously. To begin the analysis, we need to derive the expected profit of each firm. Because there is no leakage between segments, they are completely independent. For a given strategy of the rival firm, firm i 's expected profit conditional on ads and prices targeted to segment $k = o, r$ is denoted by $E\pi_i^k$ which is equal to:

$$E\pi_i^k = p_i^k D_i^k - A(\phi_i^k).$$

Firm i chooses the advertising level (ϕ_i^k) and the price (p_i^k) to be targeted to segment k in order to maximize its expected profit in this segment. Since a firm cannot identify whether or not a consumer is reached by the rival's advertising campaign, when choosing the pricing strategy, each firm takes into account the trade-off between extracting surplus from captive consumers and competing for the group of selective consumers. Following a similar reasoning as in Varian (1980) and Narasimhan (1988) it is straightforward to prove that there is no Nash price equilibrium in pure strategies. Hence the price equilibrium will be in mixed strategies. Propositions 1 and 2 characterize a symmetric equilibrium in mixed strategies in prices, with the advertising components to each segment of the market being chosen deterministically. Lemma 1 establishes the support of firms' equilibrium prices in their strong markets, i.e., $[p_{i \min}^o, p_{i \max}^o]$. Under symmetry the price support in the weak market is simply $[p_{i \min}^o - \gamma, p_{i \max}^o - \gamma]$

Lemma 1. *Define ϕ_i^{o*} and ϕ_j^{r*} to be, respectively, the equilibrium advertising intensities of firm i and firm j targeted to segment i . When*

$$\frac{v}{v - \gamma} \geq \frac{\phi_i^{o*}}{\phi_j^{r*}}, \quad (3)$$

the support of firm i 's equilibrium price targeted to its strong market is $[(v - \gamma)(1 - \phi_i^{o*}) + \gamma, v]$. However, if condition (3) does not hold, firm i 's equilibrium price support is $[v(1 - \phi_j^{r*}), v]$.

Proof. See the Appendix.

Lemma 1 shows that the properties of the equilibrium depend on the parameters v , γ and the advertising costs (through condition (3)). This condition is endogenously determined in our model since firms may strategically manipulate the ratio of consumers with information about each product through their advertising choices. In what follows, we will provide a complete characterization of the mixed strategy Nash equilibrium. First, Proposition 1 characterizes the equilibrium behavior of firms when condition (3) in Lemma 1 holds.

Proposition 1. *There is a symmetric Nash equilibrium in which:*

(i) Regarding the strong market, firm i chooses a price randomly from the distribution $F_i^o(p)$ defined in the Appendix with support $[(v - \gamma)(1 - \phi_i^{o*}) + \gamma, v]$. For $\frac{v}{v - \gamma} \geq \frac{\phi^{o*}}{\phi^{r*}}$, the advertising reach targeted to the strong market ϕ_i^{o*} is implicitly given by

$$\frac{1}{2}v - \phi_i^{o*}(v - \gamma) = A_{\phi_i^o}(\phi_i^{o*}) \quad \text{with } A_{\phi_i^o}(0) < \frac{1}{2}v. \quad (4)$$

(ii) Regarding the weak market, firm i chooses a price randomly from the distribution $F_i^r(p)$ defined in the Appendix with support $[(v - \gamma)(1 - \phi_i^{o*}), v - \gamma]$. $F_i^r(p)$ has a mass point at $v - \gamma$ with a density equal to:

$$m^r = 1 - \frac{\phi^{o*}}{\phi^{r*}} \left(\frac{v - \gamma}{v} \right). \quad (5)$$

The advertising reach targeted to the weak market ϕ_i^{r*} is implicitly given by

$$\frac{1}{2}(v - \gamma)(1 - \phi_j^{o*}) = A_{\phi_i^r}(\phi_i^{r*}), \quad (6)$$

where $A_{\phi_i^r}(0) < \frac{1}{2}(1 - \phi_j^{o*})(v - \gamma)$ and $\frac{v}{v - \gamma} \geq \frac{\phi^{o*}}{\phi^{r*}}$.

(iii) Firm i 's equilibrium profit in the strong and the weak markets are:

$$\begin{aligned} E\pi_i^{*o} &= \phi_i^{o*} A_{\phi_i^o}(\phi_i^{o*}) + \frac{1}{2}(\phi_i^{o*})^2(v - \gamma) - A(\phi_i^{o*}) \\ E\pi_i^{*r} &= \phi_i^{r*} A_{\phi_i^r}(\phi_i^{r*}) - A(\phi_i^{r*}). \end{aligned}$$

Proof. See the Appendix.

Proposition 2 now characterizes the equilibrium behavior of firms when condition (3) in Lemma 1 does not hold.

Proposition 2. *There is a symmetric Nash equilibrium in which:*

(i) *Regarding the strong market, firm i chooses a price randomly from the distribution $F_i^o(p)$ defined in the online Appendix with support $[v(1 - \phi^{r*}), v]$. $F_i^o(p)$ has a mass point at v with a density equal to $m^o = 1 - \frac{\phi^{r*}}{\phi^{o*}} \frac{v}{v-\gamma}$. For $\frac{v}{v-\gamma} < \frac{\phi^{o*}}{\phi^{r*}}$, the advertising reach targeted to the strong market ϕ_i^{o*} is implicitly given by*

$$\frac{1}{2}v(1 - \phi_j^{r*}) = A_{\phi_i^o}(\phi_i^{o*}) \text{ with } A_{\phi_i^o}(0) < \frac{1}{2}v. \quad (7)$$

(ii) *Regarding the weak market, firm i chooses a price randomly from the distribution $F_i^r(p)$ defined in the Appendix with support $[v(1 - \phi^{r*}) - \gamma, v - \gamma]$. The advertising reach targeted to the weak market ϕ_i^{r*} is implicitly given by*

$$\frac{1}{2}(v - \gamma) - v\phi_i^{r*} = A_{\phi_i^r}(\phi_i^{r*}) \text{ and } A_{\phi_i^r}(0) < \frac{1}{2}(v - \gamma). \quad (8)$$

(iii) *Firm i 's equilibrium profit in the strong and the weak markets are:*

$$\begin{aligned} E\pi_i^{*o} &= \phi_i^{o*} A_{\phi_i^o}(\phi_i^{o*}) - A(\phi_i^{o*}) \\ E\pi_i^{*r} &= \phi_i^{r*} A_{\phi_i^r}(\phi_i^{r*}) - \frac{\gamma\phi_i^{r*}}{2} - A(\phi_i^{r*}). \end{aligned}$$

Proof. See the Online Appendix.

Comparing Propositions 1 and 2, we can see that depending on the parameters v , γ and the level of advertising costs, firms' optimal advertising and pricing strategies will differ. For the quadratic advertising technology, the complete characterization of the equilibrium requires the distinction between the cases (i) $v \geq 2\gamma$ (in which the results in Proposition 1 yield $\frac{v}{v-\gamma} \geq \frac{\phi^{o*}}{\phi^{r*}}$), and (ii) $\gamma < v < 2\gamma$ (in which the equilibrium behavior is described in Proposition 2, instead). In the rest of the paper, we will say that the relative attractiveness of the weak market is high when $v - \gamma$ is high (v high compared to γ), leading to the results in Proposition 1. In contrast when $v - \gamma$ is low (v low compared to γ) we will say that the relative attractiveness of the weak market is low, leading to the results in Proposition 2.

A general result of the model is that for costless advertising, i.e. when $\lambda \rightarrow 0$, regardless of the advertising cost function under consideration, firms will provide full information only to consumers in one of the market segments. Specifically, it is optimal for each firm to inform all consumers in its strong market provided that the attractiveness of the weak market is low. The opposite result is obtained when the attractiveness of the weak market is high. In the latter case ($v - \gamma$ is high and Proposition 1 holds) each firm uses a ‘‘Hi-Lo’’ pricing strategy in its weak

market. When $\lambda \rightarrow 0$, each firm prefers to fully inform consumers in its weak market segment, i.e., $\phi^{r*} = 1$, while in its strong market, it prefers to leave some consumers uninformed, yielding $\phi^{o*} < \phi^{r*} = 1$. Each firm, by appearing weak in its strong market ($\phi^{o*} < \phi^{r*}$) invites the rival to play less aggressively in that market, mitigating price competition in that segment. When, say firm i , reduces the advertising intensity targeted to its strong market it induces the rival (firm j) to play less aggressively in that market (firm j 's weak market) as it increases the likelihood that firm j quotes the monopoly price $v - \gamma$ in the ads targeted to segment i (recall that m^r is decreasing in ϕ_i^o). Moreover, it also increases $p_{j\min}^{r*} = (v - \gamma)(1 - \phi_i^{o*})$.

In contrast, when $v - \gamma$ is low (Proposition 2 holds), each firm uses instead a ‘‘Hi-Lo’’ pricing strategy in its own strong market, quoting the highest price v , with a strictly positive probability m^o , which is increasing in ϕ^o and decreasing in ϕ^r . When $\lambda \rightarrow 0$, it is straightforward to see that $\phi^{r*} < \phi^{o*} = 1$. Now each firm intends to mitigate price competition in its weak market. When say firm i reduces the advertising intensity targeted to its weak market it induces the rival (firm j) to play less aggressively in that market (firm j 's strong market) since by doing so it increases the likelihood of firm j announcing price v in ads targeted to segment j (due to m^o). Moreover, a reduction on ϕ^{r*} also raises $p_{\min}^{o*} = v(1 - \phi^{r*})$.

3.1 Optimal Advertising Strategies

When consumers are uninformed about the product if they are not exposed to advertising, an important question that firms face in designing their advertising strategies is how should their advertising budgets be allocated between strong and weak customer segments. In this section, we investigate in which circumstances it is optimal to a firm to advertise more intensively in its strong market or rather to advertise more intensively in its weak market. In what follows we use the quadratic advertising cost function. Consider first the case where $v \geq 2\gamma$. From Proposition 1 and Lemma 1, it is straightforward to obtain that the equilibrium advertising intensity targeted to the strong market is given by equation (4) yielding:

$$\phi^{o*} = \frac{v}{4\lambda + 2(v - \gamma)}. \quad (9)$$

From equation (6), the equilibrium advertising intensity targeted to the weak market is equal to:

$$\phi^{r*} = \begin{cases} \frac{v-\gamma}{4\lambda} \frac{v+4\lambda-2\gamma}{2v+4\lambda-2\gamma} & \text{if } \lambda \geq \frac{v-\gamma}{8} \left(\sqrt{\frac{5v-9\gamma}{v-\gamma}} - 1 \right) \\ 1 & \text{if } \lambda < \frac{v-\gamma}{8} \left(\sqrt{\frac{5v-9\gamma}{v-\gamma}} - 1 \right) \end{cases}. \quad (10)$$

When $v < 2\gamma$, for a quadratic advertising technology, Proposition 2 holds. The equilibrium advertising intensity targeted to the strong market is then given by equation (7) yielding:

$$\phi^{r*} = \frac{v - \gamma}{2v + 4\lambda}, \quad (11)$$

and the equilibrium advertising intensity targeted to the weak market is given by equation (8) yielding:

$$\phi^{o*} = \begin{cases} \frac{v}{8\lambda} \frac{v+4\lambda+\gamma}{v+2\lambda} & \text{if } \lambda \geq \frac{1}{8}v \left(\sqrt{\frac{5v+4\gamma}{v}} - 1 \right) \\ 1 & \text{if } \lambda < \frac{1}{8}v \left(\sqrt{\frac{5v+4\gamma}{v}} - 1 \right) \end{cases}. \quad (12)$$

The next proposition summarizes our main findings regarding the firms' optimal advertising strategies, for the quadratic advertising cost function.

Proposition 3. (i) *When $v < 2\gamma$, regardless of the magnitude of advertising costs, each firm always advertises more intensively in its strong segment than in its weak segment, i.e. $\phi^{o*} > \phi^{r*} > 0$.*

(ii) *When $v \geq 2\gamma$, each firm advertises more intensively in its weak segment than in its strong one ($\phi^{r*} > \phi^{o*} > 0$) when advertising costs are low i.e., when $\lambda < \bar{\lambda} = \frac{(v-2\gamma)(v-\gamma)}{4\gamma}$. When $\lambda \geq \bar{\lambda}$ the reverse happens, thus $\phi^{o*} \geq \phi^{r*}$.*

Proof. See the Appendix.

Proposition 3 shows that perfect market segmentation in which firms would send ads only to their high preference segments ($\phi^{o*} = 1$ and $\phi^{r*} = 0$) never arises in the price discrimination equilibrium. In fact, firms always choose to advertise to both segments of the market. Part (i) of Proposition 3 predicts that it is always optimal for each firm to advertise more heavily to its strong market relative to its weak market when the attractiveness of the weak market is low ($v - \gamma$ low), regardless of the advertising cost level. The rationale behind this result is that by appearing “weak” in its weak market (low ϕ_i^r compared to ϕ_j^o) firm i invites the rival (firm j) to play less aggressively in that market (i.e., j 's strong market).

More interestingly, part (ii) of Proposition 3 highlights that when $v - \gamma$ is high, the standard result in the literature—i.e., $\phi^{o*} > \phi^{r*}$ —only arises in equilibrium if advertising costs are high enough. If λ is sufficiently low, then in equilibrium, each firm prefers to advertise more intensively in its weak segment of the market than in its strong segment. To the best of our knowledge this

is a new result in the context of markets in which consumers only get information about the product when exposed to advertising.¹²

When is this strategy of sending less ads to a firm's strong market (than to its weak market) profitable?¹³ For this result to occur the weak market segment needs to be sufficiently attractive (v high compared to γ) and advertising must be sufficiently cheap. In fact, when $\phi_j^r > \phi_i^o$, each firm increases the chances of getting captive consumers in its weak market and it strategically invites the rival to play less aggressively in its own strong market. Although the latter effect is clearly beneficial to both firms, the lower advertising intensity in a firm's strong market implies that each firm is leaving some of its strong customers uninformed, which represents a loss of revenue. In light of this trade-off, the strategy $\phi^{o*} < \phi^{r*}$ is only profitable when the following conditions hold. First, even if the number of consumers each firm serves in its strong market is lower (given the lower advertising intensity), the price firms can charge in the market must be high enough (v high enough). Secondly, targeting the weak market must not be too costly (γ needs to be low enough so that poaching consumers in the weak market is not too costly). Finally, advertising costs need to be sufficiently low (low λ) so that firms advertise intensively enough in the weaker market to generate a sufficient number of consumers here to make up for the loss of revenue from the stronger market, in which the firms strategically reduce the advertising intensity in order to dampen price competition. In sum, provided $v - \gamma$ is sufficiently high, the strategy $\phi^{o*} < \phi^{r*}$ works only when advertising costs are low enough.

Before proceeding it is useful to compare our results with those obtained in Iyer et al. (2005) for the case in which without advertising consumers stay out of the market. In their model each firm has an exogenous group of captive consumers and both firms compete à la Bertrand for the comparison shoppers (selective). In this framework, they show that when advertising is the only source of consumers' information then firms will always advertise more in their strong segment than in their weak segment (comparison shoppers). The reason is that under targeted advertising and price discrimination each firm enjoys a monopoly power over its captive con-

¹²As said in footnote 6, more advertising to comparison shoppers (weak segment) can also arise in Iyer et al. (2005) when a significant proportion of consumers in the strong segment (who are captive) are already informed about the firm's existence. The higher is the proportion of exogenously informed (strong) consumers the lower is the incentive of firms to employ costly advertising to their strong segment. However, in their set-up, as long as without advertising consumers stay out of the market (like in our setting), the result of more advertising in a firm's weak segment no longer arises in equilibrium.

¹³We thank a referee for urging us to think in this way.

sumers and by reducing the intensity of advertising to the shoppers, firms strategically dampen price competition in the shoppers segment. Our model considers a different demand structure (Shilony type) and the group of captive and selective consumers is endogenously determined by the firms' advertising decisions. Thus our analysis complements Iyer et al. (2005), by providing a complete picture of the market features that can explain the two possible advertising outcomes, in markets reasonably well represented by our model assumptions. We suggest that it can be optimal for firms to advertise more intensively in their strong segment than in their weak segment in industries where: (i) whatever the magnitude of advertising costs, the attractiveness of the weak segment is low, and (ii) the attractiveness of the weak segment is high and advertising costs are sufficiently high. In contrast, it can be optimal for firms to advertise less intensively to their strong than to their weak segment in markets where the attractiveness of the weak segment is high enough and advertising is sufficiently cheap (or costless).¹⁴ Therefore, when $v \geq 2\gamma$, the model predicts that an industry shock in the form of lower advertising costs, may completely alter firms' optimal advertising intensities to each segment of the market.

Finally, it is worth noting that our results shed some light on a theoretical strategic rationale behind the use of geo-conquesting advertising strategies, which are becoming increasingly popular in sectors like restaurants, retail, financial services/insurance, travel, gas & convenience stores. In particular, the equilibrium outcome in which $\phi^{o*} < \phi^{r*}$ suggests that in some industries we might even see managers spending more money on geo-conquesting advertising campaigns than on traditional geo-fencing advertising.

3.2 Prices

Another important question that firms face today (when price discrimination is possible) is whether they should target lower prices to their high preference customers or rather to their low preference customers. Most of the existing academic literature suggests that when the market exhibits best-response asymmetry—one firm's weak market is the other's strong market—the optimal choice for each firm is to offer a lower price to its low preference consumers than to its high preference consumers (e.g. Chen, 1997; Villas-Boas, 1999; Fudenberg and Tirole, 2000;

¹⁴Our previous discussion on the costless advertising case reveals that our findings are robust to alternative formulations of the advertising cost function. We have checked this for a linear advertising cost function $A(\phi^k) = \lambda\phi^k$. Using equations (4) and (6), for $v > 2\gamma$ and $\lambda < \frac{v}{2}$, we get $0 < \phi^{o*} < 1$ and $\phi^{r*} = 1$, yielding $\phi^{r*} > \phi^{o*}$.

Esteves, 2010).¹⁵

In our set-up, the firms' discriminatory prices are affected by the parameters v , γ and λ . As the most interesting outcome of the paper arises when Proposition 1 holds, we briefly discuss how advertising costs affect each firm's price decisions to each segment of the market when $v \geq 2\gamma$. Let $E(p^o)$ and $E(p^r)$ (computed in the online appendix) denote the expected prices announced in ads targeted to the strong and to the weak segments, respectively.

We find that it can be optimal for each firm to *reward its strong* customers rather than its weak customers (see the Online Appendix). Specifically, when λ is sufficiently low (i.e., when $\lambda < \bar{\lambda}$ implying $\phi^{r*} > \phi^{o*}$ (see Proposition 3)), given the symmetry of the model, each firm has a higher proportion of captive consumers in its weak market than in its strong market. Moreover, according to Proposition 1 each firm uses a "Hi-Lo" pricing strategy in its weak market. In particular, the lower is λ , the higher is the likelihood that an ad targeted to a firm's weak market announces the price $v - \gamma$ (which raises $E(p^r)$). At the same time, each firm has more incentives to compete for the group of selective consumers in its strong market, by offering them a compelling enough price (which reduces $E(p^r)$). Thus, as shown in the online appendix, there is a domain of parameters for which $E(p^o)$ is lower than $E(p^r)$. In our model, the strategy of charging less to consumers in a firm's strong market works only when the advertising costs are low enough. As advertising becomes more expensive, the outcome $\phi^{o*} > \phi^{r*}$ is obtained. In this case, a similar (but opposite) argument explains why it is optimal for each firm to reward consumers in its weak market, yielding the standard result in the literature.

4 Comparing Targeted Advertising and Price Discrimination with Targeted Advertising and Uniform Pricing

In this section we examine how the firms' advertising decisions to each segment of the market and expected equilibrium profits are affected by their ability to price discriminate. With this goal in mind, we derive the equilibrium outcomes in the benchmark case where firms have the ability to target advertising but can only compete with uniform pricing strategies. In this case, there are two components to firm i 's strategy: its advertising level to each segment of the market (ϕ_i^{ou} and ϕ_i^{ru}) and its uniform price (p_i^u). Proposition 4 characterizes the equilibrium outcomes

¹⁵An exception is Shin and Sudhir (2010) who show that firms can charge a low price to their strong customers when consumer preferences stochasticity across time and consumer heterogeneity are simultaneously high enough.

for the quadratic advertising cost function.

Proposition 4 *When advertising can be targeted and price discrimination is not permitted:*

(i) *For $v < (2 + \sqrt{2})\gamma$ the pair of prices (v, v) is an equilibrium in pure strategies and the corresponding advertising levels are $\phi_i^{ru*} = 0$ and $\phi_i^{ou*} = \frac{v}{4\lambda}$, with $\lambda < \frac{v}{4}$. Equilibrium profits are $\pi^u = \frac{v^2}{16\lambda}$.*

(ii) *For $v \geq (2 + \sqrt{2})\gamma$ and sufficiently high λ (i.e. $\lambda > \tilde{\lambda}$, whose value is defined in the online Appendix), the pair of prices $(v - \gamma, v - \gamma)$ is an equilibrium in pure strategies and the corresponding advertising levels are $\phi_i^{ou*} = \frac{v-\gamma}{4\lambda}$, with $\lambda > \frac{v-\gamma}{4}$ and $\phi_i^{ru*} = \frac{(v-\gamma)(4\lambda+\gamma-v)}{16\lambda^2}$. Equilibrium profits are $\pi^u = \frac{(v-\gamma)^2[(v-4\lambda-\gamma)^2+16\lambda^2]}{256\lambda^3}$.*

(iii) *For $v \geq (2 + \sqrt{2})\gamma$ and $\lambda < \tilde{\lambda}$ the price equilibrium is in mixed strategies and the corresponding advertising levels are $\phi^{ou*} = \phi^{ru*} = \frac{v-\gamma}{v-\gamma+4\lambda}$. The expected equilibrium profits are $E\pi^u = \frac{2\lambda(v-\gamma)^2}{(v+4\lambda-\gamma)^2}$.*

Proof. See the Online Appendix.

Proposition 4 shows that $\phi^{ou*} > \phi^{ru*}$ in cases (i)-(ii) and $\phi^{ou*} = \phi^{ru*}$ in case (iii). These results confirm the standard predictions in the literature investigating the firms' optimal allocation of advertising budgets. These findings also show that in our framework the result of more advertising to the weak market can only arise in equilibrium if firms can engage in price discrimination.

We now briefly discuss the results obtained under uniform pricing. If, for any reason, price discrimination is not permitted, when a firm advertises a lower price, as a way to serve consumers in both segments of the market, it foregoes some profits from its strong segment. Not surprisingly, when the attractiveness of the weak market is low ($v - \gamma$ is low), each firm decides to advertise its product only to its strong market, ignoring the weak market. In this case, both firms behave as monopolists in their own strong markets, charging a price equal to v . As the reservation price increases compared to γ , the same happens to the profits that firms can possibly earn by reducing the price and capturing additional consumers. In fact, as long as the condition $(v - \gamma)\phi_i^{ru*}(1 - \phi_j^{ou*}) - \gamma\phi_i^{ou*}(1 - \phi_j^{ru*}) \geq 0$ holds at equilibrium, each firm has an incentive to target ads with the same price to both segments of the market. When $v - \gamma$ is high enough, the price equilibrium $(v - \gamma, v - \gamma)$ described in case (ii) in Proposition 4 arises, as long as $\lambda > \tilde{\lambda}$. Both firms advertise more intensively to their strong than to their weak market segments. More

precisely, each firm serves all the informed consumers in its strong market and the captive consumers in the weak market. Since the weak market segment is attractive ($v - \gamma$ is high), for this case to arise, the advertising costs must be sufficiently high ($\lambda > \tilde{\lambda}$) so that firms refrain from competing for the selective consumers.

When $v - \gamma$ is high but advertising is sufficiently cheap (small λ), selective consumers in the weak segment have a non-negligible impact on firms' profit. Then, each firm's price equilibrium is in mixed strategies as an attempt to prevent the rival from systematically predicting its price. The expected equilibrium profit is the guaranteed profit a firm can realize by charging the reservation price $v - \gamma$ and selling only to the group of captive consumers in each market. Since $p_{\max}^u = v - \gamma$, firms treat the two market segments in the same way. Not surprisingly, firms send the same advertising intensity to their strong and their weak segments. More precisely, firms advertise up to the point where the cost of the last ad sent to each segment equals the expected revenue of a sale at the highest price to an uninformed consumer, i.e.,

$$\frac{1}{2}(v - \gamma)(1 - \phi_j^{-k}) = A_\phi(\phi_i^k), \quad k = o, r. \quad (13)$$

Therefore, it is straightforward to see that $\phi^{ou*} = \phi^{ru*}$.

4.1 Effects of Price Discrimination on Advertising Strategies and Profits

Important questions for marketing scholars and practitioners are the following. Does the ability to price discriminate change the intensity of advertising targeted to each segment of the market? What is the impact of price discrimination on firms' equilibrium profits?

We shed light on these questions comparing the results of the model with price discrimination to the benchmark model of targeted advertising with uniform pricing. We look at different market environments, namely those where the attractiveness of the weak market is low/high combined with low/high advertising costs.¹⁶ For the subsequent discussion, let ϕ^{od*} and ϕ^{rd*} denote the equilibrium intensity of advertising targeted respectively to the firm's own strong market and to the rival's market when price discrimination is allowed.

Consider first the case where the attractiveness of the weak market is sufficiently low such that firms behave as in Proposition 2 under price discrimination and as in part (i) of Proposition 4 under uniform pricing. In this domain of parameters, firms choose to exclusively serve the

¹⁶Although the proofs of the main conclusions are straightforward, the interested reader can obtain them from the authors upon request.

strong market only when they adopt uniform pricing. Under price discrimination, firms no longer ignore their weak market. Moreover, for $v < 2\gamma$, it is straightforward to prove that moving from uniform pricing to price discrimination reduces the intensity of advertising targeted to the strong market and increases the intensity of advertising targeted to the weak market, i.e. $\phi^{od*} < \phi^{ou*}$ and $\phi^{rd*} > \phi^{ru*} = 0$. Regarding the effect of price discrimination on profits, we find that profits *increase* when firms become able to tailor different prices to their high and low preference segments. Basically, the increase in profits is due to a demand expansion effect that more than compensates the negative effect of price competition in both segments of the market. Therefore, in markets where $v - \gamma$ is low, price discrimination is a profitable strategy regardless the level of advertising costs.

Consider next those markets where the attractiveness of the weak segment is high. Here conclusions regarding the advertising and profit effects of price discrimination are less clear cut because we need to take into account whether advertising costs are high or low. Suppose first that $v - \gamma$ is high and advertising costs are sufficiently low ($\lambda < \tilde{\lambda}$) so that firms behave as in Proposition 1, under price discrimination, and as in part (iii) of Proposition 4, under uniform pricing. If $\lambda < \bar{\lambda}$ it follows that $\phi^{rd*} > \phi^{od*}$ (see Proposition 3). Moreover, for $\lambda < \bar{\lambda}$, we also have $\phi^{rd*} > \phi^{ru*}$ and $\phi^{ou*} > \phi^{od*}$. The intuition behind this result is straightforward. With uniform pricing firms would treat the two market segments in the same way, i.e. $\phi^{ou*} = \phi^{ru*}$. However, this is not true when firms engage in price discrimination: provided advertising costs are sufficiently low, each firm prefers to reduce the advertising intensity targeted to its strong market, in order to strategically reduce price competition in that market. This yields $\phi^{od*} < \phi^{ou*}$. In the weak market, comparing equations (6) and (13) we can see that price discrimination does not directly affect each firm's advertising intensity to the segment of low valuation consumers. The effect of price discrimination on ϕ^{r*} is only indirect, through its impact on each firm's advertising choice to the strong market. Accordingly, when advertising costs are sufficiently low ($\lambda < \bar{\lambda}$), compared to uniform pricing, each firm strategically sends more ads to the weak market (and less ads to the strong market) under price discrimination, with $\phi^{rd*} > \phi^{od*}$. Instead, when $\lambda > \bar{\lambda}$, moving from uniform pricing to price discrimination leads firms to send more ads to the strong segment and less ads to the weak segment, with $\phi^{ou*} < \phi^{od*}$ and $\phi^{rd*} < \phi^{ru*}$.

Finally, when the attractiveness of the weak market is high and advertising costs are high enough ($\lambda > \tilde{\lambda}$), so that firms behave as in part (ii) of Proposition 4 (in the case of uniform

pricing) and Proposition 1 (in the case of price discrimination), each firm advertises more in its weak market and less in its strong market under price discrimination than under uniform pricing, i.e. $\phi^{rd*} > \phi^{ru*}$ and $\phi^{ou*} > \phi^{od*}$, provided that advertising costs are not too high (i.e., $\tilde{\lambda} < \lambda < \hat{\lambda} = \frac{1}{2} \frac{(v-\gamma)^2}{\gamma}$). When $\lambda > \hat{\lambda}$, the opposite result occurs. The rationale behind these results is analogous to the one explained above for part (iii) of Proposition 4.

We now discuss the profit effects of price discrimination when the attractiveness of the weak market is high. When advertising costs are sufficiently low ($\lambda < \bar{\lambda}$) overall profit with price discrimination is always *above* its counterpart with uniform pricing.¹⁷ Accordingly, in our set-up, price discrimination by means of targeted advertising does not necessarily lead to the classic prisoner's dilemma result obtained in theoretical models of competitive price discrimination.¹⁸ In our set-up, only when advertising costs are sufficiently high ($\lambda > \bar{\lambda}$), a prisoners' dilemma may occur, with firms getting lower total profits with price discrimination than with uniform pricing due to the intensification of competition for consumers in the weak market segment.¹⁹

The previous analysis highlights that price discrimination can affect the firms' advertising strategies and profits. This differs from Iyer et al. (2005), who show that, in the case of a differentiated market in a Varian (1980) type set-up, advertising decisions and profits do not change when firms move from targeted advertising and uniform pricing to targeted advertising with price discrimination.

Summing up, the paper also contributes to the ongoing debate on the profit implications of new forms of price discrimination, only made possible in the context of digital markets. We find that firms can be better off with price discrimination in markets where: (i) the attractiveness of the weak market is low, regardless the advertising costs and (ii) the attractiveness of the

¹⁷Note that $E\pi^{d*} = E\pi^{o*} + E\pi^{r*}$, with $E\pi^{o*} = \frac{v^2}{8(v+2\lambda-\gamma)}$, $E\pi^{r*} = \lambda \left(\frac{v-\gamma}{8\lambda} \frac{v+4\lambda-2\gamma}{v+2\lambda-\gamma} \right)^2$ and $E\pi^{u*} = \frac{2\lambda(v-\gamma)^2}{(v+4\lambda-\gamma)^2}$.

¹⁸Esteves (2010) shows that if consumers are fully informed about the firms' existence and firms can tailor different prices to their weak and strong segments, Bertrand competition in each segment of the market leads to equilibrium prices to $p^o = \gamma$ and $p^r = 0$.

¹⁹If we look at the impact of price discrimination on profits per segment, it is straightforward to prove that price discrimination raises profits in the strong market. This is basically due to the increase in prices when firms engage in price discrimination. We find that the expected profit in the weak market segment is higher under price discrimination than under uniform pricing if $\lambda < \bar{\lambda}$. Interestingly, when advertising costs are low, expected profits in the weak market are higher under price discrimination than under uniform pricing due to a demand expansion effect (recall that $\phi^{rd*} > \phi^{ru*}$ for $\lambda < \bar{\lambda}$). For $\lambda < \bar{\lambda}$, firms' strategic advertising choices also soften price competition, favoring profits under price discrimination vis-à-vis the situation with uniform pricing. A similar but opposite argument holds when $\lambda > \bar{\lambda}$.

weak market is high and advertising costs are sufficiently low. In contrast, firms are worse off with price discrimination when the attractiveness of the weak market is high and advertising is expensive.

5 Conclusions

This paper provides useful implications for managers and marketing practitioners developing targeted advertising strategies. It addresses the following questions: In which circumstances should a firm spend more in advertising to the low preference customers segment (geo-conquesting) than to the strong segment? Under price discrimination, which segment of customers should be rewarded? Is price discrimination with targeted advertising profitable?

Regarding the first question, the paper shows that depending on the attractiveness of the weak market and the magnitude of advertising costs (high/low) it may be optimal for each firm to advertise more intensively in its strong market (standard result in the literature) or to advertise more intensively in its weak market. The first result prevails when the attractiveness of weak market is low (regardless of the advertising costs) and when the attractiveness of the weak market is high but advertising costs are high. The reason behind this result is each firms' attempt to mitigate price competition in its weak market. We add to the literature a new result: in a set-up in which consumers remain uninformed without advertising, it can be optimal for each firm to advertise more intensively in its weak than in its strong segment. This strategy is profitable when the attractiveness of the weak market is high enough and advertising is sufficiently cheap. In this case, each firm strategically reduces the intensity of advertising targeted to its strong market as a way to dampen price competition in that segment.

The paper also sheds light on the key role that price discrimination can have both on the firms' advertising choices to each segment of the market and on the firms' equilibrium profits (compared to a world of targeted advertising and uniform pricing). Indeed, more advertising to the weak market can only arise in equilibrium if firms have the ability to target simultaneously both price and advertising content.

The model also provides useful insights regarding the profitability of price discrimination through targeted advertising, identifying the market features for which price discrimination boosts firms' equilibrium profits. Specifically, firms can be better off under price discrimination than under uniform pricing in markets where: (i) the attractiveness of the weak market is

low (regardless the advertising costs) and (ii) the attractiveness of the weak market is high and advertising costs are sufficiently low. When advertising costs are sufficiently high, price discrimination is bad for profits.

Notwithstanding the model addressed in this paper is far from covering all complex aspects of real markets, it provides a theoretical strategic rationale for the increasingly popular geo-conquesting and advertising/price discrimination strategies only possible in the context of digital markets. When firms have the possibility of targeting ads and prices to different market segments, the model suggests that as advertising costs within an industry fall, managers might find good reasons to raise the advertising budgets allocated to geo-conquesting strategies. As the theoretical model provides empirically testable hypotheses, we hope it can be used for further empirical research.

Appendix

Proof of Lemma 1. Define ϕ_i^{o*} and ϕ_j^{r*} to be the equilibrium advertising strategies of firm i as functions of v , γ and advertising costs and let us analyze the price support of firm i in its strong market segment. Here, firm i can always guarantee a profit equal to the one obtained when it serves its captive consumers at price v . Formally,

$$\frac{1}{2}p_{i \min}^o \phi_i^{o*} \geq \frac{1}{2}v\phi_i^{o*}(1 - \phi_j^{r*}) \Rightarrow p_{i \min}^o \geq v(1 - \phi_j^{r*}) \quad (14)$$

Moreover, it is a dominated strategy for firm i to price below $p_{j \min}^r + \gamma$. Since $p_{j \min}^r \geq (v - \gamma)(1 - \phi_i^{o*})$, it must be the case that

$$p_{i \min}^o \geq (v - \gamma)(1 - \phi_i^{o*}) + \gamma. \quad (15)$$

The inferior support of the price distribution of firm i is given by the lowest price simultaneously satisfying conditions (14) and (15). As the maximum price firm i is willing to charge is equal to v , we obtain that the price support of firm i in its strong market is given by $[(v - \gamma)(1 - \phi_i^{o*}) + \gamma, v]$ for $\frac{v}{v - \gamma} \geq \frac{\phi_i^{o*}}{\phi_j^{r*}}$. Instead, when $\frac{v}{v - \gamma} < \frac{\phi_i^{o*}}{\phi_j^{r*}}$, the price support of firm i in the strong market is given by $[v(1 - \phi_j^{r*}), v]$ ■

Proof of Proposition 1. As we focus on symmetric MSNE in prices, the c.d.f. are such that $F_A^o(p) = F_B^o(p) = F^o(p)$, and $F_B^r(p) = F_A^r(p) = F^r(p)$. For the sake of simplicity, with no loss of generality, we restrict our attention to firms' decisions in segment A , obtaining $F_A^o(p) = F^o(p)$ and $F_B^r(p) = F^r(p)$. Given firms' pricing and advertising strategies targeted to segment A , firms expected profits in this segment are respectively

$$\begin{aligned} E\pi_A &= \frac{1}{2}\phi_A^o p_A^o \{1 - \phi_B^r + \phi_B^r [1 - F_B^r(p_A^o - \gamma)]\} - A(\phi_A^o), \text{ and} \\ E\pi_B^r &= \frac{1}{2}\phi_B^r p_B^r \{1 - \phi_A^o + \phi_A^o [1 - F_A^o(p_B^r + \gamma)]\} - A(\phi_B^r). \end{aligned}$$

Recall that for $\frac{v}{v-\gamma} \geq \frac{\phi_A^o}{\phi_B^r}$, the support of the equilibrium prices for firm A is $[(v-\gamma)(1-\phi_A^{o*}) + \gamma, v]$ while for firm B is $[(v-\gamma)(1-\phi_A^{o*}), v-\gamma]$, since $p_{B\min}^r = (v-\gamma)(1-\phi_A^{o*})$. As usual in a MSNE each firm must be indifferent between charging any price in the support of equilibrium prices.

For firm B we must observe that for any $p_{B\min}^r \leq p_B^r \leq v-\gamma$:

$$\frac{1}{2}p_B^r \phi_B^r \{(1 - \phi_A^o) + \phi_A^o [1 - F_A^o(p_B^r + \gamma)]\} = \frac{1}{2}(v-\gamma)\phi_B^r(1 - \phi_A^o)$$

which implies that, in equilibrium,

$$F_A^o(p) = \begin{cases} 0 & \text{if } p \leq p_{B\min}^r + \gamma \\ \frac{1}{\phi_A^o} \left[1 - \frac{(v-\gamma)(1-\phi_A^{o*})}{p-\gamma} \right] & \text{if } p_{B\min}^r + \gamma \leq p \leq v \\ 1 & \text{if } p \geq v \end{cases}.$$

Analogously, for firm A we must observe that for any $p_{B\min}^r + \gamma \leq p_A^o \leq v$:

$$E\pi_A^o = \frac{1}{2}p_A^o \phi_A^o \{(1 - \phi_B^r) + \phi_B^r [1 - F_B^r(p_A^o - \gamma)]\} = \frac{1}{2}(p_{B\min}^r + \gamma)\phi_A^o \quad (16)$$

Thus, in equilibrium the corresponding distribution is

$$F_B^r(p) = \begin{cases} 0 & \text{if } p \leq p_{B\min}^r \\ \frac{1}{\phi_B^r} \left(1 - \frac{v(1-\phi_A^{o*}) + \gamma\phi_A^{o*}}{p+\gamma} \right) & \text{if } p_{B\min}^r \leq p \leq v-\gamma \\ 1 & \text{if } p \geq v-\gamma \end{cases}. \quad (17)$$

Note that $F^r(v-\gamma) = \frac{\phi_A^{o*}}{\phi_B^r} \left(\frac{v-\gamma}{v} \right)$. Since this Proposition is valid for $\frac{v}{v-\gamma} \geq \frac{\phi_A^o}{\phi_B^r}$, we obtain $F_i^r(v-\gamma)$ is smaller than 1, implying that F_i^r has a mass point at $(v-\gamma)$ equal to $m^r = 1 - \frac{\phi_i^{o*}}{\phi_j^{r*}} \left(\frac{v-\gamma}{v} \right)$. The expected profit obtained by firm A in market a when it charges any price in the support of equilibrium prices is equal to $E\pi_A = \frac{1}{2}\phi_A^o [v - (v-\gamma)\phi_A^o] - A(\phi_A^o)$. The profit-maximizing advertising intensity of firm A in its strong market is then obtained from

the condition $\frac{\partial E\pi_A}{\partial \phi_A^o} = 0$, which implies that $\frac{v}{2} - (v - \gamma)\phi_A^{o*} = A_{\phi_A^o}(\phi_A^{o*})$. Note also that the SOC hold under our assumptions about the advertising technology. To obtain the optimal advertising level ϕ_B^r , recall that firm B's expected profit in the MSNE is equal to $E\pi_B = \frac{1}{2}(v - \gamma)\phi_B^r(1 - \phi_A^o) - A(\phi_B^r)$. As the second order condition $\frac{\partial^2 E\pi_B}{\partial \phi_B^r} < 0$ is always met. From $\frac{\partial E\pi_B}{\partial \phi_B^r} = 0$ we obtain $\frac{1}{2}(v - \gamma)(1 - \phi_A^o) = A_{\phi_B^r}(\phi_B^r)$ ■

Proof of Proposition 3. Taking into account the two equations defining ϕ^{o*} and ϕ^{r*} , given by equations (7) and (8), in Proposition 2, as the RHS of both equations is the same let us compare the LHS of both equations. It is straightforward to see that $\phi^{o*} > \phi^{r*}$ as long as $\gamma > 0$, which is always true. Therefore, when $v < 2\gamma$ then $\phi^{o*} > \phi^{r*}$.

Taking into account (9) and (10), it is straightforward to see that $\phi^{o*} < \phi^{r*}$, if and only if $\lambda < \bar{\lambda} = \frac{(v-2\gamma)(v-\gamma)}{4\gamma}$. Note that $\underline{\lambda} = \frac{v-\gamma}{8} \left(\sqrt{\frac{5v-9\gamma}{v-\gamma}} - 1 \right) < \bar{\lambda}$. Thus, when $\lambda < \underline{\lambda}$, we have $\phi^{r*} = 1 > \phi^{o*}$, since $\phi^{o*} < 1$. When $\underline{\lambda} < \lambda < \bar{\lambda}$ it is always the case that $\phi^{o*} < \phi^{r*}$. Finally, when $\lambda \geq \bar{\lambda}$ then $\phi^{o*} \geq \phi^{r*}$. ■

References

- BRAHIM, N., LAHMANDI-AYED, R., AND LAUSSEL, D. (2011), "Is targeted advertising always beneficial?" *International Journal of Industrial Organization*, 29, 678-689.
- BOLTON, R. N., LEMON, K. N. (1999), "A dynamic model of customers' usage of services: Usage as an antecedent and consequence of satisfaction." *Journal of Marketing Research*, 171-186.
- BRYNJOLFSSON, E. AND SMITH, M. (2000b), "The Great Equalizer? Consumer Choice Behavior and Internet Shopbots." Working paper, MIT.
- BUTTERS, G. (1977), "Equilibrium Distribution of Sales and Advertising Prices." *Review of Economic Studies*, 44, 465-491.
- CHEN, Y. (1997), "Paying Customers to Switch." *Journal of Economics and Management Strategy*, 6, 877-897.
- CHEN, Y., AND PEARCY, J. (2010), "Dynamic Pricing: when to Entice Brand Switching and when to Reward Consumer Loyalty." *Rand Journal of Economics*, 41, 674-685.
- CORTS, K. S. (1998), "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment," *RAND Journal of Economics*." 29, 306-323.

ESTEVEES, R.B. (2010), "Pricing with customer recognition." *International Journal of Industrial Organization*, 28, 669-681.

FUDENBERG, D. AND TIROLE, J. (2000), "Customer Poaching and Brand Switching." *RAND Journal of Economics*, 31, 634-657.

FUDENBERG, D. AND VILLAS-BOAS, M. (2007), "Behavior-Based Price Discrimination and Customer Recognition," In *Handbook on Economics and Information Systems*, ed. by T. Hendershott. North-Holland, Amsterdam.

GALEOTTI A., AND MORAGA-GONZALEZ, J. L., (2008), "Segmentation, Advertising and Prices." *International Journal of Industrial Organization* , 26, 1106-1119.

IYER, G., SOBERMAN, D. AND VILLAS-BOAS, M. (2005), "The Targeting of Advertising." *Marketing Science*, 24, 461-476.

KEAVENEY, S. M. (1995), "Customer Switching Behavior in Service Industries: An exploratory study." *The Journal of Marketing*, 71-82.

KRISHNAMURTHI, L. AND RAJ , S.P. (1991), "An Empirical Analysis of the Relationship Between Brand Loyalty and Consumer Price Elasticity," *Marketing Science*, Vol. 10, No. 2, 172-83.

NARASIMHAN, C. (1988), "Competitive promotional strategies," *Journal of Business*, 427-449.

RAJU, J., SRINIVASAN, V. AND RAJIV, L. (1990), "The Effects of Brand Loyalty on Competitive Price Promotional Strategies," *Management Science*, 36, 276-304.

SHILONY, Y. (1977), "Mixed Pricing in Oligopoly." *Journal of Economic Theory*, 14, 373-388.

SHIN, J. AND SUDHIR, K. (2010). "A Customer Management Dilemma: When is it Profitable to Reward One's Own Customers?" *Marketing Science*, 29, 671-689, 2010.

STAHL, D. (1994), "Oligopolistic Pricing and Advertising." *Journal of Economic Theory*, 64, 162-177.

VARIAN, H. (1980), "A Model of Sales." *The American Economic Review*, 651-659.

VILLAS-BOAS, M. (1999), "Dynamic Competition with Customer Recognition." *RAND Journal of Economics*, 30, 604-631.