Strategic Information Acquisition and Transmission*

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First draft: December 2012, This draft: June 2014

Abstract

This paper explores the implications of costly information acquisition in a strategic communication model. We show that equilibrium decisions based on a biased expert's advice may be more precise than when information is directly acquired by the decision maker, even if the expert is not more efficient than the decision maker at acquiring information. This result bears important implications for organization design. Communication by an expert to a decision maker may often outperform delegation of the decision making authority to the expert, as well as centralization by the decision maker of both information acquisition and decision making authority.

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*We are grateful to Attila Ambrus, Luigi Balletta, Alessandro Bonatti, Wouter Dessein, William Fuchs, Faruk Gul, Johannes Hörner, Navin Kartik, Tymofiy Mylovanov, Nicola Persico, Heikki Rantakari, Joel Sobel, Dezsö Szalay, Helen Weeds and seminar participants at Carlos III, CERGE-EI, Gerzensee ESSET 2012, Hong Kong, Mannheim, Naples, Rotterdam, the 2011 SAET conference, the 2011 Petralia Sottana workshop in economics, the 2011 Economics and Music conference and the Workshop on Acquisition, Use and Transmission of Private Information in Financial Markets at the EUI for comments and suggestions.

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1 Introduction

Strategic information transmission is one of the central topics in economics of information. Starting from the seminal work of Crawford and Sobel (1982), this literature highlights the limited scope of information transmission via cheap talk messages, which generically leads to inaccurate or imprecise decisions.[1] However, the typical assumption made in this literature is that perfect information is exogenously given to the expert-sender for free, and he bears no cost of information acquisition.[2] This informational structure is clearly an extreme point in the set of feasible possibilities. Indeed, information is typically costly. Expertise and knowledge are usually obtained as a result of often time-consuming work and costly research.[3]

Departing from the previous literature, we consider a model of strategic communication where information is costly and the decision to acquire it is taken endogenously. In this context, we show that the decision-maker may be able to induce the expert to overinvest, i.e. to acquire more information than the decision-maker would acquire if the latter performed this task herself, even if the expert does not have a better technology of information acquisition. This insight yields our main finding: the decisions based on advice of a biased expert can be *more precise* than the decisions based on direct information acquisition by the decision maker. Precisely, under some parametric restriction explained later, expert advice is strictly more precise than direct information acquisition for all Pareto efficient equilibria, with the exception of the equilibrium that maximizes the expert's payoff, for

[1]See, for example, the contributions by Austen-Smith (1993), Gilligan and Krehbiel (1987, 1989), Krishna and Morgan (2001a, 2001b), Wolinsky (2002), Battaglini (2002, 2004), Ambrus and Takahashi (2008).

[2]The exceptions include Austen-Smith (1994), Ottaviani (2000) and Ivanov (2010). In Austen-Smith (1994), the sender may either acquire full information or remain completely ignorant. In Ottaviani (2000), the amount of information available to the expert is exogenously given. In the model by Ivanov (2010), informational structure can be selected costlessly by the decision-maker.

[3]For example, this is the case for investment advice by financial analyists, for policy advice by experts reporting to Congress, for expert witnesses in trials, as well as for many other real-world applications of strategic information transmission games.

which the two alternatives yield the same precision.[4,5] Our results stand in contrast with the "common wisdom" of the previous literature on communication that decisions based on the biased expert's advice are imprecise.

To explain our results, we first describe the main features of our model. Initially, both parties - the decision-maker and the expert - are uninformed about the state of the world and share a common prior. Either of them can acquire information about the state of the world, in the form of binary "trials". The cost of information acquisition increases in the quantity of information, which is measured by the number of "trials" performed.[6] This discrete model of information acquisition not only simplifies our analysis, but also captures real world situations with discrete information, such as aggregation of individual opinions from sincere voting, surveys, or experiments. Moreover, as we explain later, the main driving forces of our model extend beyond our specific model and would be present in other discrete or even continuous models.

In our baseline model, the expert acquires the information and then conveys a cheap-talk message to the decision-maker, who then takes an action. We consider two scenarios: overt information acquisition and covert information acquisition.[7] In the former, the decision maker observes the quantity of information acquired by the expert, but not its content. In the latter, the decision-maker observes neither the quantity nor the content of the information acquired by the expert. In both cases, we focus on the amount of information acquired and credibly transmitted by the expert in equilibrium, which translates into the *precision* of the final action made by the decision-maker. We then compare the outcomes of these two communication games against two alternatives: the first one is direct information acquisition by the decision-maker, the second one is delegation to the expert of both information acquisition and the choice of action.

[4]The use of Pareto efficiency as an equilibrium refinement is standard in communication games.

[5]Unlike in Crawford and Sobel (1982), equilibria are not fully Pareto ranked in our game, because information is costly, and the cost is borne solely by the expert, whereas its benefit is shared by the two players. Pareto efficient equilibria with more (less) information are preferred by the decision maker (expert).

[6]Our set-up is related to the Bernoulli-Uniform model of cheap talk analyzed by Morgan and Stocken (2008).

[7]These two games are denoted shortly as "overt game" and "covert game", in the remainder of the paper.

Our main result, the identification of parameter regions with information acquisition overinvestment, is driven by different forces in the overt and covert game. In the first one, the decision-maker induces the expert's strict overinvestment in information acquisition by ignoring her communication, unless the expert acquires the "right" amount of information. In technical terms, a babbling equilibrium is played off the equilibrium path.[8] This use of the worst credible punishment to provide the strongest incentives on path is similar to the approach adopted to find Pareto efficient equilibria in repeated games.

Moreover, this equilibrium construction finds support in the real-world, as it is fairly common to observe decision-makers who only heed advice of the experts whose qualifications or effort exceed the threshold set by the former. Consider for example expert witnesses in legal trials. In the U.S., the Federal Rules of Evidence specify that testimony by an expert witness is acceptable only if it "...is the product of sufficient facts or data," and " is the product of reliable principles and methods [9]". This rule is sufficiently broad and allows the judge to tailor her threshold of acceptability to the particular case under consideration.[10] If the judge finds that an expert had

[8]Ubiquitous in communication games, babbling equilibria are such that the decision-maker's decision is independent of the expert's message, and, thus, the expert is indifferent among sending any message, and adopts a completely uninformative communication strategy.

[9]According to the Federal Rule of Evidence 702:

"A witness who is qualified as an expert by knowledge, skill, experience, training, or education may testify in the form of an opinion or otherwise if: (a) the expert's scientific, technical, or other specialized knowledge will help the trier of fact to understand the evidence or to determine a fact in issue; (b) the testimony is based on sufficient facts or data; (c) the testimony is the product of reliable principles and methods; and (d) the expert has reliably applied the principles and methods to the facts of the case."

[10]Berlin and Williams (2000) report that a case in which: "...The Illinois Supreme Court then pointed out that it is the judge who must determine whether a potential expert witness is qualified to render opinions in a specific lawsuit" They quote the opinion of said Court in the case *Jones v. O'Young et al.* as follows: "...The trial court has the discretion to determine whether a physician is qualified and competent to state his opinion as an expert regarding the standard of care....By hearing evidence on the expert's qualifications and comparing the medical problem and the type of treatment in the case to the experience and background of the expert, the trial court can examine whether the witness has demonstrated a sufficient familiarity with the standard of care practiced in the case... [If the expert witness does not satisfy these requirements], the trial court must disallow the expert's testimony.... The requirements are a threshold beneath which the plaintiff cannot fall without failing to sustain the allegations of his complaint."

not met this threshold, (s)he would typically disqualify the expert rather than allow a limited testimony by the expert. Our results suggest that this legal procedure provides a powerful incentive for information acquisition.

Other real-world examples of what essentially is a threshold knowledge rule for admissibility of an expert's advice can be found in politics (parliamentary and congressional hearing making use of expert's advice), financial and consumer markets (financial advisors and real estate agents have rating systems and certain customers will only deal with the agents and advisors who have the highest rating category[11]) , and academia (short reference letters that do not describe in detail an academic's research are usually disregarded by hiring and tenure committees).

Furthermore, we identify a parameter region in which our strict overinvestment result extends to all Pareto efficient equilibria of the overt game, with the exception of the expert's preferred equilibrium. For the latter, our result holds weakly: the expert acquires and reveals at least as much information as the decision-maker would acquire directly. Also, that equilibrium construction does not rely on babbling equilibria off path. Instead, it is based on the most informative communication equilibrium being played on and off path.

Before turning to the covert game, it is important to underline, here, that our overinvestment results do not rely on the specific statistical structure we adopt in the overt game. The use of an off path threat to induce overinvestment does not depend on the discreteness of our model, and would be possible also in continuous information models.

In the covert game, the information acquisition investment is unobservable, and our strict equilibrium overinvestment result cannot rely on credible punishments off path. We first show that there is no loss of generality in disregarding the possibility that the expert communicates how much information he has acquired. Specifically, we show that allowing such communication would not increase the set of equilibrium outcomes. As a result, the decision-maker interprets any expert's message under the belief that the latter has acquired the equilibrium amount of information even if the expert has, in fact, deviated from it.

We refer to this property as inflexibility of the equilibrium language.

[11] J.D. Power and Associates system of rating for brokers provides one example.

In our model, this is an endogenous property of equilibria. Importantly, it limits the attractiveness for the expert of a deviation from the equilibrium level of information acquisition. In fact, the equilibrium language is optimally tailored to the equilibrium amount of information and is typically ill-suited for reporting the expert's findings after he deviated at the information acquisition stage.

This effect is not as powerful as the off path punishment in the overt game. Therefore, stronger conditions on the parameters are required for the overinvestment result to obtain. Nevertheless, one can find many examples of fixed equilibrium communication language in the real world. In particular, the language of financial advice is often standardized. Specifically, Standard and Poor's Capital IQ equity analysts rank assets on a qualitative 5-point scale (Strong Sell, Sell, Hold, Buy, Strong Buy). Similarly, consumer research firms, such as Consumer Report, J.D. Powers and Associates and others, typically rate the quality of products on a grid with a fixed number of points. Standardized restricted communication protocols can be found in public administration and in the military where communication between different units has to follow a formal fixed language. Similarly, prospective employers often ask the referees to places job candidates into one of several categories specified by the employers, rather than by the referees.

As in the overt game, it is remarkable that also in the covert game our strict overinvestment result extends to all Pareto efficient equilibria, with the exception of the equilibrium preferred by the expert, for which our result holds weakly.

It is intuitive that the main force leading to our results in the covert game, namely the inflexibility of equilibrium language, holds quite broadly beyond our specific model. In fact, consider any communication model, continuous or discrete, in which the sender's information is fixed. We know from Crawford and Sobel (1982) that unless the sender is unbiased, the set of messages used on the equilibrium path is discrete (up to outcome equivalence). In other terms, the equilibrium language is discrete and, quite generally, it depends on the amount of information held by the expert. Hence, when considering covert information acquisition, an expert deviating from the equilibrium information acquisition choice is penalized by the requirement that he uses equilibrium language to communicate; i.e., he is penalized by the inflexibility of language.

In the final part of the paper, we highlight the implications of our results for the theory of optimal organization, and the prevalence of communication-based organizations. Dessein (2002) and Ottaviani (2000) have shown that, due to the loss of information in transmission, a communication-based organization in which the principal takes decisions based on the advice of a biased expert, may be dominated by delegation of the decision-making authority to an informed expert.[12] But if an organization based on information transmission is suboptimal then, perhaps, we should not expect it to be used frequently and in important economic situations.

While the above results are derived in an environment where the expert has access to perfect and free information, our paper identifies novel economic forces that arise under costly information acquisition and which significantly raise the relative performance of communication-based organizations. Specifically, our strict overinvestment results identify plausible sufficient conditions under which information transmission from the expert to the decision maker performs better than either delegation of decision-making to the expert, or direct information acquisition by the decision-maker. Thus, more broadly, our results can be seen as providing a strong support for the prominent role of information transmission between experts and decision-makers in organizations, which has been postulated theoretically and confirmed empirically (see, e.g., Bolton and Dewatripont,1994, and Garicano, 2000).

1.1 Literature Review.

The study of information acquisition is almost entirely unexplored in the cheap talk literature. An exception is a preliminary working paper by Eso and Szalay (2010). They consider a game in which an expert, who has the same preferences as the decision-maker, is initially uninformed but can learn the exact realization of the state by paying a fixed cost. In the beginning, the decision-maker commits to a message set (equivalently, action set) that she allows the sender to choose from. They show that

[12] See also, relatedly, Aghion and Tirole (1997) and Gilligan and Krehbiel (1987).

restricting this message set and hence coarsening the language available for communication, compared to allowing the sender to communicate the exact state of the world, can induce the sender to acquire information for a larger range of costs. Similarly, Szalay (2005) shows that restricting the set of actions available to the agent in the delegation game can increase the incentive for the latter to acquire information. In both these papers, the restriction on the actions or the message set available to the expert is exogenous to communication, and the focus is on the normative question of which exogenously fixed language maximizes information acquisition. Our paper is very much distinct. Unlike in their paper, in our game the language is endogenous. Its derivation is significantly involved as the language arises within the equilibrium interaction between the players. Our motivation is also different: we study the positive question of whether information acquisition overinvestment may occur in communication games, and how it affects the relative performance of different organization structures.[13]

Another exception is Di Pei (2013), who considers a model of covert costly information acquisition and transmission which is very different from ours. In his model, the expert acquires arbitrary information partitions of the state space, to then observe the element of the acquired partition. His key assumption, which is violated in our standard and intuitive binary trial model, is that if a sender can purchase an information partition, then he can also purchase any coarsening of that partition, at a lower cost. The remarkable implication of this assumption is that all equilibria involve full revelation of the sender's private information. Indeed, there is no reason for the sender to purchase a very precise information partition and then reveal only a coarse summary of his findings, if, instead, he can directly purchase such coarser information at a lower cost. Our model is very different, and even Pareto efficient equilibria need not be fully separating. Remarkably, though, the overinvestment result that we identify here would hold a fortiori, and for a bigger parameter space, in using the information acquisition technology by Di Pei (2013).

Less closely related, Che and Kartik (2009) study information acquisi-

[13] Our model has further methodological differences: our expert is biased, he can acquire any amount of information, and his information remains imprecise, except in the limit.

tion in the context of verifiable information transmission, which has properties that are very different from cheap talk even in absence of information acquisition. In their model, the expert has the same preferences as the decision-maker but a different prior. Since information is verifiable, an informed experts can only disclose the exact signal or conceal it completely. They focus on the choice of the expert by the decision-maker, and show that the decision-maker would prefer to choose an expert with a prior different from hers, because the divergence in prior beliefs between the sender and the receiver, while stifling communication, delivers better incentives for information acquisition.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 describes information acquisition overinvestment. Section 4 derives the implications for organization design. Section 5 concludes.

2 The Model

We start by introducing our model of cheap talk with endogenous acquisition of costly information by the sender. It is a natural extension of the classic Crawford and Sobel (1982) uniform-quadratic model. There are two players, the expert and the decision maker. The decision-maker's payoff is given by

$$U^R(y,\theta) = -(y-\theta)^2, \tag{1}$$

where θ is an unknown state of the world, with uniform common prior distribution over $[0,1]$, and y is the action taken by the decision-maker.

The expert's payoff is given by

$$U^S(y,\theta,b) - c(n) = -(y-\theta-b)^2 - c(n), \tag{2}$$

where the bias $b > 0$ measures the preference discrepancy between the expert and the decision-maker and $c(n)$ is the cost of information acquisition performed by the expert.

The game unfolds as follows. First, the expert acquires costly information about θ which we model as $n \in \mathbb{N} \cup \{0\}$ i.i.d. binary trials with aggregate cost $c(n) = cn$. The number of trials n performed by the expert affects the precision of the expert's information about θ. Each trial can result either in success or failure, with probability of success equal to the true θ. Having observed the number k of successes in n trials, the expert sends a message $m \in M$ to the decision maker, where M is the message set. After receiving the message, the decision-maker chooses an action $y \in [0, 1]$.

For given n and θ, the number of successes k is distributed according to the binomial distribution:

$$f(k|n, \theta) = \frac{n!}{k!(n-k)!} \theta^k (1-\theta)^{n-k}, \text{ for } 0 \leq k \leq n,$$

while under unknown uniformly distributed θ the distribution of k is also uniform:

$$\Pr(k|n) = \int_0^1 \frac{n!}{k!(n-k)!} \theta^k (1-\theta)^{n-k} d\theta = \frac{1}{n+1}.$$

Finally, note that the posterior distribution of θ given k successes in n trials is a Beta distribution with parameters $k+1$ and $n-k+1$, and its density is given by:

$$f(\theta|k, n) = \frac{(n+1)!}{k!(n-k)!} \theta^k (1-\theta)^{n-k}, \text{ if } 0 \leq \theta \leq 1.$$

The corresponding posterior expectation of θ is $E[\theta|k, n] = \frac{k+1}{n+2}$.

We will distinguish between two cases in the analysis. In the *overt game*, prior to choosing an action the decision-maker observes the number of trials n performed by the expert. In the *covert game*, n is private unverifiable information of the expert. All proofs are relegated to the Appendix.

2.1 The Overt Game

A pure strategy Perfect Bayesian Equilibrium of the overt game is described by a tuple $\left(n, \{P_{n'}\}_{n'=0,1,...,\infty}, \{\mathbf{y}(P_{n'})\}_{n'=0,1,...,\infty}\right)$, where n is the equilibrium number of trials conducted by the expert; $P_{n'} \equiv \left(p_1^{n'}, ..., p_{\#P_{n'}}^{n'}\right)$ is the partition of the set of the expert's types $\{0, 1, ..., n'\}$ describing the information communicated by the expert after performing $n' \in \{0, 1, ..., \infty\}$ trials; and $\{\mathbf{y}(P_{n'})\}$ is the decision maker's action profile corresponding to partition $P_{n'}$.

According to this definition, if the expert performs n' trials with k successes, then he sends a message informing the decision maker that the element p_i of the communication partition $P_{n'}$ has occurred, where $k \in p_i$.[14] Correspondingly, $\{\mathbf{y}(P_{n'})\} \equiv \left(y^{n'}_{p_1}, ..., y^{n'}_{p_{\#P_{n'}}}\right)$, where $y^{n'}_{p_i} \in [0,1]$, denotes the decision-maker's action after the expert sends a message corresponding to the element p_i of the partition $P_{n'}$.

A *babbling communication partition* contains a single element, whereas in a *fully separating communication partition*, each type constitutes an element of the partition.

The following conditions must hold in an equilibrium:
(i) The action profile $\mathbf{y}(P_{n'})$ must be sequentially rational for all n' i.e., for every $p_i \in P_{n'}$, $y^{n'}_{p_i}$ maximizes the decision-maker's expected payoff given that the sender's type is in p_i:

$$y^{n'}_{p_i} \in \arg\max_y \int_0^1 U^R(y,\theta) f(\theta | k \in p_i, n) d\theta \tag{3}$$

(ii) For every $n' \in \{0, 1, ..., \infty\}$, the partition $P_{n'}$ is incentive compatible i.e., for any $k \in \{0, 1, ..., n'\}$ and $p_i \in P_{n'}$ s.t. $k \in p_i$, we have:

$$\int_0^1 U^S\left(y^{n'}_{p_i}, \theta, b\right) f(\theta | k, n')\, d\theta \geq \int_0^1 U^S(y^{n'}_q, \theta, b) f(\theta | k, n')\, d\theta, \text{ for all } q \in P_{n'}. \tag{4}$$

(iii) n maximizes the expert's expected payoff given $\{P_{n'}\}_{n'=0,1,...,\infty}$ and $\{\mathbf{y}(P_{n'})\}_{n'=0,1,...,\infty}$ i.e., letting $p^{n'}(k)$ denote the element of the partition $P_{n'}$ s.t. $k \in p^{n'}(k)$, we have:

$$n \in \arg\max_{n' \in \{0,1,...,\infty\}} \sum_{k=0}^{n'} \left(\int_0^1 U^S\left(y^{n'}_{p^{n'}(k)}, \theta, b\right) f(\theta | k, n')\, d\theta \times \Pr(k|n') \right) - c(n'). \tag{5}$$

Next, we first describe the decision-maker's optimal action, then the

[14] Note that we do not specify explicitly which message(s) $m \in M$ by the expert signal an element p_i of the partition $P_{n'}$, for any n'. Any arbitrary partition of the message space M into $\#P_{n'}$ sets $\mathcal{M}_1, ..., \mathcal{M}_{\#P_{n'}}$ such that $\cup_i \mathcal{M}_i = M$ and $\mathcal{M}_i \cap \mathcal{M}_j = \emptyset$ for $i \neq j$ will do. With any such convention, every message uniquely maps into an element of partition $P_{n'}$, for any n'.

incentive compatible partitions. Given the quadratic payoff function, the sequentially rational action $y_{p_i}^{n'}$ must be equal to the decision-maker's posterior expectation of θ given the observed number of trials n' and the element p_i of the partition $P_{n'}$ communicated by the expert. Precisely, we have:

Lemma 1 *Let $|p_i|$ denote the cardinality of p_i. Then we have:*

$$y_{p_i}^{n'} = E\left[\theta | p_i, n'\right] = \frac{1}{|p_i|} \sum_{k \in p_i} \frac{k+1}{n'+2}. \tag{6}$$

Next, we characterize the incentive-compatible (IC) communication partitions.

Proposition 1 *For any n', any incentive compatible communication partition $P_{n'}$ is such that each element of the partition consists of consecutive types.*

For each $i \in \{1, ..., I-1\}$, the cardinalities $|p_i|$ and $|p_{i+1}|$ of the elements p_i and p_{i+1} of the partition are such that the following condition holds:

$$4b\left(n'+2\right) - 2 \leq |p_{i+1}| - |p_i| \leq 4b\left(n'+2\right) + 2. \tag{7}$$

Proposition 1 has the following immediate Corollary:

Corollary 1 *For any n', a fully separating communication partition is incentive compatible if and only if $b\left(n'+2\right) \leq 1/2$. If $b \geq 1/4$ the only incentive compatible communication partition is the babbling one.*

Proposition 1 allows us to compare and contrast our model with the original model of Crawford and Sobel (1982). In the latter model, IC communication strategies are characterized by a partition of the type space such that any element of the partition is an interval (a_i, a_{i+1}), and each boundary type a_i of the expert is indifferent between the two sequentially rational actions y_i and y_{i+1} associated with the intervals (a_{i-1}, a_i) and (a_i, a_{i+1}) respectively. This implies the so-called "arbitrage condition", $a_{i+1} - a_i = a_i - a_{i-1} + 4b$, which pins down all equilibria.

Condition (7) is conceptually equivalent to the arbitrage condition. The main difference is that in our model the expert is not perfectly informed, and the type space is finite. For this reason, boundary types in an IC partition are typically not exactly indifferent between two elements of it. However, we have the following limiting result:

Lemma 2 *As $n' \to \infty$, any IC partition $P_{n'}$ of our model converges to an equilibrium partition of the Crawford and Sobel (1982) model:*

$$\lim_{n' \to \infty} |p_i| / (n'+1) \to a_i - a_{i-1} \text{ for any } i$$

Next, because we use Pareto-efficiency as a refinement concept, let us highlight the notions of Pareto-ranking of IC communication partitions and that of Pareto-efficiency of the equilibria of the overall game. For any n', when the decision-maker uses her sequentially rational strategy described in Lemma 1, the players' ex-ante expected payoffs associated with IC partition $\{P_{n'}\}$ can be rewritten as follows:

$$E\left[-\left(y(P_{n'})-\theta\right)^2 \middle| P_{n'}\right] - b^2 - cn' \tag{8}$$

$$E\left[-\left(y(P_{n'})-\theta\right)^2 |P_{n'}\right] \tag{9}$$

Notice that expressions (8) and (9) differ by $b^2 + cn'$.

Observe that at the interim stage i.e., after the number of trials n' has been chosen but the number of successes has not yet been realized, cn' is a sunk cost for the sender and the preferences of the two players are aligned. Both players would like to minimize $E\left[\left(y_{p_i}^{n'} - \theta\right)^2 |P_{n'}\right]$, the residual variance of θ associated with the equilibrium communication partition. Hence, all IC communication partitions with given number of trials n' can be Pareto-ranked according to the value of $E\left[\left(y_{p_i}^{n'} - \theta\right)^2 |P_{n'}\right]$. In particular, a fully separating IC partition, if it exists, is Pareto efficient[15]. An omitted result (Proposition A.1), available upon request provides a complete characterization of Pareto efficient IC partitions.

Consider next the notion of ex-ante Pareto efficiency of the equilibria of the overall game. We will say that equilibrium $\left(n', \{P_n\}_{n=0,1,\ldots,\infty}, \{\mathbf{y}(P_n)\}_{n=0,1,\ldots,\infty}\right)$

[15]If a partition contains a non-singleton element, pool p, then for this pool we have $E[\theta|p] = \sum_{k \in p} E[\theta|k] / |p|$. Since the quadratic function is convex, by Jensen's inequality this partition is associated with a higher residual variance than the fully separating partition.

is *ex-ante Pareto efficient* if there is no other equilibrium in which the expert's and the decision-maker's ex-ante payoffs are greater, with at least one of them strictly greater than in equilibrium $\left(n', \{P_n\}_{n=0,1,\ldots,\infty}, \{\mathbf{y}(P_n)\}_{n=0,1,\ldots,\infty}\right)$. Observe that, in contrast to the interim stage, at the ex-ante stage the preferences of the two players are not aligned. The reason is that the investment cost cn' has not been sustained yet. This creates a tension between the common interest of the two players in reducing the information loss, and the fact that the cost of information acquisition is borne entirely by the expert.

Note that when we refer to the decision-maker's objective in the game, we will typically say that the decision-maker wishes to maximize the *precision* of the decision, which is the inverse of the variance in (9), $\frac{1}{E\left[(y(P_{n'})-\theta)^2|P_{n'}\right]}$. This is equivalent to maximizing (9).

We conclude by observing that ex-ante Pareto efficiency requires that on the equilibrium path players communicate according to the Pareto efficient IC partition but it does not preclude the players from coordinating on less informative equilibria off the equilibrium path, i.e. after the expert performs a non-equilibrium number of trials.

2.2 The Covert Game

Let us now introduce the covert game. Unlike in the overt game, the decision maker does not observe the amount of information acquired by the expert, and the latter may send a cheap talk message about how many trials he performed. The other elements are the same as in our overt game. Hence, a Perfect Bayesian Equilibrium of the covert game must also specify the decision-maker's beliefs also about the expert's information acquisition choice.

Our analysis focuses on the set of equilibria in which the expert plays a pure strategy at the information acquisition stage. The next Lemma shows that we can, without loss of generality, restrict attention to equilibria in which the decision-maker's beliefs about the expert's information acquisition choice is the same on and off path.

Lemma 3 *Any outcome supported in a Perfect Bayesian Equilibrium of* *the covert game in which the expert follows a pure strategy in the choice of the number of trials can also be supported in a Perfect Bayesian Equilibrium in which the decision-maker's beliefs about the number of trials do not vary with the expert's message m.*

Relying on Lemma 3, we will focus on equilibria in which after any message, the decision-maker believes that the expert performed the equilibrium number of trials with probability 1. As in the overt game, the expert's equilibrium communication strategy is equivalent to a partition of the set of possible successes. So a pure-strategy Perfect Bayesian Equilibrium of the covert game can be represented by a triple $(n^*, P_{n^*}, \mathbf{y}(P_{n^*}))$, where n^* is the number of trials, P_{n^*} is the communication partition of the set of possible successes $\{0, 1, ..., n^*\}$, and $\mathbf{y}(P_{n^*}) \equiv \{y_{p_i}^{n^*}\}_{p_i \in P_{n^*}}$ is the decision-maker's equilibrium action profile, which satisfy the following:

(i) the action profile $\mathbf{y}(P_{n^*})$ is sequentially rational, i.e. for all $p_i' \in P_{n^*}$:

$$y_{p_i'}^{n^*} \in \arg \max_{y \in \{y_{p_i}^{n^*}\}_{p_i \in P_{n^*}}} \int_0^1 U^R(y, \theta) f(\theta | k \in p_i, n) d\theta.$$

(ii) the partition P_{n^*} is incentive compatible for the expert given the decision-maker's action profile $\mathbf{y}(P_{n^*})$, i.e. for every $p_i, p_j \in P_{n^*}$ and $k \in p_i$:

$$\int_0^1 U^S\left(y_{p_i}^{n^*}, \theta, b\right) f\left(\theta | k, n^*\right) d\theta \geq \int_0^1 U^S(y_q^{n^*}, \theta, b) f\left(\theta | k, n^*\right) d\theta.$$

(iii) n^* maximizes the expert's ex-ante expected payoff, i.e.

$$n^* \in \arg \max_{n' \in \{0,1,\ldots,\infty\}} \sum_{k=0}^{n'} \left[\max_{y_p \in \mathbf{y}(P_{n^*})} \int_0^1 U^S\left(y_p, \theta, b\right) f\left(\theta; k, n'\right) d\theta \right] \Pr\left(k; n'\right) - c(n'). \tag{10}$$

While conditions (i) and (ii) are the same as their counterparts for the overt game in (3) and (4), respectively, condition (iii) is specific to the covert game, reflecting the possibility of the expert's unobservable deviations in the number of trials. To understand it, consider the expected payoff that the expert gets by deviating to some n', $n' \neq n^*$, at the information acquisition stage. In this case, the communication game will still proceed on the basis of the equilibrium partition P_{n^*}. That is, whatever message the expert sends at the communication stage, he can only induce one of the actions in the equilibrium action profile $\mathbf{y}(P_{n^*})$. So, an expert who has obtained k successes in n' trials, will induce such action y from the action profile $\mathbf{y}(P_{n^*})$ that maximizes his payoff, as reflected in (10).

The nature of the optimality condition (10) has important implications for the analysis of the covert game. In particular, the following trade-off emerges: a more informative communication partition leads to a more precise decision. However, a higher informativeness of a (candidate equilibrium) partition could also make it more profitable for the expert to deviate at the information acquisition stage.

We conclude this section by observing that, as the overt game, the covert game also has multiple equilibria and we will focus on Pareto-efficient equilibria, which are naturally defined as follows: equilibrium $(n^*, P_{n^*}, \mathbf{y}(P_{n^*}))$ is *ex-ante Pareto efficient* if there is no other equilibrium in which the expert's and the decision-maker's expected payoffs are greater, with at least one of them strictly greater than in equilibrium $(n^*, P_{n^*}, \mathbf{y}(P_{n^*}))$.

2.3 Benchmark: Information Acquisition by the Decision Maker

One of the central results of cheap-talk communication models is that decisions based on information communicated by a biased expert are less precise, and hence less efficient, than the decisions that would be made by the decision maker if she had direct access to the information. Our paper asks whether this result continues to hold when information is costly and its acquisition is endogenous.

To this end, we need to consider the benchmark problem of a decision-maker acting without an expert and acquiring information by herself. This decision-maker chooses the number of trials n, incurs the cost $c(n) = cn$ for performing them, observes the number of successes $k \in \{0, ..., n\}$, and then chooses the action y_k^*. We maintain the same assumptions on the joint distribution of θ and k for given n and the decision-maker's payoff function as in the previous sections.

The same argument as in Lemma 1 implies that the decision-maker's optimal action y_k^* after k successes in n trials satisfies $y_k^* = E[\theta|k] = \frac{(k+1)}{n+2}$.

Moreover, we can establish the following Lemma:

Lemma 4 *If the decision-maker performs n trials, then her expected utility is equal to:*

$$E\left[-\left(y_k^*-\theta\right)^2|n\right]-cn=-\frac{1}{6(n+2)}-cn. \tag{11}$$

and the decision maker's optimal number of trials $n^*(c)$ is equal to:

$$\begin{aligned} n^*(c) &= \max\left\{n:-\frac{1}{6(n+2)}-cn-\left(-\frac{1}{6(n-1+2)}-c(n-1)\right)>0\right\} \\ &= \left\lfloor \frac{1}{2}\left(\sqrt{1+\frac{2}{3c}}-3\right)\right\rfloor. \end{aligned} \tag{12}$$

Combining (11) and (12) yields a closed form expression for the decision-maker's maximal attainable expected payoff:

$$E\left[-\left(y_{n^*}-\theta\right)^2|n^*\right]-cn^*=-\frac{1}{6\left\lfloor\frac{1}{2}\left(\sqrt{1+\frac{2}{3c}}+1\right)\right\rfloor}-c\left\lfloor\frac{1}{2}\left(\sqrt{1+\frac{2}{3c}}-3\right)\right\rfloor. \tag{13}$$

3 Overinvestment and Decision Precision

This section provides the main result of this paper: the decisions based on the advice of a biased expert can be equally or more precise than the decisions based on information directly acquired by the decision maker. This result is driven by a combination of two factors. The first factor is overinvestment in information acquisition by the expert. The second factor is the smallness of information loss in transmission.

The overinvestment is a product of a trade-off between two driving forces. On the one hand, the decision-maker has an incentive to induce the agent to acquire as much information as possible, because the cost of information acquisition is borne by the latter. On the other hand, the expert prefers to acquire less information than in the benchmark direct-acquisition case, because some information can be lost in transmission and hence it is not worth the investment to be acquired. Instead, in the direct-acquisition case all information is fully utilized by the decision-maker.

At first glance, it would appear that the expert's incentives to acquire less information should lead to underinvestment. Yet, this is not the case: in plausible equilibria the aforementioned trade-off leads to overinvestment, although the exact way in which this occurs is different in the overt and covert game, as we outlined in the introduction. Along with overinvestment, the second factor that leads to a higher decision precision in the communication game is the smallness of the loss of information in transmission. Intuitively, when information is costly, the agent has an incentive to acquire only the information that he would transmit in equilibrium. The fact that the expert's equilibrium information is fairly coarse helps the agent's incentives to transmit it in full.

Finally, we note that our overinvestment results do not rely on misalignment of preferences between the two players: They hold also when the expert is unbiased. This stand in stark contrast with the work Che and Kartik (2009) on the acquisition and transmission of verifiable information.

3.1 Decision Precision in the Overt Game

The observability of the number of trials in the overt game implies that the decision-maker can and will react to the amount of information acquired by the expert. In particular, it is quite natural that an observable deviation by the expert from the equilibrium number of trials would cause the decision-maker to lose any trust in the expert: the decision-maker would consider any message sent by the deviating expert to be non-credible and uninformative. This logic suggests that a babbling equilibrium will be played off the equilibrium path.

Such behavior by the decision-maker is credible, because babbling does, in fact, constitute an equilibrium of the communication game. It also imposes the strongest punishment on the expert for a deviation at the information acquisition stage.[16] The following Proposition demonstrates that this punishment is sufficiently strong to induce the expert to overinvest in information acquisition, compared to the single-player benchmark. This

[16] Selection of a babbling equilibria to improve the decision maker's welfare is reminiscent of the constructions in the sequential cheap talk models of Aumann and Hart (2003) and Krishna and Morgan (2004). But, unlike in our construction, babbling equilibria there are also invoked on the equilibrium path.

overinvestment, however, is not too large so that it remains consistent with full transmission of the expert's information in the communication stage. Hence, the final decision in the overt game is strictly more precise than the decision based on information directly acquired by the decision maker.

Proposition 2 *If* $b \leq \left(\sqrt{1+\frac{2}{3c}}+3\right)^{-1}$ *and* $c \leq \frac{5-\sqrt{17}}{48}$, *then the overt game has an equilibrium in which the final decision is strictly more precise than the decision based on direct information acquisition by the decision maker.*

A few remarks are in order. First, the sufficient conditions identified by the Proposition are quite intuitive. The condition on the bias is consistent with Corollary 1: full revelation of the trial outcomes is incentive compatible only if the bias is sufficiently small. The condition on the unit cost c comes from the fact that if the cost is too high, then the expert would never overinvest, not even if the decision-maker threatened to completely ignore his messages so that a babbling equilibrium would be played.

Second, for expositional simplicity we have assumed that the information acquisition cost is the same for the expert and the decision maker. The result of Proposition 2 holds a fortiori if the expert is more efficient than the decision maker at acquiring information.

Third, Proposition 2 focuses on conditions for the existence of an equilibrium with overinvestment and no loss of information in transmission. In this case, we provide a concise characterization of the conditions on the parameters b and c.

However, full revelation of information is not necessary for the decision to have higher precision in the overt game than in the case of direct information acquisition by the decision maker. What is necessary for this result is that the loss of information in the communication game should not be too large. To illustrate this, we have numerically computed equilibria for a broad range of parameters b and c and have identified a larger region of these parameter values for which there exists an equilibrium of the overt game with a higher decision precision than under direct information acquisition by the decision maker. In a large part of this parameter region the identified equilibria involve some loss of information in transmission.

We performed the analysis for $b \in [0, 0.25]$, $c \in [0, 0.027]$ and $n \leq 100$. This is the relevant parameter range, since for $b \geq 0.25$ the unique equilibrium of the communication game is uninformative, and for $c > 0.02\overline{7}$ the unique solution of the decision maker's optimization problem is $n^* = 0$. For every feasible parameter constellation in this range, we have computed the equilibria and equilibrium outcomes and then compared the minimal residual variance $E\left[(y_n - \theta)^2 \,|n\right]$ among these equilibria with the residual variance in the first term of (13).

The results of this numerical analysis are presented in Figure 1(b). Figure (1a) instead, depicts the region where the sufficient conditions in Proposition 2 are satisfied. Taken together, these two figures illustrate that, under a broad range of parameter values, even if some information is lost in communication, overinvestment more than compensates for this loss. Hence, the decision remains more precise in the communication game than in the case of direct information acquisition.

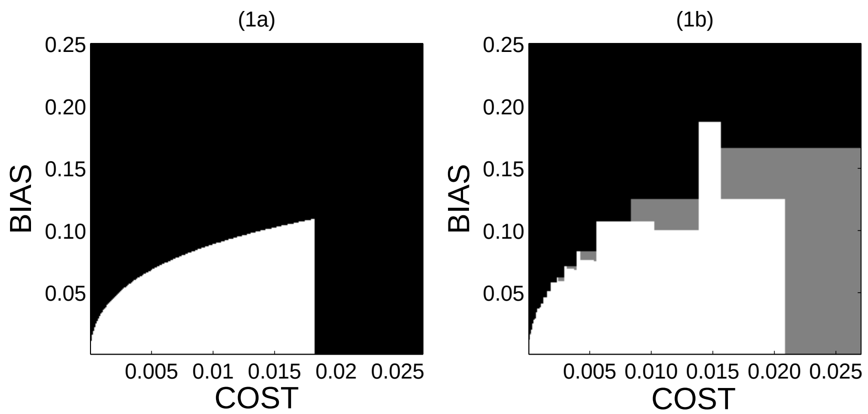

Figure 1: (1a) In the white region, the sufficient conditions in Proposition 2 are satisfied. (1b) In the white region the decision in the most informative equilibrium of the overt game is strictly more precise than with direct information acquisition. In the grey region it is as precise. In the black region it is strictly less precise.

Next, we show that reliance on babbling off the path is not necessary for weak overinvestment. The following Proposition shows the latter can be achieved even when the most informative equilibrium is played off the path, after any deviation by the expert at the information acquisition stage.

While in this equilibrium, which is the expert's preferred Pareto efficient equilibrium, the final decision is as precise as the decision based on direct information acquisition by the decision maker, in all the other Pareto efficient equilibria of the overt game, the final decision is strictly more precise. Focusing on Pareto efficient equilibria is standard practice in the communication literature. Hence, the following Proposition identifies a parameter region in which communication is generally more efficient than direct information acquisition.

Proposition 3 *If* $b \leq \left(\sqrt{1+\frac{2}{3c}}+1\right)^{-1}$*, then there exists a Pareto efficient equilibrium of the overt game in which the most informative communication equilibrium is played on and off path, and such that the final decision is as precise as with direct information acquisition by the decision maker. In any other Pareto-efficient equilibrium of the overt game the final decision is strictly more precise than the decision based on direct information acquisition.*

To prove this Proposition we first show that under its condition, full revelation of information is incentive compatible if the expert performs the same number of trials that would be acquired by the decision-maker if she did so directly, without an expert. This allows us to show that the expert would, indeed, want to acquire this amount of information in equilibrium, even if the most informative communication equilibrium is played off the path.

To understand this, recall that the preferences of the decision-maker and the expert are aligned at the stage when the number of trials have been performed but their realizations are not yet known: both of them wish to minimize $E\left[\left(y_{p_i}^{n'}-\theta\right)^2|P_{n'}\right]$, the residual variance of θ associated with the equilibrium communication partition (equivalently, maximize the decision precision). So if the expert deviates and performs a different number of trials n'', then irrespective of whether the continuation equilibrium after n'' involves full or partial revelation of information, the expert's payoff would go down by at least the same amount as the payoff of the decision maker who switches from $n^*(c)$ trials to n'' in direct information acquisition. So, since $n^*(c)$ is optimal for the decision-maker in direct information acquisition, it also constitutes an equilibrium choice for the expert.

This equilibrium is Pareto-efficient because in it the expert attains his highest ex-ante expected payoff among all equilibria. By definition, in any other Pareto-efficient equilibrium the decision maker has to obtain a higher ex-ante expected payoff, i.e. the decision has to be more precise.

3.2 Decision Precision in the Covert Game

In contrast to the overt game, in the covert game the decision maker does not observe the amount of information acquired by the expert. So the latter can make unobservable deviations in this activity. By Lemma (3) we can restrict attention to equilibria in which the expert's messages are not informative of such deviations and the decision-maker always keeps her equilibrium belief about the number of experiments performed by the expert.

The main result of this section demonstrates that equilibria with over-investment also exist in the covert game, albeit under more restrictive conditions on the parameters of the model than in the overt game. Consider an equilibrium in which the decision-maker expects the agent to perform $n^*(c) + 1$ trials and then fully reveal the outcome, where $n^*(c)$ is the optimal number of trials performed by the decision maker in the absence of an expert when the cost of a trial is equal to c, as defined in (12). A deviation to n^* trials would be beneficial for the expert with a small bias if the expert could communicate both her information acquisition decision and the observed outcome to the decision-maker. However, since in equilibrium the expert can only communicate information of the form "I have observed k successes out of $n^*(c)+1$ trials" ($k \in \{0, 1, ..., n^*(c)+1\}$) such deviation to n^* trials is no longer profitable, because any action that the expert could induce via such messages is further away from his ideal action.

The following simple example illustrates this logic.

Example 1 Suppose that $c = \frac{1}{35}$ and $b \leq \frac{17}{210}$. By Lemma 4, $n^* = 0$ i.e., the decision maker would not acquire any information, thus receiving a payoff of $-\frac{1}{12}$. However, there exists an equilibrium of the covert game in which the expert performs one trial and reveals its outcome, inducing action $y = \frac{1}{3}$ after a failure and $y = \frac{2}{3}$ after a success. The associated expected payoffs of the expert and of the decision maker are $-\frac{1}{18} - b^2 - c$

and $-\frac{1}{18}$, respectively. The decision maker achieves a higher utility than if she acquired information directly. Let us check that this is an equilibrium. If the experts deviates to zero trials, then any message he sends can only induce one of the equilibrium actions, namely $y = \frac{1}{3}$ or $y = \frac{2}{3}$. Because of his upwards bias ($b > 0$), he prefers $y = \frac{2}{3}$. The expected utility that the expert obtains by inducing $y = \frac{2}{3}$ is $-\frac{1}{9} + \frac{b}{3} - b^2$. For $b \leq \frac{17}{210}$ and $c = \frac{1}{35}$, this is less than $-\frac{1}{18} - b^2 - c$, so this deviation is unprofitable. Showing that the expert will not deviate to any $n > 1$ is straightforward and is therefore omitted.

Remarkably, while Example 1 is such that the optimal direct information acquisition choice is not to perform any trial, this feature is by no means general in our construction[17]. Example 2 considers the case when $b \leq \frac{1}{24}$ and $\frac{1}{72} < c < \frac{1}{48}$. In this case, the decision-maker acquiring information directly performs exactly one trial, and there is an equilibrium of the covert game in which the expert performs exactly two trials. This example highlights the main forces present in our equilibrium construction, in a case where expert's deviation in information acquisition generates a non-trivial information partition.

Example 2 Suppose that $b \leq \frac{1}{24}$ and $\frac{1}{72} < c < \frac{1}{48}$. By Lemma 4, the decision-maker would acquire one trial, thus receiving a payoff of $-\frac{1}{18} - c$. However, there exists an equilibrium of the covert game in which the expert performs two trials and reveals their outcomes, inducing actions $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ if the announced number of successes is zero, one, or two, respectively. . The associated expected payoffs of the expert and of the decision maker are $-\frac{1}{24} - b^2 - 2c$ and $-\frac{1}{24}$, respectively. Truthful revelation of the realization of two trials is incentive compatible for the expert because $b \leq \frac{1}{8}$ (Corollary 1). Next, we check that the expert does not have a profitable deviation in the information acquisition stage.

[17] Further, Example *1* can be extended to show that our overinvestment results hold beyond our parametric statistical model. Consider an alternative model in which the expert's information acquisition model consists in choosing the fineness of a partition of the state space $[0, 1]$, composed of equally sized intervals. I.e., the expert chooses the number n of intervals $[(k-1)/n, k/n]$, $k = 1, \ldots, n$, at cost cn, to then observe the interval to which θ belongs. It can be shown that, for $b \leq \frac{7}{60}$ and $c = \frac{1}{35}$, there exists an equilibrium of the covert game such that the decision maker achieves a higher utility than if she acquired information directly. (Details available upon request.)

If the expert deviates to zero trials, then any message he sends can only induce one of the equilibrium actions, namely $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$. The expert's payoffs from inducing these actions are $-\frac{1}{12}-\left(b+\frac{1}{4}\right)^2$, $-\frac{1}{12}-b^2$, and $-\frac{1}{12}-b^2$, respectively. Because $b^2 \leq \frac{1}{24}$, the expert's payoff from action $\frac{1}{2}$ is the highest of the three, but is smaller than his putative equilibrium payoff $-\frac{1}{24}-b^2-2c$ because $c<\frac{1}{48}$. So this deviation is unprofitable. Next, consider a deviation to $n=1$. In this case, the number of successes k could be either 0 or 1. The computations provided in the Appendix show that, when $k=0$, the expert prefers to induce action 1/4 rather than actions 1/2 or 3/4, and when $k=1$, the expert prefers to induce action 3/4 rather than actions 1/4 or 1/2. So, after a deviation to $n=1$, the expert's payoff is $Prob(k=0|n=1)\left(-2\int_0^1\left(\frac{1}{4}-\theta-b\right)^2(1-\theta)d\theta\right)+Prob(k=1|n=1)\left(-2\int_0^1\left(\frac{3}{4}-\theta-b\right)^2\theta d\theta\right)-c=-\frac{3}{48}-b^2-c$. This is smaller than his putative equilibrium payoff $-\frac{1}{24}-b^2-2c$. Finally, showing that the expert would not deviate to $n>2$ is straightforward and is therefore omitted.

Our general analysis uncovers sufficient conditions for strict overinvestment, and is presented in the following Proposition, where the symbol $I_{n=k}$ denotes the indicator function which takes the value one if $n=k$, and zero otherwise.

Proposition 4 *For any integer n, if $b \leq \frac{1}{4(n+3)}$, and $\frac{1}{6(n+2)(n+3)} < c < \frac{1}{6(n+1)(n+3)} - \max\left\{0, \left(\frac{1}{3}b\right)\mathbb{I}_{n=0}, \left(\frac{24b-1}{96}\right)\mathbb{I}_{n=1}, \left(\frac{30b-1}{450}\right)\mathbb{I}_{n=2}, \left(\frac{30b-1}{360}\right)\mathbb{I}_{n=3}, \left(\frac{63b-2}{735}\right)\mathbb{I}_{n=4}\right\}$[18], the covert game possesses an equilibrium in which the final decision is strictly more precise than the decision based on direct information acquisition by the decision maker.*

The sufficient conditions of Proposition 4 are represented graphically in Figure (2a). Consider what happens as the cost of an experiment c increases. As Figure (2a) indicates, an interval of costs for which the sufficient condition in Proposition 4 is satisfied is followed by an interval of slightly higher costs for which the condition does not hold. The latter cost interval is then again followed by an interval of higher costs where the sufficient condition holds, and so on.

[18] It is easy to check that this interval is non-empty.

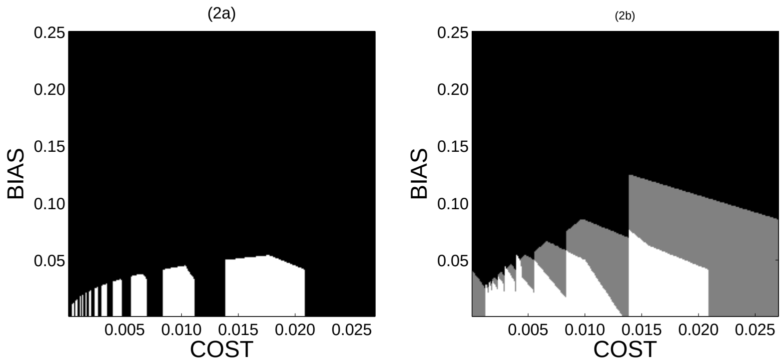

Figure 2: (2a) In the white region, the sufficient conditions in Proposition 4 hold. (2b) In the white region, the decision in the most informative equilibrium of the covert game is strictly more precise than under optimal direct information acquisition. In the grey region the precision is the same in the most informative equilibrium of the covert game and under optimal direct information acquisition. In the black region, the decision under optimal direct information is more precise than in the most informative equilibrium of the covert game.

This pattern reflects the following regularity. Let $H(n)$ be an interval of cost values for which a given n is the optimal number of trials under direct information acquisition. It can be shown that $H(n) = \left(\frac{1}{6(n+2)(n+3)}, \frac{1}{6(n+1)(n+2)}\right)$. Therefore, the cost axis can be divided into adjacent intervals $H(n)$ corresponding to different values of n. For each n, the second condition in Proposition 4 identifies a subset of $H(n)$, let us call it $L(n)$, for which the result holds. In particular, $L(n)$ has the same lower bound but a strictly smaller upper bound than $H(n)$. This explains the pattern in figure (2a). The intervals $H(n)$ are adjacent but the intervals $L(n)$ are not adjacent because each $L(n)$ only constitutes the lower part of the corresponding $H(n)$.

When the unit cost c lies in $L(n)$ and the bias is not too large, the covert communication game admits an equilibrium in which the expert runs $n+1$ trials and fully reveals their outcome. $L(n)$ is a strict subset of $H(n)$ because, if c is too close to the upper bound of $H(n)$, the expert prefers to save some cost and unobservedly deviate to n trials. The condition on the bias i.e., $b \leq \frac{1}{4(n+3)}$, guarantees that, if the expert performs $n+1$ trials, he then fully reveals their realization in the communication game.

Since the conditions of Proposition 4 are sufficient, rather than necessary, we have numerically identified the whole region of the parameter space where the precision of the decision maker's action in the most informative equilibrium of the covert game is strictly higher than under optimal direct information acquisition The results of this analysis are reported in Figure (2b). For comparison, Figure (2a) represents the region where the sufficient conditions of Proposition 4 hold. We conclude the analysis of the covert game with a result analogous to Proposition 3 for the covert game.

Proposition 5 *If* $b \leq \left(2\sqrt{1+\frac{2}{3c}}+2\right)^{-1}$*, then in the equilibrium of the covert game with the highest expert's ex ante payoff, the final decision is as precise as the decision based on direct information acquisition by the decision maker. In any other Pareto efficient equilibrium of the game, the decision is strictly more precise.*

The proof of Proposition 5 establishes that in his preferred equilibrium the expert performs $n^*(c)$ trials, where $n^*(c)$ is given by (12), and fully reveals their outcome. The key step of the proof shows that the expert cannot benefit by deviating at the information acquisition stage from this level, because after such deviation he experiences a larger loss in his expected payoff, than the loss suffered by the decision-maker if the latter made the same deviation in the benchmark case of direct information acquisition. The second part of the Proposition follows from the observation that in any other Pareto efficient equilibrium the utility of the decision maker, and hence the precision of the decision, must be higher.

We conclude this section with a brief discussion of a possible extension of our model to the case of verifiable reporting. Suppose that information acquisition is still covert, but that the expert cannot fabricate information, nor lie about its content. His only choice in the communication game is how much of the acquired information to disclose. Then, the existence of equilibria in which the expert overinvests in information and makes a full disclosure follows from Proposition 5 and the standard unravelling argument based on Milgrom (1981). We conjecture that this result would hold under a larger set of parameters than under unverifiable information, but we have not confirmed this conjecture formally.

4 Organizational Design

Our results have important implications for the issue of optimal organization and allocation of authority therein. Consider the problem of choosing an optimal decision when information acquisition is costly. The process of information acquisition and decision-making could be organized in several ways, via a number of organizational forms and methods. Particularly, let us focus on three intuitive and ubiquitous organizational forms which also approximate a number of other organizations:

- *Centralization:* the principal performs both the task of acquiring information and making a decision.
- *Delegation:* the principal delegates both information acquisition and decision-making to an agent.
- *Communication:* the principal delegates the task of information acquisition to an agent but keeps the decision-making authority.

Under centralization, the principal bears the cost of information transmission. Yet at the same time, the principal retains full control over information, and hence there is no loss or distortion of information in transmission. Delegation, instead, allows the principal to reallocate the cost of information acquisition to the agent. However, this benefit for the principal comes at the cost of a suboptimal decision taken by the agent, whose preferences are not aligned with her own. Finally, under communication the principal retains decision-making authority, but the agent's bias affects both the amount of information that he acquires and the extent to which the acquired information is communicated to the principal. The described trade-offs imply that it is not clear which organizational structure is optimal from the principal's ex ante point of view.

Relative performance of communication and delegation has been previously explored in the literature, albeit without information acquisition. Dessein (2002) and Ottaviani (2000) find that delegation tends to outperform communication. This paper contributes to this strand of literature by introducing endogenous and costly information acquisition and by also adding centralization to the set of organizational forms being compared.

To compare the three organizational forms from the point of view of the principal, we need to compute the principal's expected payoff under each of them. To this end, note that centralization is modelled by our benchmark of direct information acquisition by the decision-maker. The optimization problem solved by the agent under delegation is similar to the problem solved by the principal under centralization. In both cases, the party acquiring information (i.e., the principal under centralization, and the agent under delegation) will conduct $n^*(c)$ trials given by (12), and take the action maximizing that player's expected payoff. So, the expected payoff of the principal is given by:

Under centralization,

$$E\left[-\left(y^*-\theta\right)^2|n^*(c)\right]-cn^*(c). \tag{14}$$

Under delegation,

$$E\left[-\left(y^*-\theta\right)^2|n^*\left(c\right)\right]-b^2 \tag{15}$$

where y^* is the principal's optimal decision equal to $\frac{k+1}{n^*(c)+2}$ when k successes are observed in $n^*\left(c\right)$ trials.

Finally, communication as an organization can be modelled either via the overt or the covert game analyzed above, depending on the observability conditions. In an equilibrium of the overt or covert communication game with n trials and communication partition P, the principal earns a payoff equal to:

$$E\left[-\left(\bar{y}-\theta\right)^2|\mathcal{P}\right]. \tag{16}$$

where $\bar{y}=E(\theta|p^i)$ is the optimal decision when the expert's message signals element p^i of the partition P.

Inspection of (14)-(16) reveals that the principal achieves the highest expected payoff under communication if the precision of the decision in equilibrium of the communication game is at least as large as under delegation or centralization i.e.,

$$E\left[-\left(\bar{y}-\theta\right)^2|\mathcal{P}\right]\geq E\left[-\left(y^*-\theta\right)^2|n^*\left(c\right)\right].$$

Even if the above expression holds as equality, i.e. if the precision of the decision is the same across these organizational forms, communication dominates centralization because the cost of information acquisition is borne by the expert in the former and by the principal in the latter. Communication also dominates delegation because in the latter the expert's decision is biased against the decision-maker, as reflected by the term $-b^2$ in (15).

Propositions 3 and 5 above identify sufficient conditions under which the decisions in all Pareto efficient equilibria of the overt and covert game are at least as precise as under centralization. Based on these results, we can state the following Corollary.

Corollary 2 ***(a)*** *If* $b \leq \left(\sqrt{1+\frac{2}{3c}}+1\right)^{-1}$*, then the principal strictly prefers any Pareto efficient equilibrium of the overt game to either centralization or to delegation.*
(b) *If* $b \leq \left(2\sqrt{1+\frac{2}{3c}}+2\right)^{-1}$*, then the principal strictly prefers any Pareto efficient equilibrium of the covert game to centralization and delegation.*

Corollary 2 holds under the assumption that the agent and the principal have the same cost c of an experiment. This assumptions is relevant for the comparison between communication and centralization. However, the comparison between communication and delegation does not rely on it, since the expert bears the cost of information acquisition in both these organizational forms.

Further, the conditions in Corollary 2 are sufficient for the optimality of communication, but are not necessary. So, to present a more complete comparison of the organizational forms, we turn to numerical analysis. The results are summarized in Figures (3)-(5). Figure (3) depicts the region of the parameter space where the sufficient condition (a) of Corollary 2 holds. Figures (4a) and (4b) present the optimal organizational form computed numerically. The value for communication was calculated choosing the principal's and the agent's preferred equilibrium of the overt game in Figure (4a) and Figure (4b), respectively. Compared to Figure (3), Figures (4a) and (4b) show that communication is the best organization for a larger set of the cost/bias parameter values. In particular, communication is optimal unless the bias is sufficiently large. For small costs of information acquisition, the value of the bias plays the crucial role: any of the three organizational forms can be optimal, depending on the bias. Specifically, the principal prefers communication if the bias is small; delegation dominates if the bias is intermediate; centralization dominates if the bias is large. The region where delegation is optimal disappears as the information acquisition cost increases. Perhaps not surprisingly, centralization dominates when the bias is large, regardless of the cost of information acquisition. As we move from the expert's preferred equilibrium to the decision-maker's preferred equilibrium, communication becomes optimal under a broader set of parameter values, and mostly at the expenses of delegation, rather than centralization.

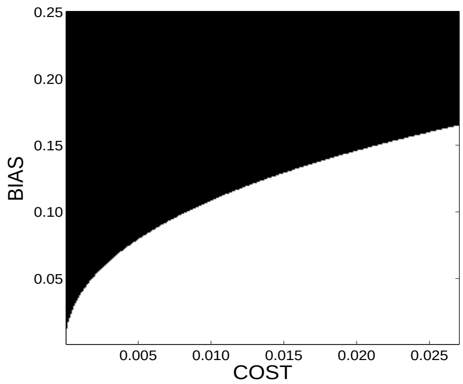

Figure 3: In the white region, the sufficient condition (a) in Corollary 2 is satisfied.

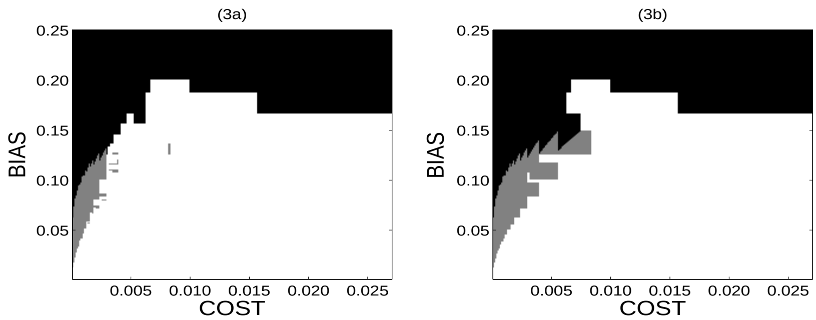

Figure 4: In the white region, the best organization is Communication. In the grey region, it is Delegation. In the black region, it is Centralization. Panel (4a) considers the equilibrium of the overt game preferred by the decision maker. Panel (4b) considers the one preferred by the expert.

Covert information acquisition is compared with centralization and delegation in Figures (5a) and (5b). Figure (5a) depicts the region of the parameter space where the sufficient condition (b) of Corollary 2 holds. Figure (5b) presents the numerical derivation of the optimal organizational form which utilizes the principal's preferred equilibrium of the covert game under communication (The results for the expert's preferred equilibrium are very similar, and hence have been omitted). Comparison of Figures (5a) and (5b) shows that Condition (b) in Corollary 2 is sufficient but not necessary, similarly to the overt game. Also, compared to Figure (4) (overt game), in Figure (5b) (covert game) the area where delegation dominates is noticeably larger.

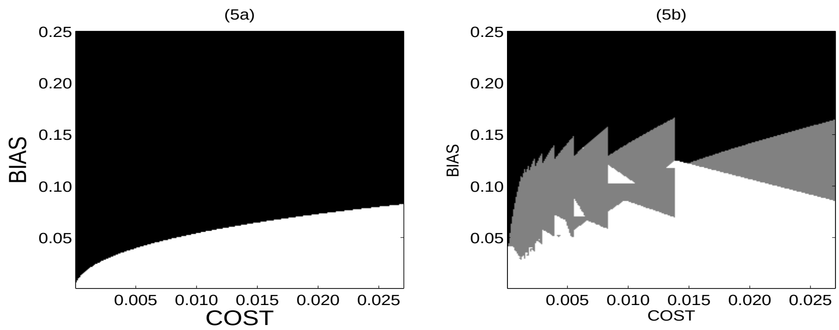

Figure 5: (5a) In the white region, the sufficient condition (b) in Corollary 2 is satisfied. (5b) In the white region, the best task allocation is Communication. In the grey region, it is Delegation. In the black region, it is Centralization.

5 Conclusion

We have developed a simple yet intuitive model of costly endogenous information acquisition with strategic communication of this information. In this context, we have shown that decisions based on a biased expert's advice may be more precise than optimal choices based on direct information acquisition, even if the expert is not more efficient than the decision maker at acquiring information. This result is important for organization design, as it implies that under certain conditions communication outperforms delegation and centralization. In this respect, our paper contributes to the literature that employs a strategic communication framework to study optimal allocation of authority in the presence of incomplete information.

A number of other interesting questions can be addressed in the framework of our model. First, suppose that the decision-maker was able to subsidize the expert's information acquisition cost. How would that affect the amount of information acquired and the precision of the decision? Second, how would the outcome of the communication game be affected if the expert acquired the information covertly but had an option to verifiably disclose the amount of information that he acquired? Would a decision maker prefer knowing the amount of information acquired by an expert, when she could not inspect its content? As shown by Austen-Smith (1994), this issue is far from being transparent. We leave these and other questions for future research.

Appendix

Proof of Lemma 1. The decision maker chooses $y_{p_i}^{n'}$ so as to maximize

$$-\int_0^1 \left(y_{p_i}^{n'} - \theta\right)^2 f\left(\theta | k \in p_i, n'\right) d\theta.$$

Taking the first-order condition, we obtain $y_{p_i}^{n'} = \int_0^1 \theta f\left(\theta | k \in p_i, n'\right) d\theta = E\left[\theta | p_i, n'\right]$. Simplifying:

$$E\left[\theta | p_i, n'\right] = E\left[E\left[\theta | k, n'\right] | k \in p_i\right] = \sum_{k \in p_i} E\left[\theta | k, n'\right] \frac{f\left(k; n'\right)}{\sum_{k \in p_i} f\left(k; n'\right)} = \frac{1}{|p_i|} \sum_{k \in p_i} \frac{k+1}{n'+2}$$

because $E\left[\theta | k, n'\right] = \frac{k+1}{n'+2}$, and $f\left(k; n'\right) = \int_0^1 f\left(k; n', \theta\right) d\theta = \dfrac{n'!}{k!\left(n'-k\right)!} \int_0^1 \theta^k \left(1-\theta\right)^{n'-k} d\theta = \dfrac{n'!}{k!\left(n'-k\right)!} \frac{k!(n'-k)!}{(n'+1)!} = \frac{1}{n'+1}$.

Proof of Proposition 1 First, we show that the incentive compatibility constraint (4) can be rewritten as

$$-\left(y_{p_i}^{n'} - y_q^{n'}\right)\left[\left(y_{p_i}^{n'} + y_q^{n'}\right) - 2E\left[\theta / k, n'\right] - 2b\right] \geq 0 \text{ for all } q \in P_{n'}.$$

For this, note the following:

$$\int_0^1 U^S\left(y_{p_i}^{n'},\theta,b\right) f\left(\theta;k,n'\right)d\theta \geq \int_0^1 U^S(y_q^{n'},\theta,b) f\left(\theta;k,n'\right)d\theta$$

$$-\int_0^1\left[\left(y_{p_i}^{n'}-\theta-b\right)^2-\left(y_q^{n'}-\theta-b\right)^2\right] f\left(\theta;k,n'\right)d\theta \geq 0$$

$$-\int_0^1\left[\left(y_{p_i}^{n'}\right)^2+(\theta+b)^2-2y_{p_i}^{n'}(\theta+b)-\left(y_q^{n'}\right)^2-(\theta+b)^2+2y_q^{n'}(\theta+b)\right] f\left(\theta;k,n'\right)d\theta \geq 0$$

$$-\int_0^1\left[\left(y_{p_i}^{n'}\right)^2-\left(y_q^{n'}\right)^2-2\left(y_{p_i}^{n'}-y_q^{n'}\right)(\theta+b)\right] f\left(\theta;k,n'\right)d\theta \geq 0$$

$$-\left(y_{p_i}^{n'}-y_q^{n'}\right)\left[\left(y_{p_i}^{n'}+y_q^{n'}\right)-2E\left[\theta/k,n'\right]-2b\right] \geq 0$$

Next, we prove that in any pure-strategy equilibrium of the communication subgame, each element of the equilibrium partition is connected. Suppose by contradiction that there exists an equilibrium where at least one element of the partition is not connected. Then, there exists at least a triple of types (k,k',k'') such that: $k<k''<k'$, k and k' belong to the same element of the partition, which we denote by p_a, and k'' belongs to a different element, which we denote by p_b. Let y_a and y_b be the equilibrium actions associated to p_a and p_b respectively. By incentive compatibility, the following inequalities must hold:

$$\begin{aligned}
(y_b-y_a)\left(y_a+y_b-\frac{2(k+1)}{n'+2}-2b\right) &\geq 0\\
(y_b-y_a)\left(y_a+y_b-\frac{2(k'+1)}{n'+2}-2b\right) &\geq 0\\
(y_a-y_b)\left(y_a+y_b-\frac{2(k''+1)}{n'+2}-2b\right) &\geq 0
\end{aligned}$$

Because the first two expressions are positive, then $y_a+y_b-\frac{2(k+1)}{n+2}-2b$ and $y_a+y_b-\frac{2(k'+1)}{n+2}-2b$ have the same sign. But then, also $y_a+y_b-\frac{2(k''+1)}{n+2}-2b$ has the same sign, because $k<k''<k$. And hence, the last expression is negative: A contradiction.

Next, we prove that incentive compatibility implies expression (7). Let k be the expert's type. Denote by y the equilibrium action associated to k, and by $\widetilde{y}$ any other equilibrium action. The incentive compatibility constraint is:

$$\left(\widetilde{y}-y\right)\left(\widetilde{y}+y-\frac{2\left(k+1\right)}{n'+2}-2b\right)\geq 0. \quad (.17)$$

First, we rule out the possibility that a type k deviates by inducing an equilibrium action $\widetilde{y}$ larger than y. This deviation is unprofitable if and only if

$$\widetilde{y}+y-\frac{2\left(k+1\right)}{n'+2}-2b\geq 0. \quad (.18)$$

Because the expression is increasing in $\widetilde{y}$ and decreasing in k, it immediately follows that the tightest incentive compatibility constraints concern the highest type $\overline{k}$ in any element p_i of the equilibrium partition, entertaining the possibility of deviating and inducing the equilibrium action $\widetilde{y}$ associated to p_{i+1}, the element of the partition immediately to the right of p.

Hence, we now consider such constraints. The explicit expression for y and $\tilde{y}$ are:

$$\begin{aligned} y &= \frac{1}{|p_i|}\left[\frac{\overline{k}+1}{n'+2}+\frac{\overline{k}-1+1}{n'+2}+...+\frac{\overline{k}-\left(|p_i|-1\right)+1}{n'+2}\right]=\frac{2\overline{k}-|p_i|+3}{2\left(n'+2\right)} \\ \widetilde{y} &= \frac{1}{|p_{i+1}|}\left[\frac{\overline{k}+1+1}{n'+2}+\frac{\overline{k}+2+1}{n'+2}+...+\frac{\overline{k}+|p_{i+1}|+1}{n'+2}\right]=\frac{2\overline{k}+|p_{i+1}|+3}{2\left(n'+2\right)} \end{aligned}$$

Hence, condition (.18) simplifies as:

$$\frac{2\overline{k}+|p_{i+1}|+3}{2\left(n'+2\right)}+\frac{2\overline{k}-|p_i|+3}{2\left(n'+2\right)}-\frac{2\left(\overline{k}+1\right)}{n'+2}-2b\geq 0,$$

or,

$$|p_{i+1}|\geq|p_i|+4b\left(n+2\right)-2. \quad (.19)$$

Proceeding in the same fashion, we prove that when $\widetilde{y}<y$, the tightest incentive compatibility constraints concern the lowest type $\underline{k}$ in any element p_i of the equilibrium partition, entertaining the possibility of deviating and inducing the equilibrium action $\widetilde{y}$ associated to p_{i-1}, the element of the partition immediately to the left of p_i. Again, letting j be the cardinality of p_i, and z be the cardinality of p_{i-1}, we obtain

$$y=\frac{1}{|p_i|}\left[\frac{\underline{k}+1}{n'+2}+\frac{\underline{k}+1+1}{n'+2}+...+\frac{\underline{k}+|p_i|-1+1}{n'+2}\right]=\frac{2\underline{k}+|p_i|+1}{2\left(n'+2\right)}$$

$$\widetilde{y}=\frac{1}{|p_{i-1}|}\left[\frac{\underline{k}-1+1}{n'+2}+\frac{\underline{k}-2+1}{n'+2}+...+\frac{\underline{k}-|p_{i-1}|+1}{n'+2}\right]=\frac{2\underline{k}-|p_{i-1}|+1}{2\left(n'+2\right)}$$

Hence, condition (.18) simplifies as:

$$\frac{2\underline{k} - |p_{i-1}| + 1}{2(n'+2)} + \frac{2\underline{k} + |p_i| + 1}{2(n'+2)} - \frac{2(\underline{k}+1)}{n'+2} - 2b \leq 0$$

which implies

$$|p_i| \leq |p_{i-1}| + 4b(n+2) + 2. \tag{.20}$$

Proof of Lemma 2 We prove that as $n' \to \infty$, for any i, $|p_i|/(n'+1) \to a_i - a_{i-1}$. In fact, condition (7) implies that

$$\frac{4b(n'+2) - 2}{n'+1} \leq \frac{|p_{i+1}| - |p_i|}{n'+1} \leq \frac{4b(n+2)+2}{n'+1},$$

and, taking limits for $n' \to \infty$, $4b \leq a_i - a_{i-1} + a_{i+1} - a_i \leq 4b$,which is exactly the arbitrage condition of Crawford and Sobel (1982).

Calculations leading to expressions (9) and (8). For any n', consider the expert's and the decision-maker's expected payoffs associated to IC partition $\{P_{n'}\}$, assuming that the decision-maker plays her sequentially rational strategy, as described by Lemma 1.:

$$\sum_{k=0}^{n'} \left(\int_0^1 U^S \left(y^{n'}_{p^{n'}(k)}, \theta, b \right) f(\theta; k, n')\, d\theta \times \Pr(k; n') \right) - c(n') \tag{.21}$$

$$\sum_{k=0}^{n'} \left(\int_0^1 U^R \left(y^{n'}_{p^{n'}(k)}, \theta, b \right) f(\theta; k, n')\, d\theta \times \Pr(k; n') \right) \tag{.22}$$

Let the operator $E[.|P_{n'}]$ denote the expectation with respect to θ and k conditional on the number of experiments n', and the associated partition $P_{n'}$. Then, using the fact that, by (6), $E[y(P_{n'})|P_{n'}] = E[\theta|P_{n'}]$, we can rewrite the expert's expected payoff in (.21) as follows:

$$\begin{aligned} E\left[-\left(y(P_{n'}) - \theta - b\right)^2 |P_{n'}\right] - cn' &= E\left[-\left(y(P_{n'}) - \theta\right)^2 + 2b\left(y(P_{n'}) - \theta\right) |P_{n'}\right] - b^2 - cn' \\ &= E\left[-\left(y(P_{n'}) - \theta\right)^2 \middle| P_{n'}\right] - b^2 - cn', \end{aligned}$$

Further, the decision-maker's expected payoffs in (.22) can be rewritten as:

$$E\left[-\left(y(P_{n'}) - \theta\right)^2 |P_{n'}\right]$$

Proof of Lemma 3: Consider an equilibrium $\mathcal{E}^1 = (n^1, m^1(n,k), B^1(.), \sigma^1)$ in which the expert performs n^1 trials, and follows message strategy $m^1(n,k)$, where n is the number of trials and k is the number of successes, the decision-maker forms beliefs $B^1(.) : M \mapsto \Delta(\{(n,k)|n,k \in \mathcal{N}, n \geq k\})$ and follows action-choice strategy $\sigma^1(.) : B^1 \mapsto \Delta([0,1])$. [19] Note that the decision maker's beliefs $B^1(.)$ is a mapping from the set of expert's messages M into the set of probability distributions $\Delta(\{(n,k)|n,k \in \mathcal{N}, n \geq k\})$, reflecting the fact that in the covert game the decision maker has to form beliefs not only about the number of successes but also about the number of experiments performed by the expert.

Let $M^e = \{m^1(n^1,k)|k = 0,1,...,n^1\}$ be the set of messages sent on the equilibrium path with a positive probability. Then $B^1_{|N}(m)$ puts probability 1 on n^1 for all $m \in M^e$. Next, fix some arbitrary $\breve{m} \in M^e$ and consider modified belief $\hat{B}(.)$ and modified strategy $\hat{\sigma}(.)$ such that for any $m \in M^e$, $\hat{B}(m) = B^1(m)$ and $\hat{\sigma}(m) = \sigma^1(m)$, while for any $m \in M \setminus M^e$, $\hat{B}(m) = B^1(\breve{m})$ and $\hat{\sigma}(m) = \sigma^1(\breve{m})$. Hence, $\hat{B}(.)$ puts probability 1 on n^1 for all $m \in M$.

Now consider a putative equilibrium $\hat{\mathcal{E}} = (n^1, m^1(n,k), \hat{B}(.), \hat{\sigma}(.))$ in which the expert performs n^1 trials and follows message strategy $m^1(n,k)$, and the decision-maker uses belief rule $\hat{B}(.)$ and strategy profile $\hat{\sigma}(.)$. With the decision-maker's belief rule $\hat{B}(.)$ in $\hat{\mathcal{E}}$, no expert's message can change the decision-maker's beliefs about the number of trials.

Furthermore, $\hat{\mathcal{E}}$ does constitute a perfect Bayesian equilibrium because $\mathcal{E}^1$ is a perfect Bayesian equilibrium, and both $\hat{\mathcal{E}}$ and $\mathcal{E}^1$ prescribe the same behavior and beliefs on the equilibrium path, with the only difference between them being in the beliefs off the equilibrium path i.e., after a message $m \in M \setminus M^e$: after such message $\hat{\mathcal{E}}$ prescribes beliefs $\hat{B}(m) = B^1(\breve{m})$, while $\mathcal{E}^1$ prescribes beliefs $B^1(m)$. However, since a message $\breve{m}$ is also available to a deviating expert in $\mathcal{E}^1$ but does not lead to a profitable deviation, there is no profitable deviation for an expert in $\hat{\mathcal{E}}$. So, $\hat{\mathcal{E}}$ is a perfect Bayesian equilibrium. *Q.E.D.*

Proof of Lemma 4: First, recall that $y^*_k = E[\theta|k] = (k+1)/(n+2)$. Using this expression below we obtain:

$$E\left[-\left(y^*_k - \theta\right)^2 |n\right] - cn = -\sum_{k=0}^{n} \Pr(k;n) \int_0^1 \left(E[\theta|k] - \theta\right)^2 f(\theta;k,n) - cn$$

[19] In this proof, we need to use a more canonical definition of perfect Bayesian equilibrium, not relying on partitions.

$$
\begin{aligned}
&= -\sum_{k=0}^{n} \frac{1}{n+1} \int_0^1 \left(\frac{k+1}{n+2} - \theta\right)^2 \frac{(n+1)!}{k!\,(n-k)!} \theta^k (1-\theta)^{n-k} \, d\theta - cn \\
&= -\sum_{k=0}^{n} \frac{1}{n+1} \int_0^1 \left[\left(\frac{k+1}{n+2}\right)^2 + \theta^2 - 2\theta \left(\frac{k+1}{n+2}\right)\right] \frac{(n+1)!}{k!\,(n-k)!} \theta^k (1-\theta)^{n-k} \, d\theta - cn \\
&= -\sum_{k=0}^{n} \frac{1}{n+1} \left[\int_0^1 \theta^2 \frac{(n+1)!}{k!\,(n-k)!} \theta^k (1-\theta)^{n-k} \, d\theta - \left(\frac{k+1}{n+2}\right)^2\right] - cn \\
&= -\sum_{k=0}^{n} \frac{1}{n+1} \left[\frac{(k+2)(k+1)}{(n+3)(n+2)} - \left(\frac{k+1}{n+2}\right)^2\right] - cn \\
&= -\frac{1}{6(n+2)} - cn. \qquad \textit{Q.E.D.}
\end{aligned}
$$

Proof of Proposition 2. We prove that there exists an equilibrium of the overt information acquisition game in which the expert runs $n^*(c)+1$ trials and fully reveals their realizations. Clearly, this equilibrium implies a decision precision higher than the benchmark of direct information acquisition by the decision maker. The result then follows because either this equilibrium is Pareto-efficient, or there exists another equilibrium which Pareto-dominates it, in which the payoff of the decision-maker, i.e. the decision precision, is even higher.

The proof proceeds as follows. First, we find the maximal number of trials $\widetilde{n}(c)$ such that, under a given investment cost c, the utility that the expert obtains by conducting $\widetilde{n}(c)$ trials and fully revealing their realizations to the decision maker is higher than the utility from running any other number of trials and playing the babbling equilibrium. Formally, $\widetilde{n}(c)$ is the highest integer that satisfies

$$
-\frac{1}{6(n+2)} - b^2 - cn \geq -\frac{1}{12} - b^2.
$$

Further, from Corollary 1 it follows that $\widehat{n}(b) \equiv \left\lfloor \frac{1}{2b} - 2 \right\rfloor$ is the maximal number of trials for which full revelation in the communication game is incentive compatible. Hence, it is an equilibrium for the expert to run $n^*(c)+1$ trials and to fully reveal the information to the decision maker whenever the following condition holds:

$$
n^*(c) + 1 \leq \max\{\widehat{n}(b), \widetilde{n}(c)\}. \tag{.23}
$$

The condition $n^*(c)+1 \leq \widetilde{n}(c)$ is satisfied if $\sqrt{\frac{2+3c}{12c}} - \frac{3}{2} + 1 \leq \frac{1}{12c} - 2$, i.e., $c \leq \frac{5-\sqrt{17}}{48}$, whereas the condition $n^*(c)+1 \leq \widehat{n}(b)$ is satisfied if

$\sqrt{\frac{2+3c}{12c}} - \frac{3}{2} + 1 \leq \frac{1}{2b} - 2$, or $b \leq \left(\sqrt{1+\frac{2}{3c}} + 3\right)^{-1}$.

If $\widehat{n}(b) \geq n^*(c) + 1$ and $\widetilde{n}(c) \geq n^*(c) + 1$, then there exists an equilibrium of the overt information acquisition game in which the expert runs $n^*(c) + 1$ trials and fully reveals their realizations, while the babbling equilibrium is played in any subgame in which $n' \neq n$ trials are run. The decision maker's utility $E\left[-(y_p - \theta - b)^2 | P_n\right]$ in this equilibrium is $-1/[6(n^* + 1 + 2)]$ which is strictly larger than the decision maker's utility $-1/[6(n^* + 2)]$ if she directly acquired information. *Q.E.D.*

Proof of Proposition 3. We prove that the expert's preferred equilibrium of the game is such that he acquires exactly $n^*(c)$ trials and fully reveals the outcome. Hence, this equilibrium yields the same decision precision as direct information acquisition by the decision maker. The result then follows from the observation that the expert's preferred equilibrium is by construction Pareto-efficient. Hence, in any other Pareto-efficient equilibrium, the ex-ante utility of the decision maker, which coincides with the precision of the decision, must be weakly larger than in this equilibrium.

Consider the expert's preferred equilibrium of the game. This equilibrium is such that the most informative (i.e. the Pareto efficient) communication equilibrium is played both on and off the equilibrium path. To prove that this is the case, notice that the maximizing the expert's payoff over all equilibria can be viewed as a two-stage maximization process. The expert's expected payoff is equal to $E\left[-\left(y_{p_i}^{n'} - \theta\right)^2 | P_{n'}\right] - b^2 - cn'$. In the first stage of the maximization, for any number n' of experiments we select an incentive compatible partition $P_{n'}$ and a profile of equilibrium actions $\mathbf{y}(P_{n'})$ that maximize $E\left[-\left(y_{p_i}^{n'} - \theta\right)^2 | P_{n'}\right] - b^2$, i.e. that maximize the precision of the information transmitted. Then, in the second stage of the maximization, we choose the number of experiments n that maximizes the maximum value of $E\left[-\left(y_{p_i}^{n'} - \theta\right)^2 | P_{n'}\right] - b^2$ as derived in the first stage, minus the cost cn'.

Next, we show that in this equilibrium, in which the Pareto-efficient incentive compatible partition is played in the communication stage on and off the equilibrium path and the expert, correctly anticipating this, selects the number of trials that maximizes his expected payoff, the equilibrium number of trials is exactly $n^*(c)$ and full revelation occurs, if the condition in the Proposition holds. First, notice that the condition $b \leq \left(\sqrt{1+\frac{2}{3c}} + 1\right)^{-1}$ implies that $\left\lfloor \frac{1}{2b} - 2 \right\rfloor \geq \left\lfloor \sqrt{\frac{2+3c}{12c}} - 1.5 \right\rfloor$, that is $\widehat{n}(b) \geq n^*(c)$. This in turn implies that fully revealing the outcome of $n^*(c)$ trials is incentive compatible. Next, consider deviations at the information acquisition stage. In equilibrium, the expert's expected utility, $E\left[U^S(\mathbf{y}^*, \theta, b) | n\right] - c(n)$ is equal to

$E\left[U^R(\mathbf{y}^*, \theta)\,|n\right] - c(n) - b^2$, the expected payoff of a decision maker who directly conducts $n^*(c)$ trials, minus b^2. Now suppose the expert deviates, and purchases n' trials and the most informative communication equilibrium is played in the ensuing communication subgame. Given this communication partition $P_{n'}$, $E\left[-\left(y_{p_i}^{n'} - \theta - b\right)^2 |P_{n'}\right] = E\left[-\left(y_{p_i}^{n'} - \theta\right)^2 |P_{n'}\right] - b^2$. For n', full separation might or might not be incentive compatible.

If it is, then the difference between equilibrium payoff and deviation payoff is equal to the payoff difference that the decision maker would receive in the single agent decision problem if he purchased n' trials rather than $n^*(c)$. This payoff difference is negative, by definition of $n^*(c)$. If some information loss occurs, the deviation gain is strictly smaller than the payoff difference that the decision maker would receive in the single agent decision problem because $E\left[-\left(y_{p_i}^{n'} - \theta\right)^2 |P_{n'}\right] < E\left[U^R(\mathbf{y}^*, \theta)\,|n'\right]$ and again the result is implied by the definition of $n^*(c)$. *Q.E.D.*

Proof of Proposition 4. We start from the observation that for any integer l, $n^*(c) = l$ for $\frac{1}{6(l+2)(l+3)} < c < \frac{1}{6(l+1)(l+2)}$, hence also for any c in the interval required by the Proposition. The proof will show that if the conditions in the proposition hold, then in equilibrium the expert acquires $n^*(c) + 1 = l + 1$ trials and fully reveals their outcome.

First, by Corollary 1, full revelation of the outcome of $l+1$ is incentive compatible for $b \leq \frac{1}{2(l+3)}$, hence it is incentive compatible for $b \leq \frac{1}{4(l+3)}$.

Next, we establish that the expert has no incentive to acquire a number of trials different from $l+1$. The expert's expected payoff from performing $l+1$ trials and fully revealing the outcome is equal to $W(l+1) = -\frac{1}{6(l+3)} - b^2$.

Because $\frac{1}{6(l+2)(l+3)} < c < \frac{1}{6(l+1)(l+2)}$ and $b \leq \frac{1}{4(l+3)}$, the proof of Proposition 5 —interchanging n^* with $l+1$ —- implies that deviating from $l+1$ trials to run $n > l+1$ trials is not profitable.

By concavity of W, $\frac{W(l+1)-W(l-j)}{j+1} > \frac{W(l+1)-W(l-1)}{2}$. Hence, requiring that $c < \frac{W(l+1)-W(l-1)}{2} = \frac{1}{6(l+1)(l+3)}$ deters all deviations from $l+1$ to $l-j$, $j = 1, ..., l$.

Finally, a deviation to l trials is not profitable for the expert if $c < W(l+1) - \hat{W}(l)$, where $\hat{W}(\cdot)$ was defined in the proof of Proposition 5. The rest of the proof establishes that for $l > 4$, $\frac{1}{6(l+1)(l+3)} < W(l+1) - \hat{W}(l)$, hence requiring that $c < \frac{1}{6(l+1)(l+3)}$ guarantees that the deviation to l trials is not profitable. Also, it establishes that for $l < 4$, the value of $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right]$ is at most $\frac{1}{3}b$ if $l = 0$, $\frac{24b-1}{96}$ if $l = 1$, $\frac{30b-1}{450}$ if $l = 2$, $\frac{30b-1}{360}$ if $l = 3$, and $\frac{63b-2}{735}$ if $l = 4$, hence the condition in the proposition guarantees that $c < \min\left\{\frac{1}{6(l+1)(l+3)}, \left[W(l+1) - \hat{W}(l)\right]\right\}$.

To calculate $W(l+1) - \hat{W}(l)$, we need to compute $\hat{W}(l)$. Denoting by y_j the action in the set $\left\{0, \frac{1}{l+3}, ..., \frac{l+1}{l+3}\right\}$ preferred by an expert who observed j successes in l trials, we obtain:

$$\begin{aligned}\hat{W}(l) &= \frac{1}{l+1}\sum_{j=0}^{l}\hat{W}(j,l;y_j) = -\sum_{j=0}^{l}\frac{1}{l+1}(y_j-b)^2 + 2\sum_{j=0}^{l}\frac{j+1}{(l+1)(l+2)}(y_j-b) - \frac{1}{3} \\ &= -\frac{1}{3} - \sum_{j=0}^{l}\frac{y_j-b}{l+1}\left[y_j - b - 2\frac{j+1}{l+2}\right].\end{aligned}$$

Hence,

$$W(l+1) - \hat{W}(l) = -\frac{1}{6(l+3)} - b^2 + \frac{1}{3} + \sum_{j=0}^{l}\frac{y_j-b}{l+1}\left[y_j - b - 2\frac{j+1}{l+2}\right]$$

$$= \frac{2k+5}{6(l+3)} - b^2 + \sum_{j=0}^{l}\frac{y_j-b}{l+1}\left[y_j - b - 2\frac{j+1}{l+2}\right].$$

Next, we characterize the expert's preferred action y_j, for $j = 0, ..., l$. First, we establish that $y_j \in \left\{\frac{j+1}{l+3}, \frac{j+2}{l+3}\right\}$. The payoff of type j is maximized by action $\frac{j+1}{l+2} + b > \frac{j+1}{l+3}$, hence the action $\frac{j+1}{l+3}$ is preferred to any smaller action. Also, $\frac{j+1}{l+2} < \frac{j+2}{l+3}$, hence the fact that in equilibrium the type whose payoff is maximized by $\frac{j+2}{l+3} + b$ is willing to truthfully reveal his type guarantees that after a deviation to l trials the action $\frac{j+2}{l+3}$ is preferred to any larger action.

Second, we observe that a sender whose payoff is maximized by $\frac{j+1}{l+2} + b$ will choose to induce action $\frac{j+1}{l+3}$ rather than $\frac{j+2}{l+3}$ if and only if $2b + \frac{2j-l}{(l+2)(l+3)} > 0$ and this quantity is increasing in j, hence for any bias such that $b \leq \frac{1}{4(l+3)}$, we can find a threshold $J = \left\lfloor -b(n+2)(n+3) + \frac{n}{2}\right\rfloor \leq \frac{n}{2}$ such that types $j \leq J$ prefer action $\frac{j+1}{n+3}$ and types $j > J$ prefer action $\frac{j+2}{n+3}$. Notice that $J = -1$ denotes the case where all types j prefer action $\frac{j+2}{n+3}$.

Then, the difference $W(l+1) - \hat{W}(l)$ can be rewritten as

$$\begin{aligned} & W(l+1) - \hat{W}(l) \\ = & \frac{2k+5}{6(l+3)} - b^2 + 2\sum_{j=0}^{J} \frac{\frac{j+1}{l+3} - b}{l+1} \left(\frac{\frac{j+1}{l+3} - b}{2} - \frac{j+1}{l+2} \right) + 2 \sum_{j=J+1}^{l} \frac{\frac{j+2}{l+3} - b}{l+1} \left(\frac{\frac{j+2}{l+3} - b}{2} - \frac{j+1}{l+2} \right) \\ = & \frac{2J^2 + 2J(12b - l + 10bk + 2bk^2 + 1) + 2 + 12b + l - 2bk + l^2 - 8bk^2 - 2bk^3}{2(l+1)(l+2)(l+3)^2} \end{aligned}$$

It it is easy to check that $\frac{1}{6(l+2)(l+3)}$ is smaller than the above expression for any J, hence the range for c identified in the statement of the proposition is nonempty.

Next, we consider the following difference:

$$W(l+1) - \hat{W}(l) - \frac{1}{6(l+1)(l+3)}$$

$$\begin{aligned} = & \frac{2J^2 + 2J(12b - l + 10bk + 2bk^2 + 1) + 2 + 12b + l - 2bk + l^2 - 8bk^2 - 2bk^3}{2(l+1)(l+2)(l+3)^2} \\ & - \frac{1}{6(l+1)(l+3)} \\ = & \frac{3J^2 + J(36b - 3k + 30bk + 6bk^2 + 3) + 18b - l - 3bk + l^2 - 12bk^2 - 3bk^3}{3(l+3)^2(l+1)(l+2)} \end{aligned}$$

The denominator is positive. The numerator is a quadratic expression in J. For $l \geq 8$, this quadratic is positive for any l and any b hence $\min\left\{\frac{1}{6(l+1)(l+3)}, \left[W(l+1) - \hat{W}(l)\right]\right\} = \frac{1}{6(l+1)(l+3)}$. Using the definition of J, we have that:

For $l = 0$, $J = -1$ and $W(l+1) - \hat{W}(l) = \frac{1-6b}{18}$, hence $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = \frac{b}{3}$.

For $l = 1$, if $b \leq \frac{1}{24}$, $J = 0$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = 0$. If instead $\frac{1}{24} < b < \frac{1}{16}$, then $J = -1$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = \frac{24b-1}{48}$.

For $l = 2$, if $b \leq \frac{1}{30}$ $J = 0$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$. For $b \in (\frac{1}{30}, \frac{1}{20}]$, $J = 0$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = \frac{30b-1}{450}$.

For $l = 3$, if $b \leq \frac{1}{60}$ $J = 1$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$. For $b \in [\frac{1}{60}, \frac{1}{24}]$, $J = 0$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = \frac{30b-1}{320}$.

For $l=4$, if $b \leq \frac{1}{42}$ $J=1$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$. For $b \in [\frac{1}{42}, \frac{1}{28}]$, $J=0$ and $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] = \frac{63b-2}{735}$.

For $l=5$, if $b \leq \frac{1}{112}$, $J=2$. If $b \in [\frac{1}{112} < b \leq \frac{3}{112}]$, $J=1$. If $b \in [\frac{3}{112} < b \leq \frac{5}{112}]$, $J=0$. In each of these three cases, $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$.

For $l=6$, from the expression for J one can see that either $J=1$ or $J=2$. In both cases, $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$.

For $l=7$, from the expression for J one can see that either $J=1$ or $J=2$ or $J=3$. In all these cases, $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right] < 0$.

We can therefore conclude that for $l > 4$, $\frac{1}{6(l+1)(l+3)} < W(l+1) - \hat{W}(l)$, hence requiring that $c < \frac{1}{6(l+1)(l+3)}$ guarantees that the deviation to l trials is not profitable. Moreover, we have established that for $l < 4$, the value of $\frac{1}{6(l+1)(l+3)} - \left[W(l+1) - \hat{W}(l)\right]$ is at most $\frac{1}{3}b$ if $l=0$, $\frac{24b-1}{96}$ if $l=1$, $\frac{30b-1}{450}$ if $l=2$, $\frac{30b-1}{360}$ if $l=3$, and $\frac{63b-2}{735}$ if $l=4$, hence the condition in the proposition guarantees that $c < \min\left\{\frac{1}{6(l+1)(l+3)}, \left[W(l+1) - \hat{W}(l)\right]\right\}$, hence guarantees that the deviation to l trials is not profitable. *Q.E.D.*

Proof of Proposition 5. Consider $n^*(c)$, the optimal number of trials under direct information acquisition by definition in (12). To prove the Proposition it is sufficient to show that there exists an equilibrium in which the sender performs $n^*(c)$ trials and fully reveals his information in the communication stage. Such an equilibrium, if it exists, would be the expert-preferred equilibrium. So, in any Pareto-efficient equilibrium the decision-maker's expected payoff has to be (at least weakly) greater than in this equilibrium.

To establish the existence of the desired equilibrium, in which the expert runs $n^*(c)$ trials and fully reveals their realizations, first, note that the condition $b \leq \left(2\sqrt{1+\frac{2}{3c}}+2\right)^{-1}$ and definition (12) together imply that $b \leq \frac{1}{2(n^*(c)+2)}$. So, by Corollary 1 full revelation is incentive compatible at the communication stage after the expert runs $n^*(c)$ trials.

Further, the expert's expected payoff after running $n^*(c)$ trials and fully revealing their realizations is equal to $-\frac{1}{6(n^*+2)} - b^2 - cn^*$. By definition, $n^*(c) \in \arg\max_n -\frac{1}{6(n+2)} - cn$. Hence, $n^*(c) \in \arg\max_n W(n) - cn \equiv -\frac{1}{6(n+2)} - b^2 - cn$.

So, to complete the proof it is sufficient to establish that for any $n \in \{0, 1, ..., \infty\}$, $W(n) \geq \hat{W}(n)$ where $\hat{W}(n)$ is the expected payoff that the expert gets after deviating to n signals.

To establish this inequality, first, note that $W(n) = \sum_{j=0}^{n} \frac{W(j,n)}{n+1}$ where

$$\begin{aligned} W(j,n) &= -\int_0^1 (E[\theta|j,n] - \theta - b)^2 f(\theta|j,n)\, d\theta \\ &= -\int_0^1 (E[\theta|j,n] - \theta)^2 f(\theta|j,n)\, d\theta - b^2 \\ &= -\left[(E[\theta|j,n])^2 - 2(E[\theta|j,n])\frac{j+1}{n+2} + \frac{(j+2)(j+1)}{(n+3)(n+2)}\right] - b^2 \\ &= -\left[\left(\frac{j+1}{n+2}\right)^2 - 2\left(\frac{j+1}{n+2}\right)\frac{j+1}{n+2} + \frac{(j+2)(j+1)}{(n+3)(n+2)}\right] - b^2 \end{aligned}$$

$$= -\left[\frac{(j+2)(j+1)}{(n+3)(n+2)} - \left(\frac{j+1}{n+2}\right)^2\right] - b^2 \qquad (.24)$$

Similarly, $\hat{W}(n) = \sum_{j=0}^{n} \frac{\hat{W}(j,n;)}{n+1}$, where

$$\hat{W}(j,n) = -\max_{y_j \in \left\{\frac{1}{n^*+2}, \frac{2}{n^*+2}, \ldots, \frac{n^*+1}{n^*+2}\right\}} \int_0^1 (y_j - \theta - b)^2 f(\theta|j,n)\, d\theta$$

$$= -\int_0^1 \left[(y_j - b)^2 + \theta^2 - 2\theta(y_j - b)\right] \frac{(n+1)!}{j!\,(n-j)!}\theta^j (1-\theta)^{n-j}\, d\theta$$

$$= -\left[(y_j - b)^2 + \int_0^1 \frac{(n+1)!}{j!\,(n-j)!}\theta^{j+2}(1-\theta)^{n-j}\, d\theta - 2(y_j - b)\int_0^1 \frac{(n+1)!}{j!\,(n-j)!}\theta^{j+1}(1-\theta)^{n-j}\, d\theta\right]$$

$$= -\left[(y_j - b)^2 + \frac{(n+1)!}{j!\,(n-j)!}\frac{(2+j)!\,(n-j)!}{(n+3)!} - 2(y_j - b)\frac{(n+1)!}{j!\,(n-j)!}\frac{(1+j)!\,(n-j)!}{(n+2)!}\right]$$

$$= -\left[(y_j - b)^2 - 2(y_j - b)\frac{j+1}{n+2} + \frac{(j+2)(j+1)}{(n+3)(n+2)}\right]. \qquad (.25)$$

Note that the message y_j optimally chosen by type j (i.e. the expert who observed j successes in n trials) has to be compatible with the equilibrium beliefs that he has acquired n^* signals, even off the equilibrium path. Therefore, $y_j \in \left\{\frac{1}{n^*+2}, \frac{2}{n^*+2}, \ldots, \frac{n^*+1}{n^*+2}\right\}$.

The proof proceeds by showing that for any $j \leq n - j$

$$D(j,n) \equiv \left[W(j,n) - \hat{W}(j,n;y_j)\right] + \left[W(n-j,n) - \hat{W}(n-j,n;y_{n-j})\right] \geq 0. \qquad (.26)$$

Since types j and $n-j$ are ex-ante equally likely after n experiments, inequality (.26) implies that $W(n) \geq \hat{W}(n)$. [20]

Before computing $D(j, n)$ let us establish the following useful property.

Claim A. Suppose that $y_j = \frac{k+1}{n^*+2}$ for some $k \in \{0, 1, ..., n^*\}$. Then either $y_{n-j} = \frac{n^*-k+1}{n^*+2}$ or $y_{n-j} = \frac{n^*-k+2}{n^*+2}$.

Proof of Claim A: For any $j \in \{0, 1, ..., n\}$, define

$$k_j \in \arg\min_{k'=0,...,n^*} \left| \frac{k'+1}{n^*+2} - \left(\frac{j+1}{n+2} + b \right) \right|. \tag{.27}$$

If for some j, the maximizer k' of the above expression is not unique, then choose one of the (two) maximizers arbitrarily and set it equal to k_j. So, $y_j = \frac{k_j+1}{n^*+2}$.

We need to distinguish two cases:
Case 1: $y_j = \frac{k_j+1}{n^*+2} \leq \frac{j+1}{n+2}$, and *Case 2:* $y_j = \frac{k_j+1}{n^*+2} > \frac{j+1}{n+2}$.

Let us start with *Case 1.* We will show that in this case, $y_{n-j} = \frac{n^*-k_j+1}{n^*+2}$.

Since $b \geq 0$, we have: $0 \leq \frac{j+1}{n+2} - \frac{k_j+1}{n^*+2} \leq \frac{k_j+2}{n^*+2} - \frac{j+1}{n+2}$. By (.27), $\left| \frac{k^j+1}{n^*+2} - \left(\frac{j+1}{n+2} + b \right) \right| \leq \left| \frac{k^j+2}{n^*+2} - \left(\frac{j+1}{n+2} + b \right) \right|$. So we have:

$$\left| \frac{n^* - k^j + 1}{n^* + 2} - \left(\frac{n-j+1}{n+2} + b \right) \right| = \left| \frac{j+1}{n+2} - \frac{k^j+1}{n^*+2} - b \right| \leq b + \left| \frac{j+1}{n+2} - \frac{k^j+1}{n^*+2} \right| \leq$$
$$b + \left| \frac{k^j+2}{n^*+2} - \frac{j+1}{n+2} \right| = \left| b + \frac{k^j+2}{n^*+2} - \frac{j+1}{n+2} \right| = \left| \frac{n^* - k^j}{n^*+2} - \left(\frac{n-j+1}{n+2} + b \right) \right|. \tag{.28}$$

Inequality (.28) implies that type $n - j$ prefers the action $\frac{n^*-k_j+1}{n^*+2}$ associated with message $n^* - k_j$ to the action $\frac{n^*-k_j}{n^*+2}$ associated with message $n^* - k_j - 1$. This, in combination with $\frac{n^*-k_j+1}{n^*+2} \geq \frac{n-j+1}{n+2}$ and the fact that the utility function of type $n - j$ is single-peaked around the maximum $\frac{n-j+1}{n+2} + b$, $b \geq 0$, implies that type $n - j$ prefers message $n^* - k_j$ to any message lower than $n^* - k_j - 1$.

Let us now show that type $n - j$ also prefers to send message $n^* - k_j$ associated with action $\frac{n^*-k_j+1}{n^*+2}$ rather than any higher message associated with a higher action. This is immediate if $\frac{n-j+1}{n+2} + b \leq \frac{n^*-k_j+1}{n^*+2}$ If, on the other hand, $\frac{n-j+1}{n+2} + b > \frac{n^*-k_j+1}{n^*+2}$, this follows from the following facts: (i)

[20] If n is odd, there is an even number of possible types $\{0, 1, ..., n+1\}$, and $\frac{n+1}{2}$ pairs of types $(j, n-j)$ with $j \leq n-j$. If n is even, then there is an odd number of possible types, and so there are $\frac{n}{2}$ pairs $(j, n-j)$ with $j < n-j$, plus the type $\frac{n}{2}$. When $j = \frac{n}{2}$, we have $n - j = j$. In this case $D\left(\frac{n}{2}, n\right) = 2\left[W\left(\frac{n}{2}, n\right) - \hat{W}(j, n; y_j)\right]$. The result then follows by showing that $D\left(\frac{n}{2}, n\right) > 0$ and that $D(j, n) > 0$ for each pair $(j, n-j)$ with $j < \frac{n}{2}$.

$\frac{n-j+1}{n+2} \leq \frac{n^*-k_j+1}{n^*+2}$, so $\frac{n-j+1}{n+2}+b-\frac{n^*-k_j+1}{n^*+2} \leq b \leq \frac{1}{2(n^*+2)}$; (ii) $\frac{n^*-k_j+2}{n^*+2}-\frac{n-j+1}{n+2}-b \geq \frac{1}{n^*+2}-b \geq \frac{1}{2(n^*+2)}$, (iii) type $n-j$'s payoff function is symmetric and single-peaked at $\frac{n-j+1}{n+2}+b$.

Next, consider *Case 2*: $y_j = \frac{k_j+1}{n^*+2} > \frac{j+1}{n+2}$. Let us show that in this case $y_{n-j} \in \left\{\frac{n^*-k_j+1}{n^*+2}, \frac{n^*-k_j+2}{n^*+2}\right\}$.

Since $\frac{n^*-k_j+1}{n^*+2} < \frac{n-j+1}{n+2}$ and $b \geq 0$, the expert of type $n-j$ gets a strictly higher payoff from action $\frac{n^*-k_j+1}{n^*+2}$ than from any lower action. Thus, it remains to show that type $n-j$'s expected utility from action $\frac{n^*-k_j+2}{n^*+2}$ is higher than her expected utility from any higher action.

Further, note that we must have $\frac{j+1}{n+2} \geq \frac{k_j}{n^*+2}$. Otherwise, since $b \leq \frac{1}{2(n^*+2)}$, type j would get a higher utility from action $\frac{k_j}{n^*+2}$ than from action $\frac{k_j+1}{n^*+2}$, which would contradict $y_j = \frac{k_j+1}{n^*+2}$.

Thus, $\frac{n-j+1}{n+2} \leq \frac{n^*-k_j+2}{n^*+2}$, and since the expected utility function of the type $n-j$ is symmetric around its maximum at $y = \frac{n-j+1}{n+2}+b$ and $b \leq \frac{1}{2(n^*+2)}$, we conclude that the type $n-j$ gets a higher expected utility from action $\frac{n^*-k_j+2}{n^*+2}$ than from any other actions. This completes the proof of Claim A.

Let us now turn back to the proof of the Proposition and compute $D(j,n)$. From (.24), (.25), (.26) we have

$$\begin{aligned} D(j,n) &= \frac{(j+1)^2}{(n+2)^2} + \frac{(n-j+1)^2}{(n+2)^2} - 2b^2 \\ &+ \left[(y_j-b)^2 + (y_{n-j}-b)^2 - 2(y_j-b)\frac{j+1}{n+2} - 2(y_{n-j}-b)\frac{n-j+1}{n+2}\right] \end{aligned}$$

$$\begin{aligned} &= \frac{(j+1)^2}{(n+2)^2} + \frac{(n-j+1)^2}{(n+2)^2} + \left[y_j^2 + y_{n-j}^2 - 2y_j\frac{j+1}{n+2} - 2y_{n-j}\frac{n-j+1}{n+2} - 2b(y_j+y_{n-j}-1)\right] \\ &= \left(y_j - \frac{(j+1)}{(n+2)}\right)^2 + \left(y_{n-j} - \frac{(n-j+1)}{(n+2)}\right)^2 - 2b(y_j+y_{n-j}-1). \quad (.29) \end{aligned}$$

If $y_{n-j} = \frac{n^*-k_j+1}{n^*+2}$, then $y_j + y_{n-j} = 1$, and hence by (.29) $D(j,n) = \left(y_j - \frac{(j+1)}{(n+2)}\right)^2 + \left(y_{n-j} - \frac{(n-j+1)}{(n+2)}\right)^2$. The latter expression is nonnegative.

If instead $y_{n-j} = \frac{n^*-k_j+2}{n^*+2}$, then $y_j + y_{n-j} = 1 + \frac{1}{n^*+2}$. So, by (.29),

$$\begin{aligned} D(j,n) &= \left(y_j - \frac{j+1}{n+2}\right)^2 + \left(y_{n-j} - \frac{n-j+1}{n+2}\right)^2 - \frac{2b}{n^*+2} \\ &= \left(\frac{j+1}{n+2} - \frac{k_j}{n^*+2}\right)^2 + \left(\frac{k_j+1}{n^*+2} - \frac{j+1}{n+2}\right)^2 - \frac{2b}{n^*+2} \quad (.30) \end{aligned}$$

In the proof of Case 2 of Claim A, we have established that $\frac{k_j}{n^*+2} \leq \frac{j+1}{n+2} \leq \frac{k_j+1}{n^*+2}$. Observe that $\frac{k_j+1}{n^*+2} - \frac{k_j}{n^*+2} = \frac{1}{n^*+2}$. So the value of the first two terms of $D(j,n)$, $\left(\frac{j+1}{n+2} - \frac{k_j}{n^*+2}\right)^2 + \left(\frac{k_j+1}{n^*+2} - \frac{j+1}{n+2}\right)^2$, depends only on $\frac{k_j+1}{n^*+2} - \frac{j+1}{n+2}$ and reaches its minimum when $\frac{j+1}{n+2} = \frac{k_j+1/2}{n^*+2}$. In this case, $\left(\frac{j+1}{n+2} - \frac{k_j}{n^*+2}\right)^2 + \left(\frac{k_j+1}{n^*+2} - \frac{j+1}{n+2}\right)^2 = \frac{1}{2(n+2)^2}$, and $D(j,n) = \frac{1}{2(n+2)^2} - \frac{2b}{n^*+2}$. Hence, $D(j,n) \geq 0$ when $b \leq \frac{1}{4(n^*+2)}$. This concludes the proof that under the given conditions on the parameters, $D(j,n) \geq 0$ hence $W(n) \geq \hat{W}(n)$. *Q.E.D.*

References

[1] Aghion, P. and J. Tirole (1997): "Formal and Real Authority in Organizations," *Journal of Political Economy*, 105, 1-29

[2] Ambrus, A. and S. Takahashi (2008), "Multi-expert cheap talk with restricted state spaces," *Theoretical Economics*, 3, 1-27.

[3] Aumann, R.J. and S. Hart (2003): "Long Cheap Talk," *Econometrica*, 71, 1619-1660.

[4] Austen-Smith, D. (1993): "Interested Experts and Policy Advice: Multiple Referrals under Open Rule," *Games and Economic Behavior*, 5, 3-43.

[5] Austen-Smith, D. (1994), "Strategic Transmission of Costly Information,". *Econometrica*. 62, 955-963.

[6] Battaglini, M. (2002), "Multiple Referrals and Multidimensional Cheap Talk," *Econometrica*, 70, 1379-1401.

[7] Berlin, L. and Williams D. M. (2000): "When an Expert Witness Is Not an Expert", *American Journal of Roentgenology*, 174, 1215-1219.

[8] Battaglini, M. (2004), "Policy Advice with Imperfectly Informed Experts," *Advances in Theoretical Economics*, 1, 1, 1.

[9] Bolton, P., and M. Dewatripont (1994): "The Firm as a Communication Network," *Quarterly Journal of Economics,* 109: 809–39.

[10] Che, Yeon-Koo and Navin. Kartik (2009): "Opinions as incentives,", *Journal of Political Economy,* 117, 815-860.

[11] Crawford V.P. and J. Sobel (1982), "Strategic Information Transmission," *Econometrica*, 50, 1431-1451.

[12] Dessein, W. (2002), "*Authority and Communication in Organizations,*" *Review of Economic Studies*, 69, 811-838.

[13] Eso, Peter, and Dezso Szalay (2010): "Incomplete Language as an Incentive Device", working paper.

[14] Garicano, Luis (2000): "Hierarchy and the Organization of Knowledge in Production," *Journal of Political Economy,* 108, 874-904.

[15] Gilligan, T. W. and K. Krehbiel (1987): "Collective Decision making and Standing Committees: An Informational Rationale for Restrictive Amendment Procedures," *Journal of Law, Economics and Organization*, 3, 287-335.

[16] Gilligan, T. W. and K. Krehbiel (1989): "Asymmetric Information and Legislative Rules with a Heterogeneous Committee," *American Journal of Political Science*, 33, 459-90

[17] Jones v. O'Young et al., 607 NE2d 224 (Ill 1992)

[18] Krishna, V. and J. Morgan (2001a): "A Model of Expertise," *Quarterly Journal of Economics*, 116, 747-75.

[19] Krishna, V. and J. Morgan (2001b): "The Art of Conversation: Eliciting Information from Experts through Multi-Stage Communication," *American Political Science Review*, 95, 435-52.

[20] Krishna, V. and J. Morgan (2004): "Asymmetric Information and Legislative Rules: Some Amendments," *Journal of Economic Theory*, 117, 147-179.

[21] Ivanov, M. (2010): "Informational Control and Organizational Design," *Journal of Economic Theory*, 145(2), 721-751.

[22] Milgrom, P. (1981) "Good News and Bad News: Representation Theorems and Applications." Bell Journal of Economics, 12(2): 380–91.

[23] Morgan, J and PC Stocken (2008), "Information Aggregation in Polls," *American Economic Review*, 98, 864-896.

[24] Ottaviani, M. (2000): "The Economics of Advice" mimeo

[25] Pei, D. (2013): "Communication with Endogenous Information Acquisition ", mimeo.

[26] Radner, R. (1993): "The Organization of Decentralized Information Processing," *Econometrica,* 61, 1109-46.

[27] Szalay, D. (2005): “The Economics of Clear Advice and Extreme Options,” *Review of Economic Studies*, 72, 1173–1198

[28] Wolinsky, A. (2002): “Eliciting Information from Multiple Experts,” *Games and Economic Behavior*, 41, 141-60.

A Results and Calculations not submitted for publication

Proposition A.1 *For any n' and b, the Pareto-efficient incentive compatible partition is $P^* = \{p_1^*, ..., p_K^*\}$ such that $K = \max\{k \in \mathbb{N} | k + \lceil 4b(n'+2) - 2 \rceil \times \frac{k(k-1)}{2}) \leq n'+1\}$. For all $i = 1, ..., K$, the element p_i^* of the equilibrium partition consists of consecutive types and has cardinality $|p_i^*| = 1 + \lceil 4b(n'+2) - 2 \rceil \times (i-1) + \lfloor \frac{r}{K} \rfloor + \mathbb{I}\left\{r - \left(\lfloor \frac{r}{K} \rfloor + 1\right) K + i > 0\right\}$, where $r \equiv n' + 1 - \left[K + \lceil 4b(n'+2) - 2 \rceil \times \frac{K(K-1)}{2}\right]$, and $\mathbb{I}$ denotes the indicator function.*

Proof. The equilibrium partition P identified in the Proposition is the one with the largest cardinality K and with the smallest difference in the cardinality of subsequent elements, subject to the incentive compatibility condition (7).

The proof is in three parts. First we show that the negative of the expected residual variance, $E\left[-\left(y_{p_i}^{n'} - \theta\right)^2 | P_{n'}\right]$ can be rewritten as $-\frac{1}{3} + E\left[E(\theta|p_i)^2\right]$. Then, we show that among the equilibrium partitions with the largest number of elements, the equilibrium with the smallest difference between the cardinalities of any two subsequent elements minimizes the expected residual variance. Third, we show that, among the equilibrium partitions with the smallest difference in the cardinality of subsequent elements, the one which minimizes the expected residual variance is the one with the largest number of elements.

Part 1: $E\left[-\left(y_{p_i}^{n'} - \theta\right)^2 | P_{n'}\right] = -\frac{1}{3} + E\left[E(\theta|p_i)^2\right]$.

By the law of iterated expectations,

$$\begin{aligned} E\left[-\left(y_{p_i}^{n'} - \theta\right)^2 | P_{n'}\right] &= -E_\theta\left[\left(E\left[\theta|p_i\right] - \theta\right)^2\right] \\ &= -E_{p_i}\left[E_\theta\left[\left(E\left[\theta|p_i\right] - \theta\right)^2 |p_i\right]\right] \\ &= -E_{p_i}\left[Var\left[\theta|p_i\right]\right]. \end{aligned}$$

Because $Var[\theta] = E_{p_i}[Var[\theta|p_i]] + Var_{p_i}[E(\theta|p_i)]$, we thus obtain:

$$\begin{aligned} E\left[-\left(y_{p_i}^{n'}-\theta\right)^2|P_{n'}\right] &= -Var[\theta] + Var_{p_i}[E(\theta|p_i)] \\ &= -Var[\theta] + E\left[E(\theta|p_i)^2\right] - E[E(\theta|p_i)]^2 \\ &= -Var[\theta] + E\left[E(\theta|p_i)^2\right] - E[\theta]^2 \\ &= -\frac{1}{12} + E\left[E(\theta|p_i)^2\right] - \left(\frac{1}{2}\right)^2 \\ &= -\frac{1}{3} + E\left[E(\theta|p_i)^2\right]. \end{aligned}$$

Part 2: Among the equilibrium partitions with the largest number of elements, the equilibrium with the smallest difference between the cardinalities of any two subsequent elements maximizes $E\left[-\left(y_{p_i}^{n'}-\theta\right)^2|P_{n'}\right] = -\frac{1}{3} + E[E(\theta|p_i)^2]$.

Suppose the number of trials is $n+1$ and the number of types is $n'+1$. Consider an equilibrium partition P with I elements $\{k_i, ..., k_{i+1}-1\}_{i=1}^{I}$, where $k_{I+1} \equiv n+1$. We obtain:

$$E\left[-\left(y_{p_i}^{n'}-\theta\right)^2|P\right] = -\frac{1}{3}+E\left[E(\theta|p_i)^2\right] = -\frac{1}{3}+\sum_{i=1}^{I}\frac{k_{i+1}-k_i}{n+1}\left(\frac{k_{i+1}+k_i+1}{2(n'+2)}\right)^2.$$

Next, consider a different equilibrium partition $P' = \left\{k_i', ..., k_{i+1}'-1\right\}_{i=1}^{I}$, such that there is a unique $i \in I$ with $k_i' = k_i + 1$, and $k_j' = k_j$ for all $j \neq i$. Denoting the associated expected residual variance by $E\left[-(y_p-\theta)^2; P\right]$ we obtain:

$$E\left[-\left(y_{p_i'}^{n'}-\theta\right)^2; P'\right] - E\left[-\left(y_{p_i}^{n'}-\theta\right)^2; P\right]$$

$$\begin{aligned} &= \frac{k_{i+1}-(k_i+1)}{n+1}\left(\frac{k_{i+1}+(k_i+1)+1}{2(n'+2)}\right)^2 + \frac{k_i+1-k_{i-1}}{n'+1}\left(\frac{k_i+1+k_{i-1}+1}{2(n'+2)}\right)^2 \\ &\quad -\frac{k_{i+1}-k_i}{n'+1}\left(\frac{k_{i+1}+k_i+1}{2(n'+2)}\right)^2 - \frac{k_i-k_{i-1}}{n'+1}\left(\frac{k_i+k_{i-1}+1}{2(n'+2)}\right)^2 \\ &= \frac{(k_{i+1}-k_{i-1})\left[(k_{i+1}-k_i)-(k_i+1-k_{i-1})\right]}{4(n'+2)^2(n'+1)} > 0. \end{aligned}$$

where the last inequality holds because P' is an equilibrium partition, hence $k_{i+1}'-k_i' > k_i'-k_{i-1}'$, which implies $k_{i+1}-k_i-1 > k_i+1-k_{i-1}$.

Part 3: Among the equilibrium partitions with the smallest difference in the cardinality of subsequent elements, the one which minimizes the expected residual variance is the one with the largest number of elements.

Denoting by $P(m)$ the best equilibrium partition among those with m elements, we prove that $P(j)$ dominates $P(j-1)$. Repeating the argument proves the statement.

To prove that $P(j)$ dominates $P(j-1)$ we describe an algorithm to construct a sequence of partitions with the following features:

(a) the first term of the sequence is $P(j)$

(b) the last term of the sequence is $P(j-1)$

(c) each term of the sequence, except for the last one, is a partition with j elements

(d) each term of the sequence is preferred by both players to the next one (i.e. has a smaller expected residual variance).

The algorithm is the following. Given the n-th term of the sequence (the n-th partition), the $(n+1)$-th is constructed as follows:

(i) If the sub-partition that includes the largest $(j-2)$ elements of n-th partition is identical to the sub-partition that includes the largest $(j-2)$ elements of $P(j-1)$, then let the $n+1$-th partition be $P(j-1)$; i.e., let the first element of the $n+1$-th partition be equal to the union of the first two elements of the n-th partition. This step concludes the algorithm, and satisfies condition (d), because, for any k_1, k_2 with $k_1 > 1$, and $k_2 > k_1 + 1$,

$$\begin{aligned} & \frac{k_2-k_1}{n'+1}\left(\frac{k_2+k_1+1}{2(n'+2)}\right)^2+\frac{k_1-1}{n'+1}\left(\frac{k_1+1+1}{2(n'+2)}\right)^2-\frac{k_2-1}{n'+1}\left(\frac{k_2+1+1}{2(n'+2)}\right)^2 \\ = \; & \frac{1}{4}\frac{(k_2-k_1)(k_2-1)(k_1-1)}{(n'+2)(n'+1)} > 0. \end{aligned}$$

(ii) If the sub-partition that includes the last $(j-2)$ elements of n-th partition is *not* identical to the sub-partition that includes the largest $(j-2)$ elements of $P(j-1)$, then the $(n+1)$-th partition is obtained from the n-th by moving the highest type included in the k-th element p_k^n into the $(k+1)$-th element p_{k+1}^n, where $k < j$ is the highest index that satisfies the following conditions:

(iia) For $l < j-2$, if the sub-partition that includes the last l elements of n-th partition is identical to the sub-partition that includes the last l

elements of $P(j-1)$, then $k<j-l$.[21]

(iib) The cardinality of p_{k+1}^n is strictly smaller than the cardinality of the k-th element of $P(j-1)$.

(iic) If the union of p_1^n and p_2^n is equal to the first element of $P(j-1)$, then $k>2$.

Because the number of types is finite, the algorithm has an end.

The type-(ii) step can be repeated exactly until the condition for the type-(i) step is satisfied because, by construction, the cardinality of the l-th element of $P(j-1)$ is weakly larger than the cardinality of the $(l+1)$-th element of $P(j)$, hence the union of the first two elements of $P(j)$ has cardinality weakly larger than the cardinality of the first element of $P(j-1)$. *Q.E.D.*

Computations omitted from Example 2: First, let us compute the expert's payoff when he performs $n=2$ trials and fully reveals his information. It is equal to $-E_k(E(\theta-E\theta|k,n=2)^2)-b^2-2c$. We may compute:

$$-E_kE((\theta-E\theta)^2|k,n=2)=Prob(k=0|n=2)E((\theta-E\theta)^2|k=0,n=2)$$
$$+Prob(k=1|n=2)E((\theta-E\theta)^2|k=1,n=2)+Prob(k=2|n=2)E(\theta-E\theta)^2|k=2,n=2) \quad \text{(A.1)}$$

Note that $Prob(k=0|n=2)=Prob(k=0|n=2)=Prob(k=2|n=2)=\frac{1}{3}$. Also, $E(\theta-E\theta)^2|k=0,n=2)=2E(\theta-E\theta)^2|k=,n=2)\frac{3}{80}$ and $E(\theta-E\theta)^2|k=1,n=2)=\frac{1}{20}$. Substituting this into (A.1) we obtain that the expert's total payoff is equal to $-\frac{1}{24}-b^2-2c$.

Next, we consider a deviation to $n=1$ trial. Let us show the following: (i) If the trial fails ($k=0$ out of $n=1$), then the expert prefers to induce action 1/4 rather than action 1/2 or action 3/4. It is enough to show that he prefers $\frac{1}{4}$ to $\frac{1}{2}$ (The argument for 3/4 follows by monotonicity), which is so if:

$$-\int_0^1\left(\frac{1}{4}-\theta-b\right)^2\frac{(n+1)!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}d\theta\geq-\int_0^1\left(\frac{1}{2}-\theta-b\right)^2\frac{(n+1)!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}d\theta \quad \text{(A.2)}$$

With $n=1$ and $k=0$, $\frac{(n+1)!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}=2(1-\theta)$, so (A.2) simplifies to:

$$-2\int_0^1\left(\frac{1}{4}-\theta-b\right)^2(1-\theta)d\theta+2\int_0^1\left(\frac{1}{2}-\theta-b\right)^2(1-\theta)d\theta\geq 0 \quad \text{(A.3)}$$

[21] For example, if $j=10$, if the last three elements of the $n-th$ partition in the sequence are identical to the last three elements of the target partition, then they shouldn't be changed anymore, hence $k<7$, so that "at most" a type is taken from the 6-th element and moved into the 7-th.

Rearranging terms and integrating, we obtain that (A.3) is equivalent to $\frac{1}{4}\left(\frac{1}{12}-2b\right)\geq 0$ which holds because $b\leq\frac{1}{24}$.

(ii) If the trial succeeds ($k=1$ out of $n=1$), then the expert prefers to induce action 3/4 rather than action 1/2 or action 1/4. It is enough to show that he prefers $\frac{3}{4}$ to $\frac{1}{2}$ (the argument for 1/4 follows by monotonicity), which is so if:

$$-\int_0^1\left(\frac{3}{4}-\theta-b\right)^2\frac{(n+1)!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}d\theta\geq-\int_0^1\left(\frac{1}{2}-\theta-b\right)^2\frac{(n+1)!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}d\theta \tag{A.4}$$

With $n=1$ and $k=1$, $\frac{(n+1)!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}=2\theta$, so (A.4) simplifies to:

$$-2\int_0^1\left(\frac{3}{4}-\theta-b\right)^2\theta d\theta+2\int_0^1\left(\frac{1}{2}-\theta-b\right)^2\theta d\theta\geq 0 \tag{A.5}$$

Rearranging terms and integrating, we obtain that (A.5) is equivalent to $\frac{1}{8}\left(\frac{1}{12}+2b\right)>0$ which is trivially satisfied.

Extension of Example *1*: We show that Example *1* can be extended to show that our overinvestment results hold beyond our parametric statistical model.

Consider an alternative model in which the expert's information acquisition model consists in choosing the fineness of a partition of the state space $[0,1]$, composed of equally sized intervals. I.e., the expert chooses the number n of intervals $[(k-1)/n,k/n]$, $k=1,\ldots,n$, at cost cn, to then observe the interval to which θ belongs. It can be shown that, for $b\leq\frac{7}{60}$ and $c=\frac{1}{35}$, there exists an equilibrium of the covert game such that the decision maker achieves a higher utility than if she acquired information directly.

Consider direct information acquisition first. The decision-maker's payoff for $n=0$ is, again, $-\frac{1}{12}$. If choosing $n=1$, the decision maker pays the cost c, to then take the action 1/4 if $\theta\in[0,1/2]$ and the action 3/4 if $\theta\in(1/2,1]$; thus her expected payoff is $-1/48-c$. Now, suppose $c=1/15$, so that the decision maker chooses $n^*=0$ if acquiring information directly. For $b\leq 7/60$, we now show that there exists an equilibrium in which the expert chooses $n=1$, i.e., "acquires" the partition $\{[0,1/2],(1/2,1]\}$ of the state space, and reveals the interval he observed, inducing action $y=\frac{1}{4}$ if seeing $[0,1/2]$ and $y=\frac{3}{4}$ if seeing $(1/2,1]$. Indeed, if the expert deviates to zero trials, then any message he sends can only induce one of the equilibrium actions, namely $y=\frac{1}{4}$ or $y=\frac{3}{4}$. Because of his upwards bias ($b>0$), he prefers $y=\frac{3}{4}$. The expected utility that the expert obtains by inducing $y=\frac{3}{4}$ is $-b^2+\frac{1}{2}b-\frac{7}{48}$. For $b\leq\frac{7}{60}$ and $c=\frac{1}{35}$, this is less than $-\frac{1}{48}-b^2-c$, so this deviation is unprofitable. Again, showing that the expert will not deviate to any $n>1$ is straightforward and is therefore omitted. Hence, the decision maker achieves a higher utility than if she acquired information directly.