Smooth, strategic communication

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September 2014

Abstract

We study strategic information transmission in a Sender-Receiver game where players' optimal actions depend on the realization of multiple signals but the players disagree on the relative importance of each piece of news. We characterize a statistical environment - featuring symmetric loss functions and elliptically distributed parameters - in which the Sender's expected utility depends only on the first moment of his posterior. Despite disagreement about the use of underlying signals, we demonstrate the existence of equilibria in differentiable strategies in which the Sender can credibly communicate posterior means. The existence of smooth communication equilibria depends on the relative usefulness of the signal structure to Sender and Receiver, respectively. We characterize extensive forms in which the quality of information is

*We thank Patrick Bolton, Peter Eső, Emir Kamenica, Kohei Kawamura, Felix Ketelaar, Botond Kőszegi, Mark Le Quement, David Martimort, Meg Meyer, Marco Ottaviani, Peter Sorensen, Peter Vida, Lucy White, and seminar participants at the University of Bonn, University of Oxford, University of Copenhagen, CEU Budapest, SYME 2013 (Aarhus), ESEM 2014 (Toulouse), the European Summer Symposium in Economic Theory 2014 (Gerzensee), VfS 2014 (Hamburg), and the SFB-TR15 conference 2014 (Mannheim). Financial support through a grant by the DFG (SFB-TR 15) is gratefully acknowledged.
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optimally designed of equal importance to Sender and Receiver so that the best equilibrium in terms of ex ante expected payoffs is a smooth communication equilibrium. The quality of smooth equilibrium communication is entirely determined by the correlation of interests. Senders with better aligned preferences are endogenously endowed with better information and therefore give more accurate advice.

JEL: D82

Keywords: strategic information transmission, multi-dimensional cheap talk, monotone strategies, endogenous information, elliptical distributions

1 Introduction

1.1 Motivation

This paper studies strategic communication when a Sender and a Receiver agree on the important determinants of optimal decisions but disagree on the relative importance of several arguments. We term this relative (dis-)agreement and show that blindly following a Sender's recommendations may be perfectly consistent with equilibrium behavior despite such disagreement.

For concreteness, imagine a politician who needs to decide whether and by how much she should raise taxes. Envision further that she would like to take her decision taking into account information about the cost of sovereign debt and the impact of a tax raise on employment, so she seeks advice from an expert. The expert's preferences are broadly aligned with those of the politician: he dislikes higher sovereign debt, but less so if interest rates remain low, and he dislikes increases in the unemployment rate. However, the relative importance the advisor attaches to these issues differs from the relative importance attached to them by the politician. The expert needs to investigate the matter before he can give an informed answer. Moreover, he can direct his investigations and so chooses to get more or less precise information about this or the other issue. The politician is free to use the advice in any way she sees fit - she cannot commit on how to use the expert's advice up-front. Her only influence on the quality of advice is through a careful selection of her advisor.

This is a natural situation of strategic communication with several differences to the one analyzed in Crawford & Sobel (1982). There are several pieces of information - more precisely, signals - that should go into the politician's and the expert's ideal choice. The politician and the expert disagree about the aggregation of the signals into a choice. Since both of them care only about a unidimensional decision that is taken, the conflict is ultimately unidimensional, despite information being higher dimensional. The way politician and expert disagree about the choice depends in non-trivial ways on the relative precision of the signals the expert gathers. If the expert's signals are relatively more (less) precise on the issues that are of primary interest to him, then the politician reacts conservatively (progressively) to the expert's advice, following his suggestions less (more) than one for one. However, provided the advisor chooses the relative precisions of the signals he observes the right way, then it becomes possible that the politician can take the advisor's recommendation at face value despite the fact that they disagree on the aggregation of individual pieces of information. Put differently, the politician can follow the advisor's conclusions blindly, although it remains pointless to ask the advisor for the reasons behind his conclusions, that is, the underlying signals he observed. Even though the last situation seems to be as knife edge as it gets, it is precisely the situation that arises if the politician selects the best advisor she can get.

In a nut shell, the politician should carefully choose her advisor, delegate investigating matters to the advisor, and then ask the advisor to state conclusions, skipping the details. On top of saving time, conclusions can be communicated without conflict, while justifications for them cannot.

Relative (dis-)agreement is common in practice. Think of a CEO hiring a personal assistant whom she plans to consult on important matters. Even if the CEO manages to select an assistant with whom she perfectly agrees for now, it is not to be expected that they will have perfectly aligned interests on all matters that will come up in the future. Suppose again that they disagree on the relative importance of several arguments shaping decisions. Suppose that the assistant does not care about such decisions per se, but starts to care only if he puts skin into the game, i.e., when he is hired. The CEO can pay the assistant up front to convince him to work for her. Moreover, the CEO can fine-tune the working conditions of her assistant in quite some detail so that she can effectively control the quality of the information that the assistant gets. However, due to time constraints the CEO cannot observe the signal realizations directly, and therefore needs the assistant to observe and communicate them to the CEO who then takes a decision.

The difference to the former situation is that the quality of information is controlled by the advisee (the CEO) rather than the advisor, as in the politician's problem. This is reminiscent of a disclosure or a persuasion game rather than an advisor controlled information acquisition story. However, in common with the first situation, the CEO optimally endows the assistant with information that makes his conclusions equally useful to both of them. Doing so ensures that the CEO can again blindly follow the assistant's advice and so selects the optimal communication procedure that maximizes the ex ante expected surplus.

1.2 Setup

The following model of strategic information transmission in the tradition of Crawford & Sobel (1982) gives rise to these insights. A Sender communicates with a Receiver before the latter takes a decision that affects both their utilities. The Sender and the Receiver prefer different decisions to be taken in almost every state of the world. We depart from Crawford & Sobel (1982) in that we assume that the Sender and the Receiver are both uncertain about their ideal policy: each player's ideal choice is equal to the realization of a random variable[1]. The conflict between the players is described by a joint distribution over the two-dimensional state. The Sender gets to observe a two-dimensional signal about the underlying state of the world and then communicates with the Receiver who finally takes an action. We exploit the advantages of elliptical distributions to give tractability to our model. Within this statistical class and for symmetric loss functions, the players would ideally want to choose an action that corresponds to the conditionally expected value of the random variable that describes their ideal choice - "their state" - given the information. Moreover, for the elliptical class, conditional means are linear functions of the signals. This allows to give a precise meaning to the direction and strength of influence of each given signal on the Sender's and the Receiver's ideal choice. The interesting case corresponds to one where the conditional expectations of both players (about their states, respectively) are increasing in both signals but each one of them has a signal that is of primary importance to him/her. In that case, we can talk about relative (dis-)agreement: they agree on the direction of influence of the signal but they disagree on how strongly the signal should affect the choice.

[1]In modern tongue, the bias is state dependent in our model.

Since information is two-dimensional and there is only one unidimensional choice to make, the model exhibits a natural source of pooling: the Sender's conditionally expected utility depends only on the posterior mean, so Sender types who have observed different signal realizations giving rise to the same conditional mean pool on the same message. This is both similar and strikingly different from the partial pooling we know from Crawford & Sobel (1982). It is similar in that the underlying Sender type is only partially revealed. However, it is completely different in that the resulting equilibrium is not described by an interval partition of the state space. In contrast, it is possible that the Sender honestly reveals his conditional mean and the Receiver rightly takes this message at face value. That is, both the Sender's message strategy and the Receiver's action strategy are strictly monotonic and differentiable functions of the players' information. In fact, the strategies are linear. It must be stressed that this equilibrium exists despite conflicts of interest, that is despite the Sender and the Receiver disagreeing on how to map signals into choices.

We term equilibria in strictly monotonic and differentiable strategies smooth communication equilibria. The existence of smooth communication equilibria depends crucially on the quality of information that the Sender gets to observe. In particular, smooth communication requires that the Sender's conditionally optimal choice correlates the same way with both the Sender's and the Receiver's underlying state. Only in this case will the Receiver - who couldn't care less about what is good for the Sender - do exactly what she is told; this is because the conditionally optimal choice of the Receiver conditional on observing the Sender's conditionally optimal choice is identically equal to the latter for an information structure that is equally useful to both Sender and Receiver in the described way. For information structures that fail to satisfy this requirement, equilibrium communication is partitional, exactly the way we know it from the existing theories.

While theoretically interesting, why should smooth communication be of any practical relevance? Information structures that induce smooth communication are optimal in various extensive forms. The intuition for this result is a simple efficiency argument. E.g., information structures that induce smooth communication arise naturally in a symmetric environment when the player in charge of shaping the environment cares about joint surplus. Think of a situation as in the CEO/personal assistant example, where the CEO-receiver needs to pay the assistant-sender to convince him to participate and suppose both the assistant and the CEO have quasilinear utility. However, payments cannot condition on advice nor on outcomes nor on both. In such a world, the CEO's payoff is equal to the sum of both players' utilities from decision making. Now suppose the CEO can control the precision of the information the assistant obtains without getting to observe the actual realizations of the signals. Then, the CEO chooses an information structure that maximizes expected joint surplus, resulting precisely in an information structure that is equally useful to the assistant and the CEO. Hence, in this environment, smooth communication is the best equilibrium in terms of ex ante expected surplus.

Envision another situation like the one in the politician's and advisor's case where the politician-receiver has less of an influence on the information that the advisor-sender observes. Imagine that the politician can only choose an advisor type. An advisor type is a joint distribution over the players' interests based on the information available at the start of the game. Such information could, e.g., be observable through publicly observable information about advisors or through their cv's and the like. Once an advisor has been selected, it is the advisor who has the right to choose the relative precision of signals, so the advisor controls the quality of the information. Clearly, each player has an interest to get information that correlates better with his/her underlying interests. However, a novel feature of our model is that the nature of conflict between advisor and politician depends on the quality of information. Moreover, the potential conflicts arising in communicating with an advisor depend on that advisor's type. When selecting a given type of advisor, the politician anticipates that this advisor will choose the information that serves him best. We show that the optimally selected advisor indeed chooses information that is equally useful to both the advisor and the politician and thus is one with whom the politician can communicate smoothly.

Smooth communication equilibria are easily tractable and thus lend themselves to comparative statics exercises. Sender types with better aligned interests end up being more competent, give more accurate advice, and thus induce a higher volatility of Receiver choices. However, the logic is subtly different from other approaches. In our model, Senders with better aligned interests are endogenously endowed with better information and are therefore rightly perceived as more competent. The loss of information through communication is the smaller the better interests are aligned and completely vanishes if ex ante interests are perfectly correlated.

1.3 Literature

We build on a vast literature on communication and information beginning with Crawford & Sobel (1982). We have already described the essential ways in which our approach differs from theirs. The state in our model is a two-dimensional vector and so there is a state dependent bias in our model. Moreover, the state remains uncertain, so the bias is not perfectly known. Moreover, information is endogenous through the signals the Sender gets to observe.

It is clearly crucial that information is two-dimensional; with one piece of information only, smooth communication equilibria exist only in the trivial case of completely aligned interests. We are not aware of any model featuring two-dimensional information and a one-dimensional choice. Battaglini (2002) studies a model with a two-dimensional state and two signals where the Receiver can perfectly extract all information. Meyer et al. (2013) study the multidimensional cheap talk problem for the case where there are exogenous restrictions on the feasible set of policies for the Receiver. The essential differences between these approaches and ours are that the choice is two-dimensional and that there are two Senders in these models. Having one Sender only and a one-dimensional choice makes it impossible to extract all information in our model. In contrast, equilibrium communication is one-dimensional and therefore involves some pooling by design of the model. Chakraborty & Harbaugh (2007) and Chakraborty & Harbaugh (2010) study comparative cheap talk where again both information and choices are at least two-dimensional. They show that the Sender can always communicate comparative statements - e.g., that one state is larger than the other. Our communication is not comparative. We derive a one-dimensional statistic as an upper bound on what can be communicated in any equilibrium in our model. The essential reason is that the dimension of choice is smaller than the dimension of information. Levy & Razin (2007) study multi-dimensional cheap talk. However, the Sender has perfect information in their paper and the dimension of choice equals the dimension of information.

Noise is an important element in our theory. Blume et al. (2007) have shown that noise in communication may improve upon equilibrium communication. We share that conclusion, for different reasons, however. In Blume et al. (2007), the Sender's information gets destroyed with some probability making the Sender more willing to share his information. In contrast, no noise is added to the Sender's message here but the Sender does not have perfect information to begin with. Noisy information is also present in Moscarini (2007) who studies competence in a model of central bank communication. Information is both unidimensional and exogenously given in that approach. Improving information has similar comparative statics effects in both Moscarini (2007) and the present approach.

The optimal design of information structures prior to communication is another important building block of our model. As we show in an extension to our main results in Theorem 2, if other information structures are selected in the first stage, then we are back in the case of partitional equilibria, as the known models feature them. There are at least two distinct approaches to endogenous information structures and we relate to both of them. In the CEO story above, the Receiver controls the quality of information that the Sender observes. Ivanov (2010) has first studied this in the strategic communication game and termed it "informational control". The optimal information structure in Ivanov (2010) remains partitional so perfect communication is impossible. Gentzkow & Kamenica (2011) analyze a model of Bayesian persuasion, where a Sender chooses the optimal information to provide a Receiver with, knowing that the Receiver chooses an action that affects both their payoffs. The key difference to our approach is the lack of commitment about messages that the Sender sends to the Receiver in our approach. In Gentzkow & Kamenica (2011), signals are more or less informative but always truthful. In contrast, in our model, for any given information structure, the Sender can decide to transmit any message to the Receiver, so information transmission is strategic as in Crawford & Sobel (1982). As a result, the precision of information that reaches the Receiver depends both on the quality of information that the Sender obtains and on the amount of information that the Sender transmits. The latter effect is absent in Gentzkow & Kamenica (2011). Moreover, we show that the Sender's willingness to transmit information depends crucially on the information that the Sender observes.[2]

The second way in which endogenous information has been addressed is through informa-

[2]The idea of endowing agents in a game with information without the principal necessarily obtaining the same information has been studied extensively in the context of auctions: see, e.g., Eső & Szentes (2007), Bergemann & Pesendorfer (2007) and Ganuza & Penalva (2010). The common idea in these papers is that the principal (auctioneer) controls the precision of the agents' information but does not get to observe the signals the agents receive.

tion acquisition by the Sender. Szalay (2005) studies the case of aligned ex post preferences with commitment to decision rules; Eső and Szalay (in preparation) study the same model without commitment to decision rules. Argenziano et al. (2013) look at endogenous information in the case of a Sender who is uniformly biased in one direction; Di Pei (2013) studies a model where the Sender can choose the partitional information that he observes. With a biased Sender, the equilibrium remains partitional. This perspective relates to our politician/advisor example. To isolate the purely strategic effects of different information structures, we assume here that all information structures are equally costly. Extensions to other cases are pursued in companion work.

Our model features conflicts whose nature depends on the quality of information. This is to the best of our knowledge new in the literature. However, there are some approaches that look at state dependent biases where for some states of the world, the Sender's preferred action is larger than the Receiver's preferred action while the reverse is true for other states. Such models are studied, e.g., by Stein (1989), Ottaviani & Sørensen (2006*a*), Ottaviani & Sørensen (2006*b*), Alonso et al. (2008), and more recently by Gordon (2010). Smooth communication equilibria do not exist in these setups, but equilibria inducing a countable infinity of Receiver actions do. Equilibria with invertible strategies in communication models have been studied in Kartik (2009), Kartik et al. (2007), and Austen-Smith & Banks (2000). The former papers involve lying costs, the latter one money-burning. We do not rely on costs of communication to obtain equilibria in invertible strategies. Li & Madarász (2008) show that a Sender of unknown bias may be willing to communicate honestly if the distribution of his bias is symmetric around zero. This is similar in our context. However, our statistical model is very different from theirs and novel to the literature on strategic communication. We relate the conflicts in the communication game to the quality of endogenous information structures and show that a symmetrically distributed bias arises as an optimum of our model.

Finally, our theory relates to models of market microstructure. While economic theorists are used to study strategic communication in the context of Crawford & Sobel (1982), finance scholars have studied communication in markets extensively. In markets, strategic communication (of demand) is often thought of in the context of rational expectations or strategic trade models in the spirit of Grossman & Stiglitz (1980), Hellwig (1980), Vives (1995), or Kyle (1989); see Vives (2008) for a survey. The reader familiar with this literature will find that our theory has much in common with these models. While the majority of papers in this literature assumes normally distributed noise terms, a small number of papers extends the results to the context of elliptically distributed noise. We borrow extensively from the ideas in that work. See in particular Nöldeke & Tröger (2006) and the references cited therein.

The remainder paper is organized as follows. In section two, we present the model and the main assumptions. In section three, we define strategies and analyze communication about signals. In section four we derive and analyze a reduced form model where wlog communication is about conditional means. We discuss in depth how conflicts between Sender and Receiver depend on the quality of information that the Sender observes. In section five, we analyze optimal noise structures from various angles. In section six, we focus on comparative statics. A final section concludes. Lengthy proofs are gathered in the Appendix.

2 Model

2.1 Timing

We consider a Sender-Receiver-game with two players and the following underlying decision problem. The Receiver - henceforth she - needs to choose a decision x that affects the utility of both the Sender - he from now on - and the Receiver. Preferences are given by

$$u^R(x-\omega) \quad \text{and} \quad u^S(x-\eta),$$

where $u^i(x-\cdot)$ for $i=R,S$ is a strictly concave, symmetric and differentiable function. The preferences only depend on the distance between the actual action, x, and the ideal action described by the bliss points $x^R(\omega)=\omega$ and $x^S(\eta)=\eta$, respectively.

At the outset, neither the Sender nor the Receiver know the realization of the random variables ω and η. However, the Sender privately observes noisy signals of their realizations according to

$$s_\omega = \omega + \varepsilon_\omega \quad \text{and} \quad s_\eta = \eta + \varepsilon_\eta,$$

where ε_ω and ε_η are independent noise terms with variances that are determined at the start of the game.[3] We discuss in section 5 in detail how this is done. Until then, we take the information structure as arbitrary and given.

After observing the realizations of s_ω and s_η, the Sender sends a message to the Receiver. The Receiver observes the message - but not the Sender's information - and then chooses x. The Receiver cannot commit ex ante how to use the information she receives from the Sender.

2.2 Feasible Information Structures

We assume that the random variables $\omega, \eta, \varepsilon_\omega, \varepsilon_\eta$ are jointly elliptically distributed, with finite first and second moments. Each of the marginals has mean zero; let σ_ω^2, σ_η^2, $\sigma_{\varepsilon_\omega}^2$, $\sigma_{\varepsilon_\eta}^2$ denote the variances of the random variables. For convenience, we assume that the joint distribution has a density. More assumptions are imposed later when needed.

Elliptical distributions can be defined as follows (e.g. Owen & Rabinovitch (1983), p.746).

Definition 1 *Let μ be an n-component vector and Σ an $(n \times n)$ positive definite symmetric matrix. Then, an n-component random vector $X = (X_1, \ldots, X_n)'$ is said to be distributed elliptically, $X \sim \mathcal{E}_n(\mu, \Sigma)$, if and only if for all nonzero n-component scalar vectors α, all the univariate random variables $\alpha' X$ such that $Var(\alpha' X)$ is constant follow the same distribution. If X has a density, then the density function of X, $f_X(x)$, can be expressed in the form $f_X(x) = c_n |\Sigma|^{-1/2} g\left((x-\mu)' \Sigma^{-1} (x-\mu)\right)$, for some constant c_n and some function g that is independent of n.*

Given that the first two moments exist, μ is the mean vector and Σ is proportional to (that is, up to a constant factor equal to) the variance-covariance matrix.

Elliptical distributions have convenient properties. We summarize the properties we use in the following lemma:

Lemma 1 *Let $X \sim \mathcal{E}_n(\mu, \Sigma)$ be elliptically distributed. Further let*

$$X = (X_1, X_2), \quad \mu = (\mu_1, \mu_2), \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

[3] The assumption of independence is innocuous since any signal structure can be decomposed in this way.

where the dimensions of X_1, μ_1 and Σ_{11} are, respectively, m, m and $m \times m$.

i) The elliptical distribution is symmetric about μ, i.e.

$$f(\mu + \Delta) = f(\mu - \Delta) \qquad \forall \Delta.$$

ii) Linear combinations of elliptically distributed random variables are again elliptical.

iii) The conditional distribution of $X_1|X_2$ is elliptical, i.e.

$$X_1|X_2 \sim \mathcal{E}_m\left(\mu_{X_1} + \Sigma_{12}\Sigma_{22}^{-1}\left(X_2 - \mu_{X_2}\right), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\right).$$

Proof. i) by definition, ii) see for example, Fang et al. (1990) Thm 2.16, iii) see for example, Cambanis et al. (1981) Cor 5.

2.3 Leading examples

The leading example for symmetric loss functions that is commonly used in the literature is the quadratic

$$u^R(x - \omega) = -(x - \omega)^2 \quad \text{and} \quad u^S(x - \eta) = -(x - \eta)^2.$$

Examples of elliptical distributions are the normal distribution, Student's–t distribution, the exponential power distribution, and the logistic distribution.

While the reader is of course welcome to replace symmetric loss by quadratic loss and elliptical by normal in what follows, all our results until and including Theorem 1 extend to the more general environment.

2.4 Ideal choices

Before discussing strategic communication, it is useful to set the stage and investigate the players' ideal choices if they had (somehow) access to the same information. The following result follows conveniently from the statistical structure we have imposed:

Lemma 2 *The ideal choice functions of the Receiver and the Sender are*

$$x^R(s_\omega, s_\eta) \equiv \arg\max_x \mathbb{E}\left[u^R(x - \omega)\middle| s_\omega, s_\eta\right] = \mathbb{E}\left[\omega | s_\omega, s_\eta\right] = \alpha^R s_\omega + \beta^R s_\eta$$

and

$$x^S(s_\omega, s_\eta) \equiv \arg\max_x \mathbb{E}\left[u^S(x-\eta) \middle| s_\omega, s_\eta \right] = \mathbb{E}\left[\eta | s_\omega, s_\eta \right] = \alpha^S s_\omega + \beta^S s_\eta,$$

where α^i, β^i *for* $i = R, S$ *are weights, independent of* s_ω, s_η.

The result is due to the symmetry of both the loss functions and the posterior distributions given the information. The lemma is straightforward to see for the case of a quadratic loss function - because first-order conditions become linear in that case. The extension to more general, symmetric loss functions follows from the fact that, if the choice x is set equal to the posterior mean, the posterior distribution is symmetric around the choice. Hence expected marginal gains to increasing the action are exactly offset by expected marginal losses. The linear expressions are simply a consequence of elliptical distributions. For future reference, we note that the weights in the Sender's ideal choice are

$$\alpha^S = \sigma^2_{\varepsilon_\eta} \frac{\rho_{\omega\eta}\sigma_\omega\sigma_\eta}{(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega})(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}) - (\rho_{\omega\eta}\sigma_\omega\sigma_\eta)^2}$$

and

$$\beta^S = \sigma^2_\eta \frac{\sigma^2_{\varepsilon_\omega} - \sigma^2_\omega\rho^2_{\omega\eta} + \sigma^2_\omega}{(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega})(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}) - (\rho_{\omega\eta}\sigma_\omega\sigma_\eta)^2}$$

and the weights in the Receiver's ideal choice are

$$\alpha^R = \sigma^2_\omega \frac{\sigma^2_{\varepsilon_\eta} + \sigma^2_\eta - \sigma^2_\eta\rho^2_{\omega\eta}}{\left(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega}\right)\left(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}\right) - \left(\rho_{\omega\eta}\sigma_\omega\sigma_\eta\right)^2},$$

and

$$\beta^R = \sigma^2_{\varepsilon_\omega} \frac{\sigma_\eta\sigma_\omega\rho_{\omega\eta}}{\left(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega}\right)\left(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}\right) - \left(\rho_{\omega\eta}\sigma_\omega\sigma_\eta\right)^2},$$

where

$$\rho_{\omega\eta} \equiv \frac{Cov(\omega, \eta)}{\sigma_\omega\sigma_\eta}$$

is the coefficient of correlation between ω and η.

2.5 Conflicts and common interests

We can now investigate whether and how the Sender and the Receiver are agreed or disagreed upon optimal actions as a function of the information they have. Since ω and η have the same prior mean, the Sender and the Receiver are agreed upon the optimal action if the Sender does not obtain any new information; formally, this is the limiting case when $\sigma^2_{\varepsilon_\omega}$ and $\sigma^2_{\varepsilon_\eta}$ both go to infinity. So, potential conflicts arise only with respect to how new information should be used. If the coefficient of correlation satisfies $\rho_{\omega\eta} \leq 0$, then the Sender and the Receiver disagree fundamentally on how news should affect choices. In particular, while the players' ideal choices always increase in the signal about their bliss point, for $\rho_{\omega\eta} \leq 0$, we have $\beta^R \leq 0$ and $\alpha^S \leq 0$. Therefore, $x^R(s_\omega, s_\eta)$ increases in s_ω and (weakly) decreases in s_η, while the exact opposite is true for $x^S(s_\omega, s_\eta)$. We conjecture that no meaningful communication is possible in that case.

Next it is illuminating to look at the opposite extreme, where players are completely agreed upon optimal actions. The following lemma states these results formally.

Lemma 3 *The Sender and the Receiver are completely agreed upon ideal choices for all* $\sigma^2_{\varepsilon_\eta}, \sigma^2_{\varepsilon_\omega} \geq 0$ *if* $\rho_{\omega\eta} = 1$ *and* $\sigma_\eta = \sigma_\omega$. *In the limiting case where* $\sigma^2_{\varepsilon_\eta} \to \infty$, *the Sender and the Receiver agree on the use of signal* s_ω *for all* $\sigma^2_{\varepsilon_\omega} \geq 0$ *if* $\frac{\sigma_\omega}{\sigma_\eta} = \rho_{\omega\eta}$. *In the limiting case where* $\sigma^2_{\varepsilon_\omega} \to \infty$, *the Sender and the Receiver agree on the use of signal* s_η *for all* $\sigma^2_{\varepsilon_\eta} \geq 0$ *if* $\frac{\sigma_\omega}{\sigma_\eta} = \frac{1}{\rho_{\omega\eta}}$.

The limits are formally identical to the case where there is only one signal available. Thus, another way to state the result is to say that only one signal is available, so that all moments exist.

Clearly, we are interested in the case where some communication is possible. On the other hand, communication becomes interesting from the strategic perspective only if there is some disagreement. Therefore, we impose the following

Assumption 1: $0 < \rho_{\omega\eta} < 1$.

Assumption 1 implies that, whenever the Sender obtains two informative signals, then the Sender and the Receiver will disagree on the ideal choice for all signal realizations but the one where both signals are equal to the prior mean.

Next, one may wonder what fundamental difference it makes that the signal is two-dimensional rather than one-dimensional. The answer depends on the model parameters. For the interesting case, that the following lemma singles out, a two-dimensional signal allows us to talk about relative disagreement in the sense that the Sender and the Receiver each have a signal that they find of primary importance to him/her.

Lemma 4 *For $\rho_{\omega\eta} \leq \frac{\sigma_\omega}{\sigma_\eta} \leq \frac{1}{\rho_{\omega\eta}}$, the Receiver is relatively more responsive to signal s_ω than the Sender is, $\alpha^R \geq \alpha^S$, and the Sender is relatively more responsive to s_η, $\beta^S \geq \beta^R$. For $\frac{\sigma_\omega}{\sigma_\eta} > \frac{1}{\rho_{\omega\eta}}$, the Receiver reacts stronger to any piece of news (that is each of the signals), for $\frac{\sigma_\omega}{\sigma_\eta} < \rho_{\omega\eta}$, the Sender reacts stronger than the Receiver to any piece of news.*

Relative disagreement and our way to model it are the essential contributions of this paper; conversely, in the case where any player is absolutely more responsive than the other one - that is, reacts stronger to any piece of news - our structure does not give rise to substantially new insights.[4] Therefore, we focus on what is new here by imposing

Assumption 2: $\rho_{\omega\eta} \leq \frac{\sigma_\omega}{\sigma_\eta} \leq \frac{1}{\rho_{\omega\eta}}$.

We are now ready to study strategic communication.

3 Strategic communication about signals

We now solve the second stage communication game under asymmetric information, taking the information structure as given. The first stage in which the information structure is determined is analyzed in section 5 below. We assume that the Receiver cannot commit to what action she will take as a function of the message she receives; moreover, information is soft, so the Sender cannot credibly commit to being honest either. The equilibrium concept we use is Perfect Bayes Nash Equilibrium. In the equilibria we study, there won't be any out of equilibrium messages, so Perfect Bayes Nash Equilibrium and Bayes Nash Equilibrium coincide.

[4] In that case, we could as well assume a one-dimensional state and signal.

3.1 Strategies

Take the information structure - that is the noise variances $\sigma^2_{\varepsilon_\omega}$ and $\sigma^2_{\varepsilon_\eta}$ - as given. After the Sender has observed his signals, he communicates a message $m \in \mathbb{M}$ to the Receiver. The message space $\mathbb{M}$ is assumed to be rich. It is enough to consider pure message strategies for the Sender.[5] A pure Sender strategy maps the Sender's information into messages $M : \mathbb{R}^2 \to \mathbb{M}$, $(s_\omega, s_\eta) \mapsto m$. A pure Receiver strategy maps messages into actions, $X : \mathbb{M} \to \mathbb{R}$, $m \mapsto x$. Let $f(\omega|\, m)$ denote the pdf of the Receiver's posterior over ω given the message sent by the Sender and let $f(\eta|\, s_\omega, s_\eta)$ denote the pdf of the Sender's posterior over η given his information.

The Receiver's optimal action given the Sender's message is

$$x^*(m) \in \arg\max_{x \in \mathbb{R}} \int u^R(x - \omega)\, f(\omega|\, m)\, d\omega.$$

Note that the Receiver solves a concave problem for arbitrary posteriors $f(\omega|\, m)$. Hence $x^*(m)$ is in fact unique and the Receiver never wishes to use a mixed strategy.

The Sender's optimal message solves

$$m(s_\omega, s_\eta) \in \arg\max_{m \in \mathbb{M}} \int u^S(x^*(m) - \eta)\, f(\eta|\, s_\omega, s_\eta)\, d\eta.$$

The Receiver updates her information about ω based on the message she receives, so the posterior distribution satisfies Bayes law.[6] Thus, there is no restriction on the message space and the equilibrium concept is the standard one.

3.2 Non-existence of full communication equilibria

Recall that Assumption 1 implies conflicting interests of the Sender and the Receiver with respect to the interpretation of new information. Under this situation, there cannot be an equilibrium where the Sender communicates all his information truthfully. Formally, we have the following result:

[5] More specifically, the *best* equilibria of our game involve pure strategies. Therefore, we abstain from introducing the notational clutter to deal formally with mixed strategies.

[6] Let $s(m) \equiv \{(s_\omega, s_\eta) : m(s_\omega, s_\eta) = m\}$, then $f(\omega|\, m) = \frac{f(\omega, m)}{\iint_{s(m)} f(s_\omega, s_\eta) ds_\omega ds_\eta}$.

Lemma 5 *Under Assumption 1, there does not exist any equilibrium where* $m(s_\omega, s_\eta) = (s_\omega, s_\eta)$.

Proof. Suppose there exists an equilibrium with $m(s_\omega, s_\eta) = (s_\omega, s_\eta)$. Then, the Receiver's posterior is $f(\omega | s_\omega, s_\eta)$ resulting in an optimal choice $x^*(s_\omega, s_\eta) = x^R(s_\omega, s_\eta) = \mathbb{E}[\omega | s_\omega, s_\eta]$. By Assumption 1, $\mathbb{E}[\omega | s_\omega, s_\eta] \neq \mathbb{E}[\eta | s_\omega, s_\eta]$ for any $(s_\omega, s_\eta) \neq (0,0)$. Hence, regardless of the signal structure, as long as the informational environment satisfies Assumption 1, there cannot be a fully informative equilibrium. ■

Even if the Sender and the Receiver agree that both signals should be used to reach a good decision, they disagree with respect to the weight they attach to the individual sources of information. Therefore, the Sender is not willing to reveal all of his information truthfully to the Receiver if the latter cannot commit to use the information the way the Sender proposes.

3.3 Sender preferences and conditional expectations

Full revelation is not an equilibrium outcome. If not all the information, how much information can in equilibrium be revealed? To answer this question, we need to address the Sender's choice between actions he can induce. The following lemma states formally that the Sender's preferences over messages (that induce distinct actions) depend only on the distance between induced actions and the Sender's conditional expectation of η given his information.

Lemma 6 *The expected utility of the Sender depends only on* $|x - \mathbb{E}[\eta | s_\omega, s_\eta]|$*, the distance between the choice and the conditional mean. Formally, we have*

$$\int u(x' - \eta) f(\eta | s_\omega, s_\eta) \, d\eta = \int u(x'' - \eta) f(\eta | s_\omega, s_\eta) \, d\eta$$

for any two choices x', x'' *satisfying* $x' - \mathbb{E}[\eta | s_\omega, s_\eta] = \mathbb{E}[\eta | s_\omega, s_\eta] - x''$.

By assumption, the Sender's vNM-utility function depends only on the distance between the action and the state η. The proof of the lemma shows that the Sender's expected utility depends only on the distance from the action to the posterior expected value of the state η. The reason is simply that the ideal choice of the Sender is equal to $\mathbb{E}[\eta | s_\omega, s_\eta]$ by Lemma

2. Moreover, by symmetry of the posterior distribution about $\mathbb{E}\left[\eta | s_\omega, s_\eta\right]$, the Sender is indifferent between any actions that are equally far away from the posterior mean. To get an intuition why Lemma 6 is true consider the case of quadratic loss functions. The Sender's expected utility is then given by

$$\int -\left(x-\eta\right)^2 f\left(\eta | s_\omega, s_\eta\right) d\eta = \int -\left(x-\mathbb{E}\left[\eta | s_\omega, s_\eta\right]\right)^2 f\left(\eta | s_\omega, s_\eta\right) d\eta - Var\left(\eta | s_\omega, s_\eta\right),$$

making the result obvious for that case.

Since the Sender's expected utility depends only on $\left|x-\mathbb{E}\left[\eta | s_\omega, s_\eta\right]\right|$, Sender types s_ω, s_η who share the same posterior mean, $\mathbb{E}\left[\eta | s_\omega, s_\eta\right]$, have the same preferences over any two distinct actions x' and x''. Therefore, such types will necessarily pool on the same messages, implying that at most the posterior mean can be communicated. Hence, without loss we now reduce the Sender's message space to the unidimensional space of conditionally expected means.

3.4 Relevant information

Define the new random variable

$$\theta \equiv \mathbb{E}\left[\eta \left| s_\omega, s_\eta\right.\right].$$

Being a (linear) function of random variables, θ is random as well. Since linear functions of elliptically distributed random variables are again elliptically distributed, θ follows an elliptical distribution $\theta \sim \mathcal{E}\left(0, \sigma_\theta^2\right)$ and the joint distribution of ω, η, and θ is elliptical too. More specifically, the variance of θ is given by

$$Var\left(\theta\right) = \left(\alpha^S\right)^2 \left(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2\right) + 2\alpha^S\beta^S\rho_{\omega\eta}\sigma_\omega\sigma_\eta + \left(\beta^S\right)^2 \left(\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2\right).$$

Substituting for α^S and β^S, we have

$$Var\left(\theta\right) = \sigma_\eta^2 \frac{\frac{\sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2} + \frac{\sigma_{\varepsilon_\eta}^2}{\sigma_\eta^2}\rho_{\omega\eta}^2 + 1 - \rho_{\omega\eta}^2}{\left(1+\frac{\sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2}\right)\left(1+\frac{\sigma_{\varepsilon_\eta}^2}{\sigma_\eta^2}\right) - \rho_{\omega\eta}^2}. \quad (1)$$

Likewise, the covariance of ω and θ is given by

$$Cov\left(\omega, \theta\right) = \alpha^S\sigma_\omega^2 + \beta^S\rho_{\omega\eta}\sigma_\omega\sigma_\eta,$$

and so we have

$$Cov\left(\omega,\theta\right)=\sigma_{\eta}\sigma_{\omega}\rho_{\omega\eta}\frac{\frac{\sigma_{\varepsilon_{\eta}}^{2}}{\sigma_{\eta}^{2}}+\frac{\sigma_{\varepsilon_{\omega}}^{2}}{\sigma_{\omega}^{2}}+1-\rho_{\omega\eta}^{2}}{\left(1+\frac{\sigma_{\varepsilon_{\omega}}^{2}}{\sigma_{\omega}^{2}}\right)\left(1+\frac{\sigma_{\varepsilon_{\eta}}^{2}}{\sigma_{\eta}^{2}}\right)-\rho_{\omega\eta}^{2}}. \tag{2}$$

Finally, we (trivially) have

$$Cov\left(\eta,\theta\right)=Var\left(\theta\right). \tag{3}$$

4 One-dimensional strategic communication

The two-dimensional model reduces without loss of generality to a one-dimensional one. We now address communication in the reduced form model. However, it is important to keep the “micro-foundations” in mind: not any reduced form model is feasible, that is, can be generated from the underlying information structures. We first derive the set of feasible reduced form communication models and discuss conflicts and common interests in the reduced form model. Then we discuss the structure of the most informative communication equilibria; that is, communication equilibria that induce the highest number of distinct Receiver responses. These are the equilibria that are commonly studied in the literature.

4.1 Feasible reduced forms and implied conflicts

Ultimately, the players do not care about the signals s_{ω} and s_{η} but about the posteriors arising from them. So, we can equivalently think of a situation where the Sender obtains a signal θ that is perfectly correlated with his bliss point η, that is $Cov\left(\eta,\theta\right)=Var\left(\theta\right)$ by (3). The correlation of ω and θ depends on the underlying noise structure. To determine the players' conflicts when communicating about θ, we again investigate the Receiver's ideal choice if she had (somehow) directly access to the information θ. Building on Lemma 1, we know that

$$\omega|\,\theta\sim\mathcal{E}\left(\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}\theta,\sigma_{\omega}^{2}(1-\rho_{\omega\theta}^{2})\right).$$

Note that the posterior mean depends crucially on the ratio $\frac{Cov(\omega,\theta)}{Var(\theta)}$, the slope of the regression of ω on θ. The intercept is zero by the fact that the priors of ω and η have the same mean. If $\frac{Cov(\omega,\theta)}{Var(\theta)} < 1$, then the Receiver is *conservative* relative to the Sender; given a positive realization of θ, the Receiver's ideal choice is smaller than the Sender's and vice versa for a negative realization. If $\frac{Cov(\omega,\theta)}{Var(\theta)} > 1$, then the Sender is relatively more conservative than the Receiver; put differently, the Receiver is relatively *progressive*; from the Sender's perspective the Receiver overreacts to news. Finally, if $\frac{Cov(\omega,\theta)}{Var(\theta)} = 1$, then neither the Sender nor the Receiver is more conservative than the other player. Before we dwell further on this issue we investigate whether and when all these cases are relevant. The following lemma characterizes the set of feasible moments $Cov(\omega,\theta)$ and $Var(\theta)$.

Lemma 7 *Any $Cov(\omega,\theta) \in (0, \sigma_\eta\sigma_\omega\rho_{\omega\eta}]$ can be generated from signals with finite variances. Moreover, for any fixed $Cov(\omega,\theta) = C$, the set of feasible variances is by Assumption 2 non-empty and given by $Var(\theta) \in \left[\frac{\sigma_\eta}{\sigma_\omega}\rho_{\omega\eta}C, \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho_{\omega\eta}}C\right]$. By implication*

$$\frac{Cov(\omega,\theta)}{Var(\theta)} \in \left[\frac{\sigma_\omega}{\sigma_\eta}\rho_{\omega\eta}, \frac{\sigma_\omega}{\sigma_\eta}\frac{1}{\rho_{\eta\omega}}\right].$$

The following figure depicts the set of feasible ratios $\frac{Cov(\omega,\theta)}{Var(\theta)}$ in three relevant cases that satisfy Assumptions 1 and 2.

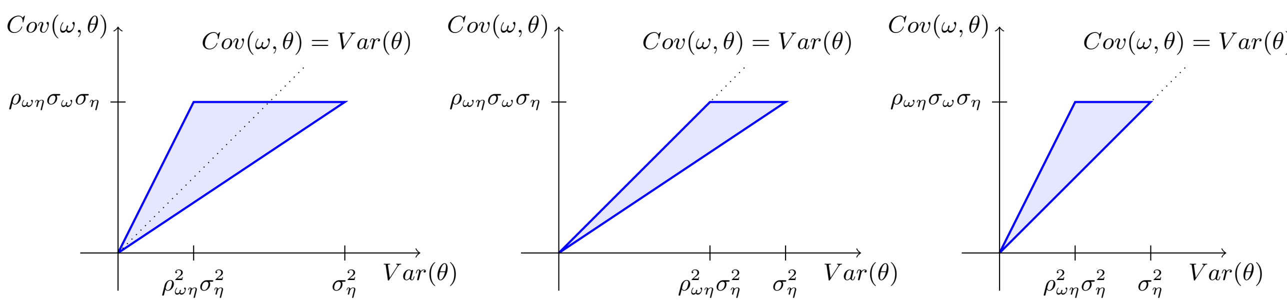

Figure 1: Case one (left panel): feasible set for $\frac{\sigma_\omega}{\sigma_\eta} \in \left(\rho_{\omega\eta}, \frac{1}{\rho_{\eta\omega}}\right)$; case two (center panel): feasible set for $\frac{\sigma_\omega}{\sigma_\eta} = \rho_{\omega\eta}$; case three (right panel): feasible set for $\frac{\sigma_\omega}{\sigma_\eta} = \frac{1}{\rho_{\omega\eta}}$.

In case one the prior uncertainty satisfies $\frac{\sigma_\omega}{\sigma_\eta} \in \left(\rho_{\omega\eta}, \frac{1}{\rho_{\eta\omega}}\right)$, cases two and three depict the extreme cases where $\frac{\sigma_\omega}{\sigma_\eta} = \rho_{\omega\eta}$ or $\frac{\sigma_\omega}{\sigma_\eta} = \frac{1}{\rho_{\omega\eta}}$, respectively.

Usually, the conflict between Sender and Receiver - the bias in terms of the standard language - is a primitive of the model. In contrast, in the present model conflicts arise endogenously as a function of the quality of the Sender's information. Recall that $Var(\theta) = Cov(\eta, \theta)$ by construction. Therefore, signal structures that imply a $\frac{Cov(\omega,\theta)}{Var(\theta)}-$ratio that is smaller than unity contain information that is relatively more useful to the Sender than to the Receiver. As a result, the Receiver obviously would want to rely less on that sort of information than the Sender would do. Vice versa for information structures that satisfy $\frac{Cov(\omega,\theta)}{Var(\theta)} > 1$. Finally, if $\frac{Cov(\omega,\theta)}{Var(\theta)} = 1$, then the information is equally useful to the Sender and the Receiver. Note that all three constellations are possible for some information structures in case one, while the Receiver is weakly conservative for any feasible information structure in case two and weakly progressive for any information structure in case three. Note finally, that for all conflicts described by Assumptions 1 and 2, there exist information structures that are equally useful to the Sender and the Receiver, that is $\frac{Cov(\omega,\theta)}{Var(\theta)} = 1$. Even though the Receiver's ideal choice based on the underlying signals differs from the Sender's ideal choice based on these signals, the Receiver's ideal choice based on θ is identically equal to θ.

To illustrate further, consider case one where $\frac{\sigma_\omega}{\sigma_\eta} \in \left(\rho_{\omega\eta}, \frac{1}{\rho_{\eta\omega}}\right)$ and imagine three of the feasible information structures with the same covariance but different levels of variance. For concreteness, let $\frac{Cov(\omega,\theta)}{Var(\theta)'} = c' > 1$, $\frac{Cov(\omega,\theta)}{Var(\theta)''} = c'' < 1$, and $\frac{Cov(\omega,\theta)}{Var(\theta)'''} = c''' = 1$. The information structures are depicted in the left panel of the following Figure 2. The right panel of the figure shows the implied ideal choice functions of the Sender and the Receiver, $x^S(\theta)$ and $x^R(\theta)$, for the three information structures. Note that by construction $x^S(\theta) = \theta$ for all three information structures, while $x^R(\theta) = c\theta$, for $c \in \{c', c'', c'''\}$.

4.2 Equilibria

We now address equilibrium communication. Our game has the standard plethora of equilibria in the communication game. It is always an equilibrium that all Sender types θ pool on the same message (or mix over all available messages) and the Receiver always chooses $x = 0$. This is the babbling equilibrium. In addition to that, partitional equilibria exist where sets of Sender types within given intervals pool on the same message. More interestingly, depending on the ratio $\frac{Cov(\omega,\theta)}{Var(\theta)}$ there can exist a class of equilibria that has not been studied so far: smooth communication equilibria where the Sender honestly states his

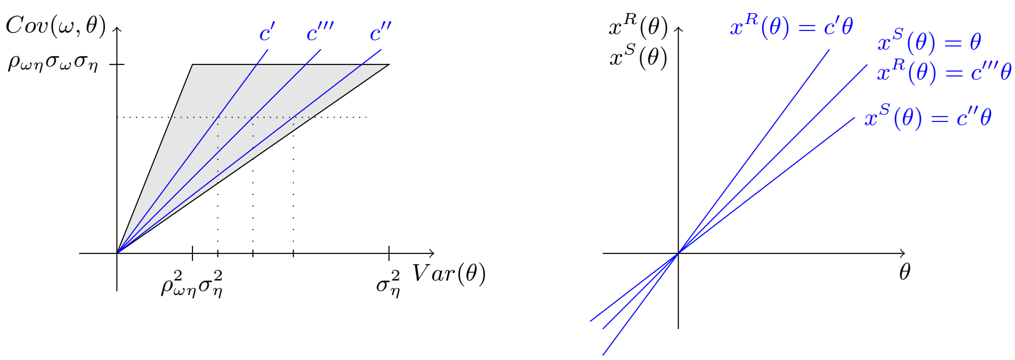

Figure 2: Conflict in terms of covariance and variance (left panel), and actions (right panel).

conclusions, that is truthfully reveals θ, and the Receiver rightly takes this recommendation at face value. These equilibria feature smooth, i.e. infinitely often differentiable Sender and Receiver strategies, and thus enable perfect communication about θ. This is possible since interests are aligned in this subspace and the underlying signals that trigger conflict remain garbled in θ.

In the following two subsections we analyze the most informative equilibria for given information structures, that is, the equilibria that induce the highest number of Receiver actions, as a function of the given information structure. First, for general symmetric preferences, we analyze the situation where the Sender and the Receiver are equally conservative, that is $Cov\left(\omega,\theta\right)=Var\left(\theta\right)$. Second, for quadratic preferences and log-concave densities, we look at the cases where one player is more conservative and only partitional equilibria exist.

4.2.1 Perfect communication about conditional means

Suppose the Sender communicates his conditionally expected value of η given his information truthfully to the Receiver, that is

$$m\left(\theta\right)=\theta \text{ for all } \theta.$$

The Receiver's best reply given information θ is

$$\mathbb{E}\left[\omega|\,\theta\right]=\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}\theta.$$

Clearly, the Sender will only be happy to communicate θ truthfully if the Receiver does exactly what the Sender would want to do, so

$$\mathbb{E}\left[\omega|\,\theta\right] = \theta \text{ for all } \theta,$$

which requires that

$$Cov\left(\omega,\theta\right) = Var\left(\theta\right), \tag{4}$$

corresponding to exactly the case where the Sender and the Receiver are equally conservative. The following theorem states this result formally and relates equation (4) to the underlying noise structure:

Theorem 1 *Under Assumptions 1 and 2, there exist information structures, defined by* $\frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2_\omega}$ *and* $\frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2_\eta}$*, such that* $Cov\left(\omega,\theta\right) = Var\left(\theta\right)$ *so that a perfectly informative equilibrium in conditional expectations* θ *exists. More specifically, for* $\frac{\sigma_\omega}{\sigma_\eta} \in \left(\rho_{\omega\eta}, \frac{1}{\rho_{\omega\eta}}\right]$, $Cov\left(\omega,\theta\right) = Var\left(\theta\right)$ *if and only if* $\frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2_\omega}$ *and* $\frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2_\eta}$ *satisfy*

$$\frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2_\eta} = \frac{\left(1-\frac{\sigma_\omega}{\sigma_\eta}\rho_{\omega\eta}\right)}{\rho_{\omega\eta}\left(\frac{\sigma_\omega}{\sigma_\eta}-\rho_{\omega\eta}\right)}\frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2_\omega} + \frac{\left(1-\frac{\sigma_\omega}{\sigma_\eta}\rho_{\omega\eta}\right)}{\rho_{\omega\eta}\left(\frac{\sigma_\omega}{\sigma_\eta}-\rho_{\omega\eta}\right)}\left(1-\rho^2_{\eta\omega}\right). \tag{5}$$

For $\frac{\sigma_\omega}{\sigma_\eta} = \rho_{\omega\eta}$*, we have* $Cov\left(\omega,\theta\right) = Var\left(\theta\right)$ *iff* $\frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2_\omega} = 0$ *and only signal* s_ω *is available (corresponding to the limiting case with two signals where* $\sigma^2_{\varepsilon_\eta} \to \infty$*).*

Our previous discussion makes the proof of the theorem trivial: equation (5) arises from plugging α^S and β^S into (4) and solving. For $\frac{\sigma_\omega}{\sigma_\eta} \in \left(\rho_{\omega\eta}, \frac{1}{\rho_{\omega\eta}}\right)$, (5) describes a linear locus with positive intercept and slope in the signal-noise ratio space. In the extreme case where $\frac{\sigma_\omega}{\sigma_\eta} = \frac{1}{\rho_{\omega\eta}}$, the locus coincides with the horizontal axis: since the Sender is only interested in η, he does not use signal s_ω, so the noise ratio $\frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2_\omega}$ becomes irrelevant. In the other extreme case where $\frac{\sigma_\omega}{\sigma_\eta} = \rho_{\omega\eta}$, the Sender needs to observe ω without noise and to obtain no information about η to ensure $Cov\left(\omega,\theta\right) = Var\left(\theta\right)$.

Perfect communication about conditional expectations is possible provided that the Sender's and the Receiver's preferences are perfectly aligned in that subspace of the Sender's information. In turn, interests are perfectly aligned in that subspace if the regression of

ω on θ has a slope of unity: even though she would want to, the Receiver cannot extract more than θ from what she is told. Notice that there is still bunching of Sender types; the Sender makes garbled statements about the underlying information. However, in contrast to partitional equilibria, the bunches are very easy to characterize. The surprising element is that smooth communication strategies can be part of the equilibrium even though the players disagree, which is clearly the case for $\frac{\sigma_\omega}{\sigma_\eta} \in \left(\rho_{\omega\eta}, \frac{1}{\rho_{\omega\eta}}\right)$.

4.2.2 Partitional equilibria

Consider now experiments for which $Cov(\omega, \theta) \neq Var(\theta)$. For these experiments, there exists no equilibrium involving invertible message strategies in θ. We make the following assumptions:

Assumption 3: $u^R(x, \omega) = -(x-\omega)^2$ and $u^S(x, \omega) = -(x-\eta)^2$.

Assumption 4: The distribution of θ has a log-concave density.

Assumption 3 is the leading example of symmetric loss functions that is adopted throughout the literature. The leading class of distributions that is both elliptical and has a log-concave density is the normal distribution. These assumptions are convenient to study partitional equilibria. We can characterize such equilibria with a countable number of messages m_i for $i = 1, \ldots, n$. Under Assumption 3, if Sender types $\theta \in (\theta_{i-1}, \theta_i]$ pool on message m_i, then the Receiver optimally responds with the action

$$x_i^* = \mathbb{E}\left[\omega \mid \theta \in (\theta_{i-1}, \theta_i]\right].$$

The reason is that under Assumption 3, marginal utility becomes linear. Since preferences satisfy the single crossing condition, we can characterize the optimal Sender responses by the marginal types θ_i who are indifferent between sending message m_i and message m_{i+1}. The indifference condition for type θ_i is

$$\theta_i - \mathbb{E}\left[\omega \mid \theta \in (\theta_{i-1}, \theta_i]\right] = \mathbb{E}\left[\omega \mid \theta \in (\theta_i, \theta_{i+1}]\right] - \theta_i.$$

For elliptical distributions, we have the following convenient property:

$$\begin{aligned}
\mathbb{E}\left[\omega \mid \theta \in (\theta_{i-1}, \theta_i]\right] &= \mathbb{E}\left[\mathbb{E}\left[\omega \mid \theta \in (\theta_{i-1}, \theta_i]\right] \mid \theta\right] = \mathbb{E}\left[\mathbb{E}\left[\omega \mid \theta\right] \mid \theta \in (\theta_{i-1}, \theta_i]\right] \\
&= \frac{Cov(\omega, \theta)}{Var(\theta)} \mathbb{E}\left[\theta \mid \theta \in (\theta_{i-1}, \theta_i]\right],
\end{aligned}$$

where the equalities in the first line use the law of iterated expectations and the second line uses the linearity of conditional expectations in the elliptical class. The system of indifference conditions thus simplifies to

$$\theta_i - \frac{Cov(\omega,\theta)}{Var(\theta)}\mathbb{E}[\theta|\,\theta \in (\theta_{i-1},\theta_i]] = \frac{Cov(\omega,\theta)}{Var(\theta)}\mathbb{E}[\theta|\,\theta \in (\theta_i,\theta_{i+1}]] - \theta_i.$$

Log-concavity ensures stability of this system of equations.[7]

As pointed out above and stated formally in Theorem 2 below, the number of induced Receiver actions crucially depends on which player is more conservative. For $\frac{Cov(\omega,\theta)}{Var(\theta)} < 1$, the Receiver is more conservative than the Sender. For $\frac{Cov(\omega,\theta)}{Var(\theta)} > 1$, the Sender is more conservative than the Receiver.

Theorem 2 *Under Assumptions 1 - 4, we have:*
i) for any $\frac{Cov(\omega,\theta)}{Var(\theta)}$*, there always exists an equilibrium inducing two distinct actions,*
ii) for $\frac{Cov(\omega,\theta)}{Var(\theta)} < 1$*, there is no upper bound on the number of induced actions,*
iii) for $\frac{Cov(\omega,\theta)}{Var(\theta)} \geq 2$ *the maximum number of induced actions in equilibrium is* $n = 2$*.*

It is always an equilibrium if the Sender says either "up" or "down" and the Receiver responds by taking an action equal to the upward and downward truncated means, respectively, where the point of truncation is the prior mean. This is simply due to the symmetry of the distributions within the elliptical class and the fact that the Sender and the Receiver agree based on prior information. This is in stark contrast to models with a unidirectional bias between the Sender and the Receiver, where communication completely breaks down if the bias becomes too large.

[7] It is well known for distributions with log-concave densities that $\frac{\partial}{\partial\theta_{i-1}}\mathbb{E}[\theta|\,\theta \in (\theta_{i-1},\theta_i]] \leq 1$ and $\frac{\partial}{\partial\theta_i}\mathbb{E}[\theta|\,\theta \in (\theta_{i-1},\theta_i]] \leq 1$. Moreover, (as shown, e.g. by Szalay (2012))

$$\frac{\partial}{\partial\theta_{i-1}}\mathbb{E}[\theta|\,\theta \in (\theta_{i-1},\theta_i]] + \frac{\partial}{\partial\theta_i}\mathbb{E}[\theta|\,\theta \in (\theta_{i-1},\theta_i]] \leq 1.$$

Szalay (2012) uses these properties to demonstrate uniqueness of equilibria inducing a given number of induced actions in his model. The details how log-concavity enters into the present model can be found in the proof of Theorem 2.

Communication with a conservative Receiver works pretty well in the sense that there exists an equilibrium inducing countably infinitely many actions. Communication is arbitrarily precise around the prior mean. In contrast, partition elements farther away from the prior mean have some size. The reason is simply that the ideal choice functions of the Sender and the Receiver intersect at the prior mean; moreover, the distance between ideal choices increases linearly in the distance from the prior mean. These effects are in line with results in the literature. Gordon (2010) finds that a countably infinite equilibrium exists in case of an "outward biased" Sender or in other words in case of a conservative Receiver.

The case of a relatively conservative Sender is substantially different. In particular, if the Receiver wishes to react more than twice as much to new information than the Sender wishes, then communication almost breaks down. Intuitively, for information structures where $\frac{Cov(\omega,\theta)}{Var(\theta)} \geq 2$ the Sender's information is considerably more precise about ω than about η. Clearly, in that case θ contains primarily information about ω. Therefore, the Sender is reluctant to give precise information about θ.

We have tried but were unable to get a complete characterization of finite versus infinite equilibria. The reason is that the case $\frac{Cov(\omega,\theta)}{Var(\theta)} > 1$ counteracts the assumptions that give regularity to the problem (log-concavity). For the same reason, we have not been able to show that the number of induced actions in any finite equilibrium is decreasing in $\frac{Cov(\omega,\theta)}{Var(\theta)}$; however, this is a reasonable conjecture. We pursue these questions in ongoing companion work.

5 Choosing information structures

In this section, we characterize optimal information structures in various settings. To obtain such characterizations, we continue to invoke Assumption 3, i.e., assume that both the Sender and the Receiver have quadratic loss functions. Exploiting that assumption, we begin by deriving explicit expressions for both players' equilibrium expected utilities. Then, we analyze the cases where each player is interested in her decision payoff only and either the Sender or the Receiver has the right to choose the information structure. This corresponds to the perspectives taken in the persuasion and disclosure literatures (if the Receiver chooses the information structure) and the literature on information acquisition (if the Sender chooses the information structure). Then, we allow for ex ante payments between the Sender and the Receiver. More specifically, the Receiver hires a Sender and needs to convince the Sender to participate by an ex ante payment to the Sender. Moreover, the Receiver can commit to an information structure, again the perspective taken in the persuasion literature. Finally, we look at the case where the Receiver can only select the type of Sender, where a Sender's type is described by the joint distribution of ω and η. Subsequently, it is the Sender who has the right to choose the information structure. Perhaps surprisingly, under the latter two extensive forms, optimal information structures are such that they enable smooth communication.

5.1 Equilibrium payoffs

In an equilibrium where an uncountable infinity of messages is sent, $m(\theta) = \theta$ for all θ, the Receiver's optimal decision schedule is $x^*(\theta) = \theta$ and her expected utility for the quadratic loss case simplifies to

$$\begin{aligned} \int \int u^R(x^* - \omega) f(\omega|\theta) d\omega f(\theta) d\theta &= -Var(\theta) + 2Cov(\theta, \omega) - Var(\omega) \\ &= Cov(\theta, \omega) - Var(\omega), \end{aligned}$$

where the second equality follows from the fact that $Cov(\theta, \omega) = Var(\theta)$ in an equilibrium where $x^*(\theta) = \theta$. If an equilibrium with a countable number of n messages m_i for $i = 1, \dots, n$ is played, types $\theta \in \Theta_i \equiv [\theta_{i-1}, \theta_i]$ pool on message m_i and thereby induce an optimal Receiver action $x_i^* = \mathbb{E}[\omega|\theta \in \Theta_i]$. The Receiver's expected payoff in such an equilibrium is

$$\begin{aligned} &\sum_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \int u^R(x_i^* - \omega) f(\omega|\theta) d\omega f(\theta) d\theta \\ = \; &\frac{Cov(\theta, \omega)^2}{Var(\theta)^2} \left(Var(\theta) - \sum_{i=1}^{n} p_i Var(\theta | \theta \in \Theta_i) \right) - Var(\omega). \end{aligned}$$

where $p_i \equiv \Pr(\theta \in \Theta_i)$. Note that the two expressions have the common representation

$$\frac{Cov(\theta, \omega)^2}{Var(\theta)^2} \left(Var(\theta) - \mathbb{I}_p \sum_{i=1}^{n} p_i Var(\theta | \theta \in \Theta_i) \right) - Var(\omega), \tag{6}$$

where $\mathbb{I}_p$ is an indicator variable taking value one if the equilibrium involves a countable number of actions and zero otherwise. Moreover, a smooth communication equilibrium exists only if $\frac{Cov(\theta,\omega)}{Var(\theta)} = 1$. Thus, we can think of the Receiver's expected utility as of an upper bound - the expected utility when θ is communicated smoothly - minus an expected loss term that arises when and only when information is lost through partitional communication.

Likewise, the Sender's expected utility in any equilibrium inducing $x^*(\theta) = \theta$ is

$$\begin{aligned} \int \int u^S (x^* - \eta) f(\eta | \theta) d\eta f(\theta) d\theta &= -Var(\theta) + 2Cov(\eta, \theta) - Var(\eta) \\ &= Var(\theta) - Var(\eta), \end{aligned}$$

where the second equality follows from the fact that $\mathbb{E}[\eta | \theta] = \theta$ (which is equivalent to $Cov(\eta, \theta) = Var(\theta)$). The Sender's expected utility in an equilibrium that induces a countable number of actions $x_i^* = \mathbb{E}[\omega | \theta \in \Theta_i]$ is

$$\begin{aligned} &\sum_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \int u^S (x_i^* - \eta) f(\eta | \theta) d\eta f(\theta) d\theta \\ = &\left(2\frac{Cov(\omega, \theta)}{Var(\theta)} - \frac{Cov(\omega, \theta)^2}{Var(\theta)^2}\right) \left(Var(\theta) - \sum_{i=1}^{n} p_i Var(\theta | \theta \in \Theta_i)\right) - Var(\eta). \end{aligned}$$

Thus, the Sender's expected utility in any equilibrium can be written conveniently as

$$\left(2\frac{Cov(\omega, \theta)}{Var(\theta)} - \frac{Cov(\omega, \theta)^2}{Var(\theta)^2}\right) \left(Var(\theta) - \mathbb{I}_p \sum_{i=1}^{n} p_i Var(\theta | \theta \in \Theta_i)\right) - Var(\eta). \tag{7}$$

Again, we can decompose expected utility into the upper bound, which is reached if communication is smooth in θ, and expected losses arising when communication is non-smooth. For convenience of the reader, the elementary calculations behind these simplifications are gathered in Lemma 8 in the Appendix.

We are now ready to discuss optimal information structures.

5.2 Privately optimal information structures

Suppose first the Receiver has the right to determine the information structure. This is the perspective taken in the disclosure literature, where a principal chooses the information that some agent obtains without the principal getting to see the realizations of the agent's signals. In the same spirit, suppose the Receiver gets to choose the information structure, but the Sender alone observes the realizations of signals.

An optimal information structure from the Receiver's perspective solves the problem

$$\max_{Cov(\omega,\theta),Var(\theta)} (6) \tag{P$_R$}$$
$$s.t.\ Cov(\omega,\theta), Var(\theta) \text{ feasible.}$$

A feasible information structure is one that is generated by some given noise-signal ratios.

Likewise, suppose the Sender has the right to choose the information structure. That is the perspective taken in the literature on information acquisition. An optimal information structure from his perspective solves the problem

$$\max_{Cov(\omega,\theta),Var(\theta)} (7) \tag{P$_S$}$$
$$s.t.\ Cov(\omega,\theta), Var(\theta) \text{ feasible.}$$

In line with the tradition in the strategic communication literature, we assume that the most informative equilibrium is played in each continuation game induced by a given information structure. Problems (P_R) and (P_S) are both pretty involved. The reason is as follows. While the upper bound depends on the information structure in straightforward fashion, the expected losses due to partitional communication depend in very intricate ways on the information structure. More specifically, the information structure influences the equilibrium partition, the distribution of θ within any given partition element, and the probability distribution over the partition elements Θ_i for $i = 1, \ldots, n$. For this reason, we defer a complete characterization of solutions to these problems to a separate paper. Here, we contend ourselves in establishing qualitative properties of the solutions. An information structure is said to be optimal from the Receiver's or the Sender's perspective if it solves problem (P_R) or (P_S), respectively.

Theorem 3 *Under Assumptions 1 - 3, an information structure is optimal from the Receiver's (Sender's) perspective only if it makes her (him) relatively progressive, that is if* $Cov(\omega,\theta) \geq (\leq) Var(\theta)$.

Put differently, an information structure cannot be optimal for the Receiver if it makes her look relatively more conservative than the Sender. The formal proof works as follows. Consider an information structure one, $(Cov_1(\omega,\theta), Var_1(\theta))$, satisfying $Cov_1(\omega,\theta) < Var_1(\theta)$.

Such an information structure is dominated from the Receiver's perspective. In particular, it is true that for any such information structure, there exists another information structure two, $(Cov_2(\omega,\theta), Var_2(\theta))$ say, such that $Var_2(\theta) = Var_1(\theta)$ and $Cov_2(\omega,\theta) = Var_2(\theta)$, so that the Receiver's expected payoff under the latter information structure is strictly higher.

Intuitively, suppose the Receiver reacts strictly conservatively to the Sender's advice. Then, there are two sources of gains when the information structure is changed to one where the Sender and the Receiver perfectly agree on how θ is mapped into actions. For the changed information structure, there is no more loss due to partitional communication, because the upper bound is tight. Moreover, the new information structure can always be generated from the former one by increasing $Cov(\omega,\theta)$ while leaving $Var(\theta)$ unchanged. However, such a change induces a further increase in the upper bound under smooth communication, which further boosts the Receiver's utility. The case where the Sender's perspective is taken corresponds simply to the mirror image of the Receiver's problem, except that in considering the alternative information structure, we leave the covariance unchanged and change the variance instead.

The conclusion is very natural: the one who controls the information structure is in equilibrium the one who is more enthusiastic when it comes to using the information, simply because she/he chooses information that is relatively more useful to her/him. To see this, recall again that each of them ultimately cares about how useful this piece of information is with respect to their underlying motive, that is ω or η, respectively. Since θ is defined as the conditional expected value of η given the available information, the conditional expectation of η given θ is identically equal to θ. Hence, $Var(\theta) = Cov(\eta,\theta)$. Thus, the Receiver prefers information structures where $Cov(\omega,\theta) \geq Cov(\eta,\theta)$, because these information structures contain relatively more information about the Receiver's bliss point rather than the Sender's bliss point. Vice versa, the Sender prefers information structures where θ correlates better with η rather than with ω.

5.3 Joint Surplus-optimal information structures

Consider now a situation where the Sender can opt out, in which case the Receiver has to choose the action based on prior information alone. Suppose that the Receiver can pay the Sender ex ante to convince him to participate. However, payments cannot depend on the advice nor on outcomes (nor on both advice and outcomes). Suppose further that both the Sender and the Receiver are risk neutral with respect to money payments. Formally, the Receiver's payoff is now

$$\frac{Cov\left(\theta,\omega\right)^2}{Var\left(\theta\right)^2}\left(Var\left(\theta\right)-\mathbb{I}_p\sum\nolimits_{i=1}^{n}p_iVar\left(\theta\left|\theta\in\Theta_i\right.\right)\right)-Var\left(\omega\right)-t,$$

while the Sender's payoff is

$$\left(2\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}-\frac{Cov\left(\omega,\theta\right)^2}{Var\left(\theta\right)^2}\right)\left(Var\left(\theta\right)-\mathbb{I}_p\sum\nolimits_{i=1}^{n}p_iVar\left(\theta\left|\theta\in\Theta_i\right.\right)\right)-Var\left(\eta\right)+t.$$

If the Sender cares only about the decision when he participates - say, because only then he puts skin into the game - then he will indeed participate if the latter expression is at least equal to zero. If the Sender cares about the decision in any case, then his equilibrium payoff needs to be at least equal to $-Var\left(\eta\right)$. Since expected utilities in the two situations differ only by a constant, it does not matter which way we go, so assume the former situation for the sake of the argument. Clearly, the Receiver chooses the smallest payment that convinces the Sender to participate. As a result, the Receiver effectively cares about the sum of Receiver and Sender expected utility from decision making, that is the sum of (6) and (7). Performing this summation, and simplifying we obtain the Receiver's problem:

$$\max_{Cov(\omega,\theta),Var(\theta)} 2Cov\left(\omega,\theta\right)-2\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}\mathbb{I}_p\sum\nolimits_{i=1}^{n}p_iVar\left(\theta\left|\theta\in\Theta_i\right.\right)-Var\left(\omega\right)-Var\left(\eta\right) \tag{P$_J$}$$

$$s.t.\ Cov\left(\omega,\theta\right),Var\left(\theta\right)\ \text{feasible}.$$

Very conveniently, joint expected utility adds up to the sum of the maximally feasible individual expected utilities under smooth communication, $2Cov\left(\omega,\theta\right)$, minus an expected loss term arising from partitional communication. The following theorem is now immediate:

Theorem 4 *Under Assumptions 1 - 3, the unique solution to the problem of maximizing joint expected payoff with respect to the information structure, satisfies* $Var\left(\theta\right)=Cov\left(\theta,\omega\right)=Cov\left(\eta,\omega\right)$. *Equivalently, for* $\frac{\sigma_\omega}{\sigma_\eta}\in\left(\rho_{\omega\eta},\frac{1}{\rho_{\omega\eta}}\right]$, *in terms of the underlying noise variances,* *the solution is*

$$\left(\sigma^2_{\varepsilon_\omega}, \sigma^2_{\varepsilon_\eta}\right) = \left(0, \frac{\left(1 - \frac{\sigma_\omega}{\sigma_\eta}\rho_{\omega\eta}\right)}{\rho_{\omega\eta}\left(\frac{\sigma_\omega}{\sigma_\eta} - \rho_{\omega\eta}\right)}\left(1-\rho^2_{\omega\eta}\right)\sigma^2_\eta\right).$$

The proof is very simple, so we keep the discussion heuristic. Since $\sum_{i=1}^n p_i Var\left(\theta \left| \theta \in \Theta_i\right.\right) \geq 0$, a smooth communication equilibrium - where $\mathbb{I}_p = 0$ - is preferable to a partitional equilibrium, provided that the highest $Cov\left(\omega, \theta\right)$ is reached in a smooth communication equilibrium. Indeed, the highest $Cov\left(\theta, \omega\right)$ is equal to $Cov\left(\eta, \omega\right)$, the covariance between the underlying motives.

Intuitively, the motives we have identified in the discussion of privately optimal information structures exactly offset each other. The Receiver prefers situations in which the information structure is relatively more useful to her - that is, where $Cov\left(\theta, \omega\right) \geq Cov\left(\theta, \eta\right)$ - while the Sender prefers situations where the opposite is true - that is, where $Cov\left(\theta, \eta\right) \geq Cov\left(\theta, \omega\right)$. Since the Sender and the Receiver have the same loss functions, these forces exactly offset each other. Now, once attention is restricted to information structures that are equally useful to both of them, so $Cov\left(\theta, \omega\right) = Cov\left(\theta, \eta\right)$, the Sender and the Receiver have completely aligned interests with respect to the quality of information. They both want to improve the informational content of the information structure. To see this, note that for information structures where $\frac{Cov(\omega,\theta)}{Var(\theta)} = 1$, the Receiver's expected utility gross of payments is $Cov\left(\theta, \omega\right) - Var\left(\omega\right)$, while the Sender's expected utility gross of payments is $Cov\left(\theta, \omega\right) - Var\left(\eta\right)$, so both expressions are monotonically increasing in $Cov\left(\theta, \omega\right)$.

5.4 Selecting the advisor

Suppose the Sender has the right to choose the information structure. This is the case if we think of the Sender as conducting investigations by himself which likely implies that he also influences the precision of his information. However, the Receiver can select a Sender out of the set defined by Assumptions 1 and 2. More specifically, take the Receiver's prior uncertainty, σ_ω, and any $\rho_{\omega\eta}$ satisfying Assumption 1 as given and think of the set of available Sender types, σ_η, as the ones satisfying Assumption 2. For any given σ_ω and $\rho_{\omega\eta}$, the set of available Sender types thus forms a closed set $\sigma_\eta \in \left[\sigma_\omega\rho_{\omega\eta}, \frac{\sigma_\omega}{\rho_{\omega\eta}}\right]$, with type $\sigma_\eta = \frac{\sigma_\omega}{\rho_{\omega\eta}}$ being the most uncertain Sender type and type $\sigma_\eta = \sigma_\omega\rho_{\omega\eta}$ being the Sender type facing the smallest uncertainty ex ante. Once selected, the Sender chooses an optimal information structure, that is one that solves problem (P_S). Finally, the Sender and the Receiver communicate in the most informative way for the chosen conflict and the chosen information structure. The equilibrium outcome in this extensive form has the following structure:

Theorem 5 *Under Assumptions 1 - 3, the Receiver can implement the optimal outcome from her perspective by choosing the most certain advisor,* $\sigma_\eta^* = \sigma_\omega \rho_{\omega\eta}$*, who then chooses to observe* η *without noise,* $\sigma_{\varepsilon_\eta} = 0$*. In the most informative equilibrium, communication is smooth with* $Var\left(\theta; \sigma_\eta^*\right) = Cov\left(\theta, \omega; \sigma_\eta^*\right) = Cov\left(\eta, \omega; \sigma_\eta^*\right) = \sigma_\omega^2 \rho_{\omega\eta}^2$*.*

The theorem follows straightforwardly from our previous results, so we state the proof heuristically. Recall that privately optimal information structures are such that each player would like to end up being relatively progressive. From our discussion of feasible information structures, recall that the feasible set for any type $\sigma_\eta > \sigma_\omega \rho_{\omega\eta}$ contains elements with $Cov\left(\theta, \omega\right) > Var\left(\theta\right)$. Eliminating such information structures never hurts the Receiver. In contrast, by choosing type $\sigma_\eta = \sigma_\omega \rho_{\omega\eta}$, the Receiver effectively chooses to be in case two of Figure 1. Thereby, she ensures that the optimal information structures from that Sender's perspective within his feasible set satisfies $Var\left(\theta\right) = Cov\left(\theta, \omega\right)$ and thus the necessary condition for optimality from the Receiver's perspective from Theorem 3. Indeed, this type can observe everything of interest to him without noise and can still communicate smoothly with the Receiver. Note that the Sender of type $\sigma_\eta = \sigma_\omega \rho_{\omega\eta}$ could also choose information structures that would not enable the players to communicate smoothly. However, the point is that the Sender has no interest to do so. By choosing the appropriate advisor, the Receiver eliminates conflicts of interests with respect to the choice of information structure.

Note that there is no way the Receiver could do better. Any other Sender type would choose an information structure that would make the Receiver relatively conservative, $\frac{Cov(\theta,\omega)}{Var(\theta)} \leq 1$, possibly strictly so. Moreover, there is no way to make the equilibrium choices more variable than by having the most certain advisor.

6 Comparative statics of smooth communication equilibria

We have argued that smooth communication equilibria have some practical relevance, because they are induced by information structures that reflect optima of certain problems. Moreover, smooth communication equilibria are analytically extremely tractable, allowing us to address questions such as: How does the quality of advice and of decision-making depend on conflicts of interests? How does prior uncertainty impact on decision-making? While these questions are far from trivial in other approaches we are familiar with, our model gives very clear and simple answers.

The equilibrium quality of advice is measured by $Var(\theta)$ in a smooth communication equilibrium. The higher the variance of induced choices, the better off are both the Sender and the Receiver. In the most informative equilibrium for the optimal information structure, we have

$$Var(\theta) = Cov(\omega, \eta) = \rho_{\omega\eta}\sigma_\omega\sigma_\eta.$$

For the case of equal prior uncertainty, $\sigma_\omega = \sigma_\eta$, the alignment of interests can simply be measured by the correlation between ω and η, $\rho_{\omega\eta}$. Both the Receiver and the Sender are better off if interests are better aligned, that is when $\rho_{\omega\eta}$ is increased. The reason is that a Sender with better aligned preferences can be given access to better information. Therefore, an advisor who is more trustworthy will appear more competent: his equilibrium information is of better quality. The limiting case is perfectly aligned interests, which is of course ruled out by Assumption 1.

Similarly, if the Receiver manages to select the ideal advisor among those satisfying Assumptions 1 and 2, then

$$Var(\theta) = \sigma_\omega^2\rho_{\omega\eta}^2,$$

supporting the same conclusion. The case where $\rho_{\omega\eta} \to 1$ corresponds to the ideal world where the best available advisor can be completely trusted. In that limiting case, the Receiver manages to get perfect information through the Sender.

It is interesting to note that for a given correlation $\rho_{\omega\eta}$, higher prior uncertainty has a positive impact on the quality of decision-making. The reason is that with a high level of prior uncertainty, the weight attached to new information tends to be higher. Since the variance of ideal choices based on prior information only is zero, placing more weight on new information unambiguously increases the variance.

7 Conclusions

We analyze a Sender-Receiver-Game of strategic communication in which the most informative equilibrium involves smooth communication and is therefore extremely tractable. The key ingredients are as follows. The Sender and the Receiver are uncertain about but agreed upon their most preferred action ex ante; the Sender observes a two-dimensional signal about the underlying two-dimensional state of nature; while the Sender and the Receiver agree that both pieces of information are useful to reach a decision, they disagree with respect to the weight they would ideally attach to each piece of information. Relative, in contrast to absolute disagreement leaves room for perfectly informative communication about a statistic of the Sender's information, his conditionally expected mean. Since his preferences for given information depend only on conditional means, it is impossible to extract more information from the Sender. Thus, we obtain the standard partial pooling result of cheap talk equilibria; however, pooling is much easier to characterize in the most efficient equilibrium of our game, because only Sender types with the same conditional expectation pool and so we obtain communication that is perfectly revealing in that space.

The upshot of our theory is that with relative (dis-)agreement, it becomes possible to talk about conclusions, while it still remains pointless to ask the Sender to justify his conclusions. E.g., communication by central banks about their anticipated inflation and things the like is much easier than to discuss how they reached this conclusion. Finally, the equilibrium is the more informative the better interests are aligned, with a slightly different twist than other theories predict. The Sender is endogenously endowed with better information when his interests are better aligned with those of the Receiver. Thus, more trustworthy Senders are better informed and therefore give more accurate advice.

8 Appendix

Conditioning for elliptical distributions

The expectation and covariance matrix are as follows

$$E[\omega, \eta, s_\omega, s_\eta] = (0, 0, 0, 0)'$$

and

$$Cov(\omega, \eta, s_\omega, s_\eta) = \begin{pmatrix} \sigma_\omega^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\omega^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta \\ \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\eta^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\eta^2 \\ \sigma_\omega^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta \\ \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\eta^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2 \end{pmatrix}.$$

By Lemma 1 the conditional variances can be calculated via

$$E[\mathbf{X} | \mathbf{Y}] = \mu_X + \Sigma_{12}\Sigma_{22}^{-1}(\mathbf{Y} - \mu_Y)$$

with

$$Cov(X, Y) = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}.$$

Proof of Lemma 2. Let (s_ω, s_η) be the information available and let $z = \omega, \eta$. Consider the problem

$$\max_x \int_{-\infty}^{\infty} u(x - z) f(z | s_\omega, s_\eta) \, dz.$$

Since the utility depends only on the distance between x and z we have $u'(x - z) > 0$ for $z < x$, $u'(x - z) = 0$ for $x = z$, and $u'(x - z) < 0$ for $z > x$.

Consider the candidate solution $x^* = \mu \equiv \mathbb{E}[z | s_\omega, s_\eta]$. The first-order condition can be written as

$$\int_{-\infty}^{\infty} u'(x^* - z) f(z | s_\omega, s_\eta) \, dz = \int_{-\infty}^{\infty} u'(\mu - z) f(z | s_\omega, s_\eta) \, dz = 0.$$

Consider two points $z_1 = \mu - \Delta$ and $z_2 = \mu + \Delta$ for arbitrary $\Delta > 0$. By symmetry of u around its bliss point and symmetry of the distribution around μ, we have

$$u'(\Delta) f(\mu - \Delta | s_\omega, s_\eta) = -u'(-\Delta) f(\mu + \Delta | s_\omega, s_\eta).$$

Since this holds point-wise for each Δ, it also holds if we integrate over Δ. Thus, the first-order condition applies at $x^* = \mu$. By concavity of u in x, only one value of x solves the first-order condition. Hence, the solution is the one stated in the Lemma. ■

Proof of Lemma 3. The conditional expectations are

$$\mathbb{E}\left[\eta | s_\omega, s_\eta\right] = \alpha^S s_\omega + \beta^S s_\eta \tag{8}$$

and

$$\mathbb{E}\left[\omega | \, s_\omega, s_\eta\right] = \alpha^R s_\omega + \beta^R s_\eta, \tag{9}$$

where $\alpha^S, \beta^S, \alpha^R, \beta^R$ are defined in the text. Suppose $\sigma^2_{\varepsilon_\eta}$ and $\sigma^2_{\varepsilon_\omega}$ are both positive and finite. Equations (8) and (9) are identical for all s_ω and s_η if and only if

$$\sigma^2_{\varepsilon_\eta} \rho_{\omega\eta} \sigma_\omega \sigma_\eta = \sigma^2_\omega \left(\sigma^2_{\varepsilon_\eta} + \sigma^2_\eta - \sigma^2_\eta \rho^2_{\omega\eta} \right)$$

and

$$\sigma^2_\eta \left(\sigma^2_{\varepsilon_\omega} - \sigma^2_\omega \rho^2_{\omega\eta} + \sigma^2_\omega \right) = \sigma_\eta \sigma_\omega \rho_{\omega\eta} \sigma^2_{\varepsilon_\omega}.$$

This requires that

$$\sigma^2_\eta \left(1 - \rho^2_{\omega\eta}\right) = \left(\frac{\rho_{\omega\eta} \sigma_\eta}{\sigma_\omega} - 1 \right) \sigma^2_{\varepsilon_\eta}$$

and

$$\sigma^2_\omega \left(1 - \rho^2_{\omega\eta}\right) = \left(\frac{\sigma_\omega \rho_{\omega\eta}}{\sigma_\eta} - 1 \right) \sigma^2_{\varepsilon_\omega}.$$

A necessary condition for these two conditions to apply is that $\rho_{\omega\eta} \geq \frac{\sigma_\omega}{\sigma_\eta}$ and $\rho_{\omega\eta} \geq \frac{\sigma_\eta}{\sigma_\omega}$. Since $\rho_{\omega\eta} \in [-1, 1]$, this implies that $\rho_{\omega\eta} = \frac{\sigma_\eta}{\sigma_\omega} = \frac{\sigma_\omega}{\sigma_\eta}$ and therefore, $\rho^2_{\omega\eta} = 1$ and $\sigma_\eta = \sigma_\omega$.

Consider now the limiting cases where one of the variances goes out of bounds. Applying l'Hôpital's rule to (8) and (9), we get in the limit as $\sigma^2_{\varepsilon_\eta} \to \infty$

$$\mathbb{E}\left[\omega | s_\omega\right] = \frac{\sigma^2_\omega}{\sigma^2_\omega + \sigma^2_{\varepsilon_\omega}} s_\omega \qquad \text{and} \qquad \mathbb{E}\left[\eta | s_\omega\right] = \frac{\rho_{\omega\eta} \sigma_\omega \sigma_\eta}{\sigma^2_\omega + \sigma^2_{\varepsilon_\omega}} s_\omega,$$

so that

$$\mathbb{E}\left[\omega | s_\omega\right] \equiv \mathbb{E}\left[\eta | s_\omega\right] \qquad \Leftrightarrow \qquad \rho_{\omega\eta} \sigma_\eta = \sigma_\omega.$$

Likewise, for the case where $\sigma^2_{\varepsilon_\omega} \to \infty$, we get

$$\mathbb{E}\left[\omega|s_\eta\right] = \frac{\rho_{\omega\eta}\sigma_\omega\sigma_\eta}{\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}} s_\eta \qquad \text{and} \qquad \mathbb{E}\left[\eta|s_\eta\right] = \frac{\sigma^2_\eta}{\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}} s_\eta,$$

so

$$\mathbb{E}\left[\omega|s_\eta\right] \equiv \mathbb{E}\left[\eta|s_\eta\right] \qquad \Leftrightarrow \qquad \rho_{\omega\eta}\sigma_\omega = \sigma_\eta.$$

■

Proof of Lemma 4. Straightforward manipulations of $\alpha^R, \beta^R, \alpha^S, \beta^S$ show that

$$\alpha^R \geq \alpha^S \Leftrightarrow \sigma^2_\omega \left(\sigma^2_\eta \left(1 - \rho^2_{\omega\eta}\right)\right) \geq \sigma^2_{\varepsilon_\eta}\sigma_\omega \left(\rho_{\omega\eta}\sigma_\eta - \sigma_\omega\right)$$

which is true for arbitrary $\sigma^2_{\varepsilon_\eta}$ if and only if $\frac{\sigma_\omega}{\sigma_\eta} \geq \rho_{\omega\eta}$; and likewise,

$$\beta^S \geq \beta^R \Leftrightarrow \sigma^2_\eta\sigma^2_\omega \left(1 - \rho^2_{\omega\eta}\right) \geq \sigma^2_{\varepsilon_\omega}\sigma_\eta \left(\sigma_\omega\rho_{\omega\eta} - \sigma_\eta\right)$$

which again is true for arbitrary $\sigma^2_{\varepsilon_\omega}$ if and only if $\frac{\sigma_\omega}{\sigma_\eta} \leq \frac{1}{\rho_{\omega\eta}}$. ■

Proof of Lemma 6. Let $x' - \mathbb{E}\left[\eta|\, s_\omega, s_\eta\right] = \mathbb{E}\left[\eta|\, s_\omega, s_\eta\right] - x'' \equiv z > 0$, then

$$\int u\left(x' - \eta\right) f\left(\eta|\, s_\omega, s_\eta\right) d\eta = \int u\left(z - \left(\eta - \mathbb{E}\left[\eta|\, s_\omega, s_\eta\right]\right)\right) f\left(\eta|\, s_\omega, s_\eta\right) d\eta.$$

The random variable $\hat{\eta} \equiv \eta - \mathbb{E}\left[\eta|\, s_\omega, s_\eta\right]$ has mean zero and symmetric distribution. Let $\hat{f}\left(\hat{\eta}|\, s_\omega, s_\eta\right)$ denote the standardized distribution (which has mean zero), then we have

$$f\left(\eta|\, s_\omega, s_\eta\right) = \hat{f}\left(\eta - \mathbb{E}\left[\eta|\, s_\omega, s_\eta\right]|\, s_\omega, s_\eta\right) = \hat{f}\left(\hat{\eta}|\, s_\omega, s_\eta\right) \text{ and } d\eta = d\hat{\eta}.$$

Take two realizations $\hat{\eta}', e\hat{t}a''$ of $\hat{\eta}$. By symmetry of u around 0, if $\hat{\eta}' > 0$ and $\hat{\eta}'' = -\hat{\eta}'$, then

$$u\left(z - \hat{\eta}'\right) = u\left(-z - \hat{\eta}''\right),$$

and symmetry of the distribution gives

$$\hat{f}\left(\hat{\eta}'|\, s_\omega, s_\eta\right) = \hat{f}\left(\hat{\eta}''|\, s_\omega, s_\eta\right).$$

Therefore,

$$u\left(z - \hat{\eta}'\right) \hat{f}\left(\hat{\eta}'|\, s_\omega, s_\eta\right) = u\left(-z - \hat{\eta}''\right) \hat{f}\left(\hat{\eta}''|\, s_\omega, s_\eta\right)$$

holds true over the entire support for all pairs of $(\hat{\eta}', -\hat{\eta}')$, implying that

$$\begin{aligned}\int u\left(z-\hat{\eta}\right)\hat{f}\left(\hat{\eta}\right|s_{\omega},s_{\eta}\right)d\hat{\eta} &= \int u\left(-z-\hat{\eta}\right)\hat{f}\left(\hat{\eta}\right|s_{\omega},s_{\eta}\right)d\hat{\eta}\\ &= \int u\left(-z-\left(\eta-\mathbb{E}\left[\eta\right|s_{\omega},s_{\eta}\right]\right)\right)f\left(\eta\right|s_{\omega},s_{\eta}\right)d\eta\\ &= \int u\left(x''-\eta\right)f\left(\eta\right|s_{\omega},s_{\eta}\right)d\eta.\end{aligned}$$

■

Proof of Lemma 7. Recall that

$$Cov\left(\omega,\theta\right)=\sigma_{\eta}\sigma_{\omega}\rho_{\omega\eta}\frac{a+b+1-\rho_{\omega\eta}^{2}}{\left(1+a\right)\left(1+b\right)-\rho_{\omega\eta}^{2}},$$

and

$$Var\left(\theta\right)=\sigma_{\eta}^{2}\frac{a+b\rho_{\omega\eta}^{2}+1-\rho_{\omega\eta}^{2}}{\left(1+a\right)\left(1+b\right)-\rho_{\omega\eta}^{2}},$$

where $a\equiv\frac{\sigma_{\varepsilon_{\omega}}^{2}}{\sigma_{\omega}^{2}}$ and $b\equiv\frac{\sigma_{\varepsilon_{\eta}}^{2}}{\sigma_{\eta}^{2}}$.

For future reference, note that

$$\frac{\partial Cov\left(\omega,\theta\right)}{\partial a}=\sigma_{\eta}\sigma_{\omega}\rho_{\omega\eta}\frac{-b\left(-\rho_{\eta\omega}^{2}+b+1\right)}{\left(\left(1+a\right)\left(1+b\right)-\rho_{\omega\eta}^{2}\right)^{2}},$$

$$\frac{\partial Cov\left(\omega,\theta\right)}{\partial b}=\sigma_{\eta}\sigma_{\omega}\rho_{\omega\eta}\frac{-a\left(-\rho_{\eta\omega}^{2}+a+1\right)}{\left(\left(1+a\right)\left(1+b\right)-\rho_{\omega\eta}^{2}\right)^{2}},$$

$$\frac{\partial Var\left(\theta\right)}{\partial a}=\frac{\partial}{\partial a}\sigma_{\eta}^{2}\frac{a+b\rho_{\omega\eta}^{2}+1-\rho_{\omega\eta}^{2}}{\left(1+a\right)\left(1+b\right)-\rho_{\omega\eta}^{2}}=\frac{-b^{2}\rho_{\eta\omega}^{2}\sigma_{\eta}^{2}}{\left(\left(1+a\right)\left(1+b\right)-\rho_{\omega\eta}^{2}\right)^{2}},$$

and

$$\frac{\partial Var\left(\theta\right)}{\partial b}=\frac{\partial}{\partial b}\sigma_{\eta}^{2}\frac{a+b\rho_{\omega\eta}^{2}+1-\rho_{\omega\eta}^{2}}{\left(1+a\right)\left(1+b\right)-\rho_{\omega\eta}^{2}}=\sigma_{\eta}^{2}\frac{-\left(-\rho_{\eta\omega}^{2}+a+1\right)^{2}}{\left(\left(1+a\right)\left(1+b\right)-\rho_{\omega\eta}^{2}\right)^{2}}.$$

It is most convenient to characterize the set of feasible information structures by taking a level of covariance as given and computing the set of feasible variance levels for the given level of covariance.

Thus, let

$$\sigma_\eta \sigma_\omega \rho_{\omega\eta} \frac{a+b+1-\rho_{\omega\eta}^2}{(1+a)(1+b)-\rho_{\omega\eta}^2} = C.$$

Note that for $a=0$ or $b=0$, the covariance is constant and equal to $\sigma_\eta\sigma_\omega\rho_{\omega\eta}$. Moreover, the covariance is decreasing in a for given b and decreasing in b for given a. We have

$$\lim_{b\to\infty} \frac{a+b+1-\rho_{\omega\eta}^2}{(1+a)(1+b)-\rho_{\omega\eta}^2} = \frac{1}{1+a},$$

and

$$\lim_{a\to\infty} \frac{a+b+1-\rho_{\omega\eta}^2}{(1+a)(1+b)-\rho_{\omega\eta}^2} = \frac{1}{1+b}.$$

So, letting both a and b (in whatever order) go to infinity results in a covariance of zero. Hence, any $C \in (0, \sigma_\eta\sigma_\omega\rho_{\omega\eta}]$ can be generated by finite a, b.

For $a, b \neq 0$, along a locus where the covariance is equal to C, we must have

$$\sigma_\eta\sigma_\omega\rho_{\omega\eta} \frac{-b\left(-\rho_{\eta\omega}^2+b+1\right)}{\left((1+a)(1+b)-\rho_{\omega\eta}^2\right)^2} da + \sigma_\eta\sigma_\omega\rho_{\omega\eta} \frac{-a\left(-\rho_{\eta\omega}^2+a+1\right)}{\left((1+a)(1+b)-\rho_{\omega\eta}^2\right)^2} db = 0$$

so that

$$-\frac{b\left(-\rho_{\eta\omega}^2+b+1\right)}{a\left(-\rho_{\eta\omega}^2+a+1\right)} da = db,$$

and a and b are inversely related. Consider now how the variance changes along a locus where the covariance is constant. Totally differentiating the variance, and substituting for db as a function of da, we obtain

$$\begin{aligned} & \frac{-b^2\rho_{\eta\omega}^2\sigma_\eta^2}{\left((1+a)(1+b)-\rho_{\omega\eta}^2\right)^2} da + \sigma_\eta^2 \frac{-\left(-\rho_{\eta\omega}^2+a+1\right)^2}{\left((1+a)(1+b)-\rho_{\omega\eta}^2\right)^2} db \\ = & \frac{-b^2\rho_{\eta\omega}^2\sigma_\eta^2}{\left((1+a)(1+b)-\rho_{\omega\eta}^2\right)^2} da - \sigma_\eta^2 \frac{-\left(-\rho_{\eta\omega}^2+a+1\right)^2}{\left((1+a)(1+b)-\rho_{\omega\eta}^2\right)^2} \frac{b\left(-\rho_{\eta\omega}^2+b+1\right)}{a\left(-\rho_{\eta\omega}^2+a+1\right)} da. \end{aligned}$$

Note that

$$-b^2\rho_{\eta\omega}^2 + b\left(-\rho_{\eta\omega}^2+b+1\right)\frac{\left(-\rho_{\eta\omega}^2+a+1\right)}{a} \geq 0$$

since

$$-ab\rho_{\eta\omega}^2 + \left(-\rho_{\eta\omega}^2+b+1\right)\left(-\rho_{\eta\omega}^2+a+1\right) > 0.$$

Therefore, the variance is increasing in a and is thus minimal for the smallest possible value of a that generates the desired covariance level and is highest for the highest possible value of a that generates the covariance.

Consider first the smallest level of a. Since a and b are inversely related, this is the level associated to $b \to \infty$.

$$\lim_{b\to\infty} \sigma_\eta \sigma_\omega \rho_{\omega\eta} \frac{a+b+1-\rho_{\omega\eta}^2}{(1+a)(1+b)-\rho_{\omega\eta}^2} = \sigma_\eta \sigma_\omega \rho_{\omega\eta} \frac{1}{1+a}.$$

Note that any covariance smaller than $\sigma_\eta \sigma_\omega \rho_{\omega\eta}$ can be generated this way. a is then determined by the condition

$$\sigma_\eta \sigma_\omega \rho_{\omega\eta} \frac{1}{1+a} = C$$

and so

$$\frac{\sigma_\eta \sigma_\omega \rho_{\omega\eta}}{C} - 1 = a.$$

Let $\underline{V}(C)$ denote the smallest possible value of the variance for a given level of C. Substituting $\frac{\sigma_\eta \sigma_\omega \rho_{\omega\eta}}{C} - 1 = a$ into the variance and taking the limit as $b \to \infty$, we obtain

$$\underline{V}(C) = \sigma_\eta^2 \frac{\rho_{\omega\eta}^2}{\frac{\sigma_\eta \sigma_\omega \rho_{\omega\eta}}{C}} = \frac{\sigma_\eta}{\sigma_\omega} \rho_{\omega\eta} C.$$

Likewise, for $b = \frac{\sigma_\eta \sigma_\omega \rho_{\omega\eta}}{C} - 1$ and taking the limit as $a \to \infty$ we obtain the highest value of the variance

$$\overline{V}(C) = \sigma_\eta^2 \frac{1}{\frac{\sigma_\eta \sigma_\omega \rho_{\omega\eta}}{C}} = \frac{\sigma_\eta}{\sigma_\omega} \frac{1}{\rho_{\omega\eta}} C.$$

It follows that a level of variance is feasible for a given level of $C < \sigma_\eta \sigma_\omega \rho_{\omega\eta}$ if

$$V \in \left[\frac{\sigma_\eta}{\sigma_\omega} \rho_{\omega\eta} C, \frac{\sigma_\eta}{\sigma_\omega} \frac{1}{\rho_{\omega\eta}} C \right].$$

Consider now the case where $C = \sigma_\eta \sigma_\omega \rho_{\omega\eta}$. Since

$$\frac{a+b+1-\rho_{\omega\eta}^2}{(1+a)(1+b)-\rho_{\omega\eta}^2} = 1$$

iff $ab = 0$, the level of covariance requires that either $a = 0$ or $b = 0$ or both. Hence, for $C = \sigma_\eta \sigma_\omega \rho_{\omega\eta}$, we have for $b = 0$ and $a \geq 0$

$$Var(\theta) = \sigma_\eta^2 \frac{a + b\rho_{\omega\eta}^2 + 1 - \rho_{\omega\eta}^2}{(1+a)(1+b)-\rho_{\omega\eta}^2} = \sigma_\eta^2$$

and for $a = 0$ and $b \geq 0$

$$Var(\theta) = \sigma_\eta^2 \frac{a + b\rho_{\omega\eta}^2 + 1 - \rho_{\omega\eta}^2}{(1+a)(1+b) - \rho_{\omega\eta}^2} = \sigma_\eta^2 \frac{b\rho_{\omega\eta}^2 + 1 - \rho_{\omega\eta}^2}{1 + b - \rho_{\omega\eta}^2}.$$

Since $\lim_{b\to\infty} \frac{b\rho_{\omega\eta}^2+1-\rho_{\omega\eta}^2}{1+b-\rho_{\omega\eta}^2} = \rho_{\omega\eta}^2$, the expression $\sigma_\eta^2 \frac{b\rho_{\omega\eta}^2+1-\rho_{\omega\eta}^2}{1+b-\rho_{\omega\eta}^2}$ ranges from $\rho_{\omega\eta}^2 \sigma_\eta^2$ to σ_η^2. ■

Proof of Theorem 1. The statement follows from solving $Cov(\omega, \theta) = Var(\theta)$ for the underlying noise structure. Recall that

$$Cov(\omega, \theta) = \sigma_\eta \sigma_\omega \rho_{\omega\eta} \frac{a + b + 1 - \rho_{\omega\eta}^2}{(1+a)(1+b) - \rho_{\omega\eta}^2}$$

and

$$Var(\theta) = \sigma_\eta^2 \frac{a + b\rho_{\omega\eta}^2 + 1 - \rho_{\omega\eta}^2}{(1+a)(1+b) - \rho_{\omega\eta}^2},$$

where $a \equiv \frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2}$ and $b \equiv \frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}$. Solving $Cov(\omega, \theta) = Var(\theta)$, we obtain

$$\frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta} \left(a + b + 1 - \rho_{\omega\eta}^2\right) = \left(a + b\rho_{\omega\eta}^2 + 1 - \rho_{\omega\eta}^2\right).$$

Rearranging, we have

$$\left(\frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta} - \rho_{\omega\eta}^2\right) b = a \left(1 - \frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta}\right) + \left(1 - \frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta}\right) \left(1 - \rho_{\omega\eta}^2\right).$$

For $\frac{\sigma_\omega}{\sigma_\eta} \in \left(\rho_{\omega\eta}, \frac{1}{\rho_{\omega\eta}}\right]$ we have $\frac{\sigma_\omega}{\sigma_\eta} - \rho_{\omega\eta} \neq 0$, hence we can divide through by $\frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta} - \rho_{\omega\eta}^2$ and obtain

$$b = a \frac{\left(1 - \frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta}\right)}{\rho_{\omega\eta} \left(\frac{\sigma_\omega}{\sigma_\eta} - \rho_{\omega\eta}\right)} + \frac{\left(1 - \frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta}\right)}{\rho_{\omega\eta} \left(\frac{\sigma_\omega}{\sigma_\eta} - \rho_{\omega\eta}\right)} \left(1 - \rho_{\omega\eta}^2\right).$$

Consider next the case where $\frac{\sigma_\omega}{\sigma_\eta} - \rho_{\omega\eta} = 0$. Note that

$$a \left(1 - \frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta}\right) + \left(1 - \frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta}\right) \left(1 - \rho_{\omega\eta}^2\right) > 0,$$

and that for finite b

$$\left(\frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta} - \rho_{\omega\eta}^2\right) b = 0.$$

Hence, $Cov(\omega, \theta) = Var(\theta)$ only if b is unbounded. Therefore, consider now the limiting case

$$\lim_{b\to\infty} \sigma_\eta^2 \frac{a + b\rho_{\omega\eta}^2 + 1 - \rho_{\omega\eta}^2}{(1+a)(1+b) - \rho_{\omega\eta}^2} = \sigma_\eta^2 \frac{\rho_{\omega\eta}^2}{1+a}.$$

For $a = 0$ and in the limit as $b \to \infty$

$$Var(\theta) = \rho_{\omega\eta}^2 \sigma_\eta^2.$$

Likewise, for $a = 0$ and in the limit as $b \to \infty$

$$Cov(\omega, \theta) = \sigma_\eta \sigma_\omega \rho_{\omega\eta}.$$

Clearly, $\rho_{\omega\eta}^2 \sigma_\eta^2 = \sigma_\eta \sigma_\omega \rho_{\omega\eta}$ for $\frac{\sigma_\omega}{\sigma_\eta} = \rho_{\omega\eta}$. ■

Proof of Theorem 2. We prove each part in sequence.

i) There is always an equilibrium inducing two distinct actions. Suppose there are just two messages, up or down, and the Receiver chooses $x = \mathbb{E}[\omega | \theta \geq \theta_0]$ if she hears up, and $x = \mathbb{E}[\omega | \theta < \theta_0]$ if she hears down. For this to be an equilibrium, type θ_0 needs to be indifferent between the two actions, that is

$$\theta_0 - \mathbb{E}[\omega | \theta < \theta_0] = \mathbb{E}[\omega | \theta \geq \theta_0] - \theta_0.$$

By symmetry of the distribution, this condition is always satisfied for $\theta_0 = 0$, since

$$-\mathbb{E}[\omega | \theta < 0] = \mathbb{E}[\omega | \theta \geq 0].$$

ii) We show that there is a class of equilibria within which there is no upper bound on the number of induced actions. Since we want to characterize equilibria with the largest number of induced actions, and there are such equilibria within the considered class, there is no loss of generality in focusing on equilibria of that class for our purposes. It proves convenient to relabel the indices on marginal types θ_i. Consider a symmetric equilibrium with thresholds $\theta_1, \ldots, \theta_n$ on the upper half and thresholds $\theta_{-1}, \ldots, \theta_{-n}$, with $\theta_{-i} = -\theta_i$ for all i. For n even we additionally have a threshold $\theta_0 = 0$. The indifference condition of types θ_i for $i = 2, \ldots, n-1$ are given by

$$\theta_i - \mathbb{E}[\omega | \theta \in (\theta_{i-1}, \theta_i]] = \mathbb{E}[\omega | \theta \in (\theta_i, \theta_{i+1}]] - \theta_i, \tag{10}$$

while the last threshold satisfies

$$\theta_n - \mathbb{E}\left[\omega|\, \theta \in (\theta_{n-1}, \theta_n]\right] = \mathbb{E}\left[\omega|\, \theta \geq \theta_n\right] - \theta_n. \tag{11}$$

We proceed as follows: in a first step we show by induction that $\frac{d\theta_{i+1}}{d\theta_i} \geq 1$. Secondly, we show that given a symmetric equilibrium with n threshold values, we can always construct a symmetric equilibrium with $n+1$ threshold values. Note that for existence it suffices to consider the case where n is odd, and that because of symmetry we only consider the threshold values above zero, the same reasoning holds for the mirror images, so that in total we construct an equilibrium inducing $2(n+1)$ actions.

Define

$$c \equiv \frac{Cov(\omega, \theta)}{Var(\theta)}$$

and

$$\mu(\theta_{i-1}, \theta_i) \equiv \mathbb{E}\left[\theta|\, \theta \in (\theta_{i-1}, \theta_i]\right].$$

Totally differentiating the indifference condition of type θ_1, i.e. $\theta_1 = \mathbb{E}\left[\omega|\, \theta \in (\theta_1, \theta_2]\right] - \theta_1$, we find that

$$\frac{d\theta_2}{d\theta_1} = \frac{2 - c\mu_{\theta_1}(\theta_1, \theta_2)}{c\mu_{\theta_2}(\theta_1, \theta_2)}.$$

By log-concavity of the distribution, we have $\mu_{\theta_1}(\theta_1, \theta_2) \leq 1$ and $\mu_{\theta_2}(\theta_1, \theta_2) \leq 1$, so for $c \leq 1$, we have $\frac{d\theta_2}{d\theta_1} > 0$. Moreover, we also have $\frac{d\theta_2}{d\theta_1} > 1$, since

$$\frac{2 - c\mu_{\theta_1}(\theta_1, \theta_2)}{c\mu_{\theta_2}(\theta_1, \theta_2)} > 1 \Leftrightarrow 2 > c\left(\mu_{\theta_1}(\theta_1, \theta_2) + \mu_{\theta_2}(\theta_1, \theta_2)\right)$$

and the latter inequality is true because the sum in brackets is at most unity for log-concave distributions.

Totally differentiating condition (10), we have

$$\frac{d\theta_{i+1}}{d\theta_i} = \frac{1 - \frac{c}{2}\left(\frac{d\theta_{i-1}}{d\theta_i}\mu_{\theta_{i-1}}(\theta_{i-1}, \theta_i) + \mu_{\theta_i}(\theta_{i-1}, \theta_i) + \mu_{\theta_i}(\theta_i, \theta_{i+1})\right)}{\frac{c}{2}\mu_{\theta_{i+1}}(\theta_i, \theta_{i+1})}.$$

We now show the induction hypothesis $\frac{d\theta_i}{d\theta_{i-1}} \geq 1$ implies that $\frac{d\theta_{i+1}}{d\theta_i} \geq 1$. In fact, the latter is equivalent to

$$1 \geq \frac{c}{2}\left(\frac{d\theta_{i-1}}{d\theta_i}\mu_{\theta_{i-1}}(\theta_{i-1}, \theta_i) + \mu_{\theta_i}(\theta_{i-1}, \theta_i) + \mu_{\theta_i}(\theta_i, \theta_{i+1}) + \mu_{\theta_{i+1}}(\theta_i, \theta_{i+1})\right).$$

By the induction hypothesis and the log-concavity of the distribution, (which implies that $\mu_{\theta_{i-1}}(\theta_{i-1},\theta_i)+\mu_{\theta_i}(\theta_{i-1},\theta_i)\leq 1$ and $\mu_{\theta_i}(\theta_i,\theta_{i+1})+\mu_{\theta_{i+1}}(\theta_i,\theta_{i+1})\leq 1$), it follows that the inequality is satisfied for $c\leq 1$.

We can construct an equilibrium with $n+1$ threshold values (above zero), if we can satisfy the following two conditions. Firstly, condition (10) must be satisfied for $i=2,\ldots,n$. Secondly, θ_{n+1} must satisfy condition (11), i.e.

$$\theta_{n+1}-\mathbb{E}\left[\omega\right|\theta\in(\theta_n,\theta_{n+1}]]=\mathbb{E}\left[\omega\right|\theta\geq\theta_{n+1}]-\theta_{n+1}. \tag{12}$$

Let θ_1^n denote the value of the first threshold θ_1 that is consistent with the equilibrium in n threshold values. (One can show that θ_1^n is unique.) Since the last threshold θ_n^n in this equilibrium satisfies condition (11), it is impossible that there exists a threshold $\theta_{n+1}<\infty$ satisfying condition (12). We must therefore reduce θ_1 below θ_1^n.

Let $\theta_i(\theta_1)$ denote solutions to condition (10) as a function of the initial condition θ_1. The critical one among conditions (10) for the equilibrium with $n+1$ threshold values is

$$\theta_n(\theta_1)-\mathbb{E}\left[\omega\right|\theta\in(\theta_{n-1}(\theta_1),\theta_n(\theta_1)]]=\mathbb{E}\left[\omega\right|\theta\in(\theta_n(\theta_1),\theta_{n+1}(\theta_1)]]-\theta_n(\theta_1). \tag{13}$$

The sequence $(\theta_1,\ldots,\theta_n(\theta_1))$ is increasing in θ_1. We can find a value $\theta_{n+1}(\theta_1)$ that solves (13) provided that $\theta_{n-1}(\theta_1),\theta_n(\theta_1)$ are low enough and provided that

$$\theta_n(\theta_1)-\mathbb{E}\left[\omega\right|\theta\in(\theta_{n-1}(\theta_1),\theta_n(\theta_1)]]>\lim_{\theta_{n+1}\to\theta_n(\theta_1)}\mathbb{E}\left[\omega\right|\theta\in(\theta_n(\theta_1),\theta_{n+1}]]-\theta_n(\theta_1) \tag{14}$$

and

$$\theta_n(\theta_1)-\mathbb{E}\left[\omega\right|\theta\in(\theta_{n-1}(\theta_1),\theta_n(\theta_1)]]<\lim_{\theta_{n+1}\to\infty}\mathbb{E}\left[\omega\right|\theta\in(\theta_n(\theta_1),\theta_{n+1}]]-\theta_n(\theta_1). \tag{15}$$

The former condition, (14), can be written as

$$(2-c)\,\theta_n(\theta_1)>c\mu\left(\theta_{n-1}(\theta_1),\theta_n(\theta_1)\right).$$

This condition is always satisfied for $c\leq 1$. To see this, note that

$$(2-c)\,\theta_n(\theta_1)\geq\theta_n(\theta_1)>\mu\left(\theta_{n-1}(\theta_1),\theta_n(\theta_1)\right)\geq c\mu\left(\theta_{n-1}(\theta_1),\theta_n(\theta_1)\right),$$

where the critical inequality follows from the fact that $\theta_n(\theta_1)>\theta_{n-1}(\theta_1)$.

Consider the latter condition, (15). The left-hand side is increasing in θ_1. To see this, note that

$$\begin{aligned}
& \frac{d\theta_n(\theta_1)}{d\theta_1} - c\left(\mu_{\theta_{n-1}}(\theta_{n-1}(\theta_1), \theta_n(\theta_1)) \frac{d\theta_{n-1}(\theta_1)}{d\theta_1} + \mu_{\theta_n}(\theta_{n-1}(\theta_1), \theta_n(\theta_1)) \frac{d\theta_n(\theta_1)}{d\theta_1}\right) \\
= & \frac{d\theta_n(\theta_1)}{d\theta_1}\left(1 - c\left(\mu_{\theta_{n-1}}(\theta_{n-1}(\theta_1), \theta_n(\theta_1)) \frac{d\theta_{n-1}(\theta_1)}{d\theta_n(\theta_1)} + \mu_{\theta_n}(\theta_{n-1}(\theta_1), \theta_n(\theta_1))\right)\right) \\
\geq & \ 0.
\end{aligned}$$

Thus, reducing θ_1 below θ_1^n reduces the value of the left-hand side. On the other hand, the right-hand side

$$\mathbb{E}[\omega|\, \theta \geq \theta_n(\theta_1)] - \theta_n(\theta_1)$$

is decreasing in $\theta_n(\theta_1)$ since $c \leq 1$ and $\mathbb{E}[\theta|\, \theta \geq \theta_n(\theta_1)] - \theta_n(\theta_1)$ is decreasing in $\theta_n(\theta_1)$ due to log-concavity. Thus, reducing θ_1 below θ_1^n increases the value of the right hand side of (15). It follows that for θ_1 small enough, we can satisfy (13).

It remains to be shown that θ_{n+1} satisfies condition (12), i.e. that we can chose θ_1 small enough. Define

$$D \equiv \theta_{n+1} - \mathbb{E}[\omega|\, \theta \in (\theta_n(\theta_1), \theta_{n+1}]] - (\mathbb{E}[\omega|\, \theta \geq \theta_{n+1}] - \theta_{n+1}),$$

and note that D is increasing in θ_{n+1} (due to log-concavity and $c \leq 1$). Hence, we can find θ_{n+1} that sets D equal to zero if on the one hand

$$\lim_{\theta_{n+1} \to \theta_n(\theta_1)} D < 0,$$

and on the other hand

$$\lim_{\theta_{n+1} \to \infty} D > 0.$$

The latter condition does not constrain us in any way. Now consider the former. Rearranging, we have

$$\begin{aligned}
\lim_{\theta_{n+1} \to \theta_n(\theta_1)} D &= (1-c)\,\theta_n(\theta_1) - (\mathbb{E}[\omega|\, \theta \geq \theta_n(\theta_1)] - \theta_n(\theta_1)) \\
&= (2-c)\,\theta_n(\theta_1) - \mathbb{E}[\omega|\, \theta \geq \theta_n(\theta_1)].
\end{aligned}$$

So for the former condition we need to have

$$\theta_n(\theta_1) < \frac{c}{2-c}\mu(\theta_n(\theta_1), \infty).$$

For θ_1 small enough, this inequality is satisfied, since $\frac{c}{2-c} \in (0,1]$, $\theta_n(\theta_1)$ goes to zero as θ_1 does, in which case the right-hand side remains bounded away from zero.

We conclude that we can set θ_1 low enough to find a new equilibrium value θ_1^{n+1} so that for $\theta_1 = \theta_1^{n+1}$, a new equilibrium inducing $2(n+1)$ actions is constructed. It follows that there is no upper bound on n for $c \leq 1$.

iii) For $c \geq 2$, the maximum number of induced actions is two. From part i), we know that there always exists an equilibrium with two distinct induced actions. Consider a candidate equilibrium inducing three actions, the middle one being equal to zero. The indifference condition for type θ_1 is

$$\frac{1}{2}\mathbb{E}[\omega|\,\theta > \theta_1] = \theta_1.$$

Using $\mathbb{E}[\omega|\,\theta > \theta_1] = c\mathbb{E}[\theta|\,\theta > \theta_1]$, we have

$$\frac{c}{2}\mathbb{E}[\theta|\,\theta > \theta_1] = \theta_1.$$

Since $\frac{c}{2} \geq 1$ and $\mathbb{E}[\theta|\,\theta > \theta_1] > \theta_1$ for any finite θ_1, no solution can exist. Similarly, the equilibrium condition for θ_1 in any equilibrium with an odd number of induced actions and $n \geq 5$ is

$$\theta_1 = \frac{c}{2}\mathbb{E}[\theta|\,\theta \in (\theta_1, \theta_2]],$$

so, by the same argument, there is no such solution.

The indifference condition for the first threshold in any equilibrium with an even number of induced actions and $n \geq 4$ for arbitrary θ_2 is given by

$$\theta_1 - \mathbb{E}[\omega|\,\theta \in (0, \theta_1]] = \mathbb{E}[\omega|\,\theta \in (\theta_1, \theta_2]] - \theta_1.$$

For $c \geq 2$ we immediately get the following contradiction

$$2\theta_1 = c\left(\mathbb{E}[\theta|\,\theta \in (0,\theta_1]] + \mathbb{E}[\theta|\,\theta \in (\theta_1, \theta_2]]\right) > c(0 + \theta_1).$$

Hence there does not exist any θ_1 and no equilibrium with n even and $n \geq 4$. ■

Lemma 8 *In a partitional equilibrium*

$$\begin{aligned} &\sum\nolimits_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \int u^R(x_i^* - \omega) f(\omega|\,\theta)\, d\omega f(\theta)\, d\theta \\ = &\frac{Cov(\theta,\omega)^2}{Var(\theta)^2}\left(Var(\theta) - \sum\nolimits_{i=1}^{n} p_i Var(\theta\,|\theta \in \Theta_i)\right) - Var(\omega) \end{aligned}$$

and

$$\begin{aligned} & \sum\nolimits_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \int u^R \left(x_i^* - \omega\right) f\left(\omega | \theta\right) d\omega f\left(\theta\right) d\theta \\ = & \frac{Cov\left(\theta, \omega\right)^2}{Var\left(\theta\right)^2} \left(Var\left(\theta\right) - \sum\nolimits_{i=1}^{n} p_i Var\left(\theta \left| \theta \in \Theta_i\right.\right)\right) - Var\left(\omega\right). \end{aligned}$$

Proof of Lemma 8. The Sender's payoff is

$$\sum\nolimits_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \int u^S \left(x_i^* - \eta\right) f\left(\eta | \theta\right) d\eta f\left(\theta\right) d\theta.$$

Expanding the square around $\mathbb{E}\left[\omega | \theta\right]$, we can rewrite this as

$$\begin{aligned} & -\sum\nolimits_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \int \left(x_i^* - \mathbb{E}\left[\omega | \theta\right] + \mathbb{E}\left[\omega | \theta\right] - \eta\right)^2 f\left(\eta | \theta\right) d\eta f\left(\theta\right) d\theta \\ = & -\sum\nolimits_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \int \left(\left(x_i^* - \mathbb{E}\left[\omega | \theta\right]\right)^2 + 2\left(x_i^* - \mathbb{E}\left[\omega | \theta\right]\right)\left(\mathbb{E}\left[\omega | \theta\right] - \eta\right) + \left(\mathbb{E}\left[\omega | \theta\right] - \eta\right)^2\right) f\left(\eta | \theta\right) d\eta f\left(\theta\right) d\theta \\ = & -\sum\nolimits_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \left(x_i^* - \mathbb{E}\left[\omega | \theta\right]\right)^2 f\left(\theta\right) d\theta - \sum\nolimits_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} 2\left(x_i^* - \mathbb{E}\left[\omega | \theta\right]\right)\left(\mathbb{E}\left[\omega | \theta\right] - \theta\right) f\left(\theta\right) d\theta \\ & -\sum\nolimits_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \int \left(\mathbb{E}\left[\omega | \theta\right] - \eta\right)^2 f\left(\eta | \theta\right) d\eta f\left(\theta\right) d\theta \end{aligned}$$

We now compute each term more explicitly, proceeding backwards. Term three can be written as

$$\begin{aligned} & -\sum\nolimits_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \int \left(\mathbb{E}\left[\omega | \theta\right] - \eta\right)^2 f\left(\eta | \theta\right) d\eta f\left(\theta\right) d\theta \\ = & -\sum\nolimits_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} \int \left(\mathbb{E}\left[\omega | \theta\right]^2 - 2\theta \mathbb{E}\left[\omega | \theta\right] + \eta^2\right) f\left(\eta | \theta\right) d\eta f\left(\theta\right) d\theta \\ = & -\left(\frac{Cov\left(\omega, \theta\right)^2}{Var\left(\theta\right)^2} Var\left(\theta\right) - 2\frac{Cov\left(\omega, \theta\right)}{Var\left(\theta\right)} Var\left(\theta\right) + Var\left(\eta\right)\right) \\ = & -\left(\frac{Cov\left(\omega, \theta\right)^2}{Var\left(\theta\right)} - 2Cov\left(\omega, \theta\right) + Var\left(\eta\right)\right). \end{aligned}$$

Term two can be rewritten as

$$
\begin{aligned}
& -\sum\nolimits_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i} 2\left(x_i^* - \mathbb{E}\left[\omega|\,\theta\right]\right)\left(\mathbb{E}\left[\omega|\,\theta\right]-\theta\right) f\left(\theta\right) d\theta \\
= & -\sum\nolimits_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i} 2\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}\left(\mathbb{E}\left[\theta|\,\theta\in\left[\theta_{i-1},\theta_i\right]\right]-\theta\right)\left(\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}-1\right)\theta f\left(\theta\right) d\theta \\
= & -2\left(\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}-1\right)\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}\sum\nolimits_{i=1}^{n} p_i\int_{\theta_{i-1}}^{\theta_i}\left(\mathbb{E}\left[\theta|\,\theta\in\left[\theta_{i-1},\theta_i\right]\right]\theta-\theta^2\right)\frac{f\left(\theta\right)}{p_i} d\theta \\
= & \ 2\left(\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}-1\right)\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}\sum\nolimits_{i=1}^{n} p_i Var\left(\theta|\,\theta\in\left[\theta_{i-1},\theta_i\right]\right) \\
& +2\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}\left(\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}-1\right)\sum\nolimits_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i}\theta^2 f\left(\theta\right) d\theta \\
= & \ 2\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}\left(\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}-1\right)\left(Var\left(\theta\right)-\sum\nolimits_{i=1}^{n}\Pr\left(\theta\in\left[\theta_{i-1},\theta_i\right]\right)\mathbb{E}\left[\theta|\,\theta\in\left[\theta_{i-1},\theta_i\right]\right]^2\right)
\end{aligned}
$$

Finally, term one can be written as

$$
\begin{aligned}
& -\sum\nolimits_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i}\left(x_i^* - \mathbb{E}\left[\omega|\,\theta\right]\right)^2 f\left(\theta\right) d\theta \\
= & -\sum\nolimits_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i}\frac{Cov\left(\omega,\theta\right)^2}{Var\left(\theta\right)^2}\left(\mathbb{E}\left[\theta|\,\theta\in\left[\theta_{i-1},\theta_i\right]\right]-\theta\right)^2 f\left(\theta\right) d\theta \\
= & -\frac{Cov\left(\omega,\theta\right)^2}{Var\left(\theta\right)^2}\sum\nolimits_{i=1}^{n} p_i Var\left(\theta|\,\theta\in\left[\theta_{i-1},\theta_i\right]\right).
\end{aligned}
$$

Summing up, we can write

$$
\begin{aligned}
& \sum\nolimits_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i}\int u^S\left(x_i^*-\eta\right) f\left(\eta|\,\theta\right) d\eta f\left(\theta\right) d\theta \\
= & \left(\frac{Cov\left(\omega,\theta\right)^2}{Var\left(\theta\right)^2}-2\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}\right)\sum\nolimits_{i=1}^{n} p_i Var\left(\theta|\,\theta\in\left[\theta_{i-1},\theta_i\right]\right) \\
& -\left(\frac{Cov\left(\omega,\theta\right)^2}{Var\left(\theta\right)}-2Cov\left(\omega,\theta\right)+Var\left(\eta\right)\right).
\end{aligned}
$$

Likewise, the Receiver's expected utility, $\sum_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i}\int u^R\left(x_i^*-\omega\right) f\left(\omega|\,\theta\right) d\omega f\left(\theta\right) d\theta$, takes the form

$$-\sum_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i}\int\left(x_i^*-\mathbb{E}\left[\omega\right|\theta]+\mathbb{E}\left[\omega\right|\theta]-\omega\right)^2 f\left(\omega\right|\theta)\,d\omega f\left(\theta\right)d\theta$$

$$=-\sum_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i}\int\left(\left(x_i^*-\mathbb{E}\left[\omega\right|\theta]\right)^2+2\left(x_i^*-\mathbb{E}\left[\omega\right|\theta]\right)\left(\mathbb{E}\left[\omega\right|\theta]-\omega\right)+\left(\mathbb{E}\left[\omega\right|\theta]-\omega\right)^2\right)f\left(\omega\right|\theta)\,d\omega f\left(\theta\right)d\theta$$

$$=-\sum_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i}\left(x_i^*-\mathbb{E}\left[\omega\right|\theta]\right)^2 f\left(\theta\right)d\theta-\left(\frac{Cov\left(\theta,\omega\right)^2}{Var\left(\theta\right)^2}Var\left(\theta\right)-2\frac{Cov\left(\theta,\omega\right)^2}{Var\left(\theta\right)}+Var\left(\omega\right)\right)$$

$$=-\sum_{i=1}^{n}\int_{\theta_{i-1}}^{\theta_i}\left(x_i^*-\mathbb{E}\left[\omega\right|\theta]\right)^2 f\left(\theta\right)d\theta+\frac{Cov\left(\theta,\omega\right)^2}{Var\left(\theta\right)^2}Var\left(\theta\right)-Var\left(\omega\right)$$

$$=-\frac{Cov\left(\omega,\theta\right)^2}{Var\left(\theta\right)^2}\sum_{i=1}^{n}p_iVar\left(\theta\left|\theta\in\Theta_i\right.\right)+\frac{Cov\left(\theta,\omega\right)^2}{Var\left(\theta\right)^2}Var\left(\theta\right)-Var\left(\omega\right).$$

■

Proof of Theorem 3. The proof is split into two parts. The first part establishes the Receiver's preferences over information structures 1 and 2 described in the main text; the second part adapts the analysis to the Sender's problem.

The Receiver prefers information structure 2, since

$$\begin{aligned}
& Var_2\left(\theta\right)-Var\left(\omega\right)\\
= & \frac{Cov_2\left(\theta,\omega\right)^2}{Var_2\left(\theta\right)^2}Var_2\left(\theta\right)-Var\left(\omega\right)=\frac{Cov_2\left(\theta,\omega\right)^2}{Var_1\left(\theta\right)^2}Var_1\left(\theta\right)-Var\left(\omega\right)\\
> & \frac{Cov_1\left(\theta,\omega\right)^2}{Var_1\left(\theta\right)^2}\left(Var_1\left(\theta\right)-\mathbb{I}_p\sum_{i=1}^{n}p_iVar_1\left(\theta\left|\theta\in\Theta_i\right.\right)\right)-Var\left(\omega\right),
\end{aligned}$$

where $Var_1\left(\theta\left|\theta\in\Theta_i\right.\right)$ is the conditional variance (conditional on the truncated set Θ_i) under information structure 1. The left-most expression is the Receiver's expected utility in the truthful and smooth equilibrium, which is feasible under structure 2. It follows that - provided we can find $\left(Cov_2\left(\omega,\theta\right),Var_2\left(\theta\right)\right)$ satisfying $Var_2\left(\theta\right)=Var_1\left(\theta\right)$ and $Cov_2\left(\omega,\theta\right)=Var_2\left(\theta\right)$ - that information structure $\left(Cov_1\left(\omega,\theta\right),Var_1\left(\theta\right)\right)$ is dominated. Note that under Assumptions 1 and 2, an information structure 2 with the said properties indeed exists.

Consider now the Sender's problem. The Sender's expected payoff is

$$\left(2\frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)}-\frac{Cov\left(\omega,\theta\right)^2}{Var\left(\theta\right)^2}\right)\left(Var\left(\theta\right)-\mathbb{I}_p\sum_{i=1}^{n}p_iVar\left(\theta\left|\theta\in\Theta_i\right.\right)\right)-Var\left(\eta\right).$$

Note that the term $2c - c^2$ is maximized at $c = 1$. Therefore,

$$\begin{aligned} & Var_2(\theta) - Var(\eta) \\ = & \left(2\frac{Cov_2(\omega,\theta)}{Var_2(\theta)} - \frac{Cov_2(\omega,\theta)^2}{Var_2(\theta)^2}\right) Var_2(\theta) - Var(\eta) \\ > & \left(2\frac{Cov_1(\omega,\theta)}{Var_1(\theta)} - \frac{Cov_1(\omega,\theta)^2}{Var_1(\theta)^2}\right) \left(Var_1(\theta) - \sum_{i=1}^{n} p_i Var(\theta \,|\theta \in \Theta_i)\right) - Var(\eta). \end{aligned}$$

The left-most expression is the equilibrium utility in the smooth and truthful equilibrium under information structure 2; note that the truthful equilibrium is feasible under this structure. The right-hand side of the inequality is the equilibrium expected utility under the best feasible equilibrium under information structure 1. ■

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