What Do Matching Models Predict?

Bernard Salanié
Columbia University

June 8, 2015

Rationalizing, Identifying, Testing

Rationalizing, Identifying, Testing

Observed: some data distributed as $X \simeq F$
Theory: $F_X \in \mathcal{F} = (F^\theta)$

Rationalizing, Identifying, Testing

Observed: some data distributed as $X \simeq F$
Theory: $F_X \in \mathcal{F} = (F^\theta)$

Rationalizing the data: is F in $\mathcal{F}$, if so for which (set of values) $\theta(F)$?

- Just/exact identification: $\theta(F)$ is a singleton

Rationalizing, Identifying, Testing

Observed: some data distributed as $X \simeq F$
Theory: $F_X \in \mathcal{F} = (F^\theta)$

Rationalizing the data: is F in $\mathcal{F}$, if so for which (set of values) $\theta(F)$?

- Just/exact identification: $\theta(F)$ is a singleton
- under/partial identification: it is a proper set

Rationalizing, Identifying, Testing

Observed: some data distributed as $X \simeq F$
Theory: $F_X \in \mathcal{F} = (F^\theta)$

Rationalizing the data: is F in $\mathcal{F}$, if so for which (set of values) $\theta(F)$?

- Just/exact identification: $\theta(F)$ is a singleton
- under/partial identification: it is a proper set
- overidentification: for some F it is empty.

Rationalizing, Identifying, Testing

Observed: some data distributed as $X \simeq F$
Theory: $F_X \in \mathcal{F} = (F^\theta)$

Rationalizing the data: is F in $\mathcal{F}$, if so for which (set of values) $\theta(F)$?

- Just/exact identification: $\theta(F)$ is a singleton
- under/partial identification: it is a proper set
- overidentification: for some F it is empty.

Rationalizing, Identifying, Testing

Observed: some data distributed as $X \simeq F$
Theory: $F_X \in \mathcal{F} = (F^\theta)$

Rationalizing the data: is F in $\mathcal{F}$, if so for which (set of values) $\theta(F)$?

- Just/exact identification: $\theta(F)$ is a singleton
- under/partial identification: it is a proper set
- overidentification: for some F it is empty.

Testing the theory: for some statistic $S(X)$, for some F,

$$F^S \notin (F_S^\theta).$$

(Basic) Matching Models

(Basic) Matching Models

Frictionless (including perfect information)

(Basic) Matching Models

Frictionless (including perfect information)
mostly 1-to-one and bipartite (the *marriage problem*)

(Basic) Matching Models

Frictionless (including perfect information)
mostly 1-to-one and bipartite (the *marriage problem*)
with or without transfers.

(Basic) Matching Models

Frictionless (including perfect information)
mostly 1-to-one and bipartite (the *marriage problem*)
with or without transfers.

Rationalizability: infer the primitives of the model from observed mathing patterns (and possibly transfers and/or outcomes)

(Basic) Matching Models

Frictionless (including perfect information)
mostly 1-to-one and bipartite (the *marriage problem*)
with or without transfers.

Rationalizability: infer the primitives of the model from observed mathing patterns (and possibly transfers and/or outcomes)

Testing: what features of the data could reject the model?

The NTU model

The NTU model

Strict preferences P (everything can be rationalized under indifference)
may be represented by $U_m(w)$ for men, $V_w(m)$ for women
0=single, utilities $U_m(0), V_w(0)$.

The NTU model

Strict preferences P (everything can be rationalized under indifference)
may be represented by $U_m(w)$ for men, $V_w(m)$ for women
0=single, utilities $U_m(0), V_w(0)$.

stable matching: no blocking coalitions of 1 or 2 members.
denote u_m the utility of man m, v_w for woman w

The NTU model

Strict preferences P (everything can be rationalized under indifference)
may be represented by $U_m(w)$ for men, $V_w(m)$ for women
0=single, utilities $U_m(0), V_w(0)$.

stable matching: no blocking coalitions of 1 or 2 members.
denote u_m the utility of man m, v_w for woman w
$u_m \geq U_m(0)$, $v_w \geq V_w(0)$
and if $U_m(w) > u_m$ then $V_w(m) < v_w$ and vice-versa.

NTU Coupling Equations

NTU Coupling Equations

$$u_m = \max_w \{U_m(w) | V_w(m) \geq v_w\}$$

$$v_w = \max_m \{V_w(m) | U_m(w) \geq u_m\}.$$

NTU Coupling Equations

$$u_m = \max_w \{ U_m(w) | V_w(m) \geq v_w \}$$
$$v_w = \max_m \{ V_w(m) | U_m(w) \geq u_m \} .$$

$\{ m | V_w(m) \geq v_w \}$ is the *acceptance set* of woman w.

Transferable Utility

Transferable Utility

Now $U_m(w)$ and $V_w(m)$ are pre-transfer utilities; transfers clear the market.

man m gets $U_m(w) - t_{mw}$, woman w gets $V_w(m) + t_{mw}$.

Transferable Utility

Now $U_m(w)$ and $V_w(m)$ are pre-transfer utilities; transfers clear the market.

man m gets $U_m(w) - t_{mw}$, woman w gets $V_w(m) + t_{mw}$.

stable matching: no blocking coalitions of 1 or 2 members *after transfers*

utilities after transfers u_m, v_w

Transferable Utility

Now $U_m(w)$ and $V_w(m)$ are pre-transfer utilities; transfers clear the market.
man m gets $U_m(w) - t_{mw}$, woman w gets $V_w(m) + t_{mw}$.
stable matching: no blocking coalitions of 1 or 2 members *after transfers*
utilities after transfers u_m, v_w
$u_m \geq U_m(0)$, $v_m \geq V_w(0)$
and:
$\tilde{\Phi}(m, w) \equiv U_m(w) + V_w(m) \leq u_m + v_w$.

TU Coupling Equations

TU Coupling Equations

$$u_m = \max_w \left(\tilde{\Phi}(m, w) - v_w \right)$$
$$v_w = \max_m \left(\tilde{\Phi}(m, w) - u_m \right).$$

TU Coupling Equations

$$u_m = \max_w \left(\tilde{\Phi}(m, w) - v_w \right)$$
$$v_w = \max_m \left(\tilde{\Phi}(m, w) - u_m \right).$$

All men belong to the acceptance set of woman w.

One Market

One Market

We only observe “who marries whom” on one marriage market;
we do not restrict unobserved heterogeneity (“universal domain.”)

One Market

We only observe “who marries whom” on one marriage market;
we do not restrict unobserved heterogeneity (“universal domain.”)

NTU: rationalizes anything *and usually much more*
if (m, w) are matched, then put w at the top of P_m and vice versa.

One Market

We only observe “who marries whom” on one marriage market; we do not restrict unobserved heterogeneity (“universal domain.”)

NTU: rationalizes anything *and usually much more*
if (m, w) are matched, then put w at the top of P_m and vice versa.

TU: rationalizes anything as *unique equilibrium* with $\tilde{\Phi}$ in normal cone of convex polytope at the observed matching.

Several Identical Markets

Several Identical Markets

Echenique *Math OR* 2006: if we observe different matchings they must all be stable.

Several Identical Markets

Echenique *Math OR* 2006: if we observe different matchings they must all be stable.

Take a collection H of matchings, when is there a P that rationalizes it, i.e. all matchings in H are stable for P?

Several Identical Markets

Echenique *Math OR* 2006: if we observe different matchings they must all be stable.
Take a collection H of matchings, when is there a P that rationalizes it, i.e. all matchings in H are stable for P?

(TU: theory almost-rejected since stable matching is generically unique.)

Not everything goes

Not everything goes

Suppose H=all feasible matchings is rationalizable by some P.

Not everything goes

Suppose H=all feasible matchings is rationalizable by some P.
Take m and w such that (with ≥ 3 agents on each side) neither the men-preferred stable matching nor the woman-preferred one match m and w.

Not everything goes

Suppose H=all feasible matchings is rationalizable by some P.
Take m and w such that (with ≥ 3 agents on each side)
neither the men-preferred stable matching nor the woman-preferred one match m and w.
Now take a matching that gives the women-preferred match to m and the men-preferred match to w.

Not everything goes

Suppose H=all feasible matchings is rationalizable by some P.
Take m and w such that (with ≥ 3 agents on each side)
neither the men-preferred stable matching nor the woman-preferred one match m and w.
Now take a matching that gives the women-preferred match to m and the men-preferred match to w.
That matching is blocked by m and w pairing.

Not everything goes

Suppose H=all feasible matchings is rationalizable by some P.
Take m and w such that (with ≥ 3 agents on each side) neither the men-preferred stable matching nor the woman-preferred one match m and w.
Now take a matching that gives the women-preferred match to m and the men-preferred match to w.
That matching is blocked by m and w pairing.

Therefore with ≥ 3 agents on each side not every set of feasible matchings is rationalizable.

Partial Results

Partial Results

if the matchings in H all assign different partners to each agent, then H is rationalizable.

Partial Results

if the matchings in H all assign different partners to each agent, then H is rationalizable.

Proof:

1. take any order $\mathcal{O}$ on the matchings in H.

Partial Results

if the matchings in H all assign different partners to each agent, then H is rationalizable.
Proof:

1. take any order $\mathcal{O}$ on the matchings in H.
2. Take any man m; order according to $\mathcal{O}$ the group of women he never matches with, and the group of those he is observed to match with in one matching.

Partial Results

if the matchings in H all assign different partners to each agent, then H is rationalizable.

Proof:

1. take any order $\mathcal{O}$ on the matchings in H.
2. Take any man m; order according to $\mathcal{O}$ the group of women he never matches with, and the group of those he is observed to match with in one matching.
3. For woman w, do the same but reverse order $\mathcal{O}$.

Partial Results

if the matchings in H all assign different partners to each agent, then H is rationalizable.

Proof:

1. take any order $\mathcal{O}$ on the matchings in H.
2. Take any man m; order according to $\mathcal{O}$ the group of women he never matches with, and the group of those he is observed to match with in one matching.
3. For woman w, do the same but reverse order $\mathcal{O}$.

Partial Results

if the matchings in H all assign different partners to each agent, then H is rationalizable.

Proof:

1. take any order $\mathcal{O}$ on the matchings in H.
2. Take any man m; order according to $\mathcal{O}$ the group of women he never matches with, and the group of those he is observed to match with in one matching.
3. For woman w, do the same but reverse order $\mathcal{O}$.

if H is rationalizable then it can be rationalized in **a lot** of ways.

Aggregate Matchings

Aggregate Matchings

Echenique-Lee-Shum Yenmez Eca 2013:

Aggregate Matchings

Echenique-Lee-Shum Yenmez Eca 2013:

M_x identical men of type $x = 1, \ldots, X$, with identical preferences
W_y identical women of type $y = 1, \ldots, Y$, with identical preferences

Aggregate Matchings

Echenique-Lee-Shum Yenmez Eca 2013:

M_x identical men of type $x = 1, \ldots, X$, with identical preferences
W_y identical women of type $y = 1, \ldots, Y$, with identical preferences

An *aggregate matching* is a feasible matrix $n(x, y)$ of numbers of matches per type.
E.g: $x, y = 1, 2, 3,$

$$\begin{pmatrix} 11 & 0 & 10 \\ 0 & 22 & 41 \\ 13 & 91 & 0 \end{pmatrix}$$

Cycles and Rationalizability

Cycles and Rationalizability

Link all non-zero elements on same row or column of matrix n:

Cycles and Rationalizability

Link all non-zero elements on same row or column of matrix n:

n is rationalizable in NTU iff graph has no connected distinct minimal cycles.

Cycles and Rationalizability

Link all non-zero elements on same row or column of matrix n:

n is rationalizable in NTU iff graph has no connected distinct minimal cycles.
n is rationalizable in TU iff graph has no minimal cycle.

Cycles and Rationalizability

Link all non-zero elements on same row or column of matrix n:

n is rationalizable in NTU iff graph has no connected distinct minimal cycles.
n is rationalizable in TU iff graph has no minimal cycle.

More interesting:
n is rationalizable in TU iff it is rationalizable in NTU as the men-preferred or the woman-preferred matching
(as with the Gale-Shapley deferred acceptance mechanism).

Intuition for NTU

Intuition for NTU

Strict preferences P_x imply that if two cells y and y' in row x are nonzero, x must prefer one of these women to the other, say y to y'

Intuition for NTU

Strict preferences P_x imply that if two cells y and y' in row x are nonzero, x must prefer one of these women to the other, say y to y'
if in column y cells x and x' are nonzero, then for stability women y must prefer x' to x

Intuition for NTU

Strict preferences P_x imply that if two cells y and y' in row x are nonzero, x must prefer one of these women to the other, say y to y'

if in column y cells x and x' are nonzero, then for stability women y must prefer x' to x

$\rightarrow$ cycles must be flows.

Intuition for NTU

Strict preferences P_x imply that if two cells y and y' in row x are nonzero, x must prefer one of these women to the other, say y to y'

if in column y cells x and x' are nonzero, then for stability women y must prefer x' to x

$\rightarrow$ cycles must be flows.

But connected cycles cannot be flows.

Intuition for TU

Intuition for TU

NTU extremal matchings: cannot be a cycle because we could improve by shifting one side in the direction of their flow.

Intuition for TU

NTU extremal matchings: cannot be a cycle because we could improve by shifting one side in the direction of their flow.
TU matchings cannit be a cycle for the same reason.

NTU

Aggarwal (forthcoming *AER*): the NMRP (again!)

NTU

Aggarwal (forthcoming *AER*): the NMRP (again!)

identical preferences on one side of the market: all hospitals agree on ranking of residents.

NTU

Aggarwal (forthcoming *AER*): the NMRP (again!)

identical preferences on one side of the market: all hospitals agree on ranking of residents.
then each hospital's acceptance set is a quality threshold.

TU

TU

m has observed type x_m, w has observed type y_w
there is also unobserved payoff-relevant heterogeneity.

TU

m has observed type x_m, w has observed type y_w
there is also unobserved payoff-relevant heterogeneity.
ANOVA decomposition:

$$\tilde{\Phi}(m, w) = \Phi(x_m, y_w) + \varepsilon_m(y_w) + \eta_w(x_m) + u_{mw}.$$

TU

m has observed type x_m, w has observed type y_w
there is also unobserved payoff-relevant heterogeneity.
ANOVA decomposition:

$$\tilde{\Phi}(m, w) = \Phi(x_m, y_w) + \varepsilon_m(y_w) + \eta_w(x_m) + u_{mw}.$$

u_{mw} interacts unobservables $\to$ restrict it to be 0: *separability*.
Choo and Siow 2006, Chiappori, Salani'e and Weiss 2015,
Galichon–Salanié 2015.

TU

m has observed type x_m, w has observed type y_w
there is also unobserved payoff-relevant heterogeneity.
ANOVA decomposition:

$$\tilde{\Phi}(m, w) = \Phi(x_m, y_w) + \varepsilon_m(y_w) + \eta_w(x_m) + u_{mw}.$$

u_{mw} interacts unobservables $\rightarrow$ restrict it to be 0: *separability*.
Choo and Siow 2006, Chiappori, Salani'e and Weiss 2015, Galichon–Salanié 2015.
Content: no complementarities across unobservables (conditional on observables.)

TU

m has observed type x_m, w has observed type y_w
there is also unobserved payoff-relevant heterogeneity.
ANOVA decomposition:

$$\tilde{\Phi}(m, w) = \Phi(x_m, y_w) + \varepsilon_m(y_w) + \eta_w(x_m) + u_{mw}.$$

u_{mw} interacts unobservables $\rightarrow$ restrict it to be 0: *separability*.
Choo and Siow 2006, Chiappori, Salani'e and Weiss 2015, Galichon–Salanié 2015.
Content: no complementarities across unobservables (conditional on observables.)
Does *not* exclude matching over unobservables; but restricts its form.

Identifying Joint Surplus

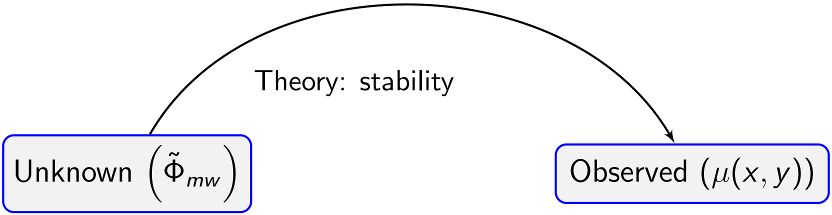
Unknown $\left(\tilde{\Phi}_{mw}\right)$

Identifying Joint Surplus

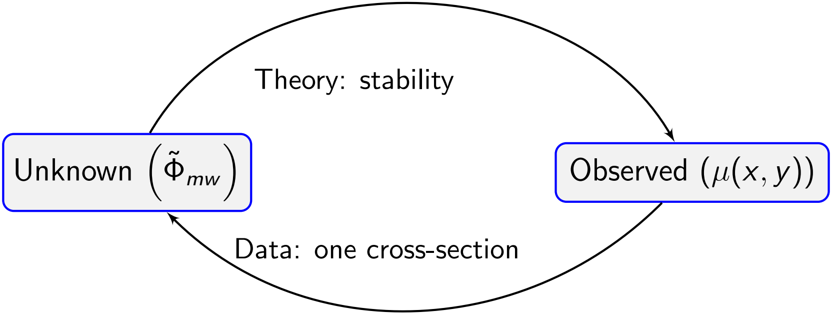
Unknown $\left(\tilde{\Phi}_{mw}\right)$

Observed $(\mu(x, y))$

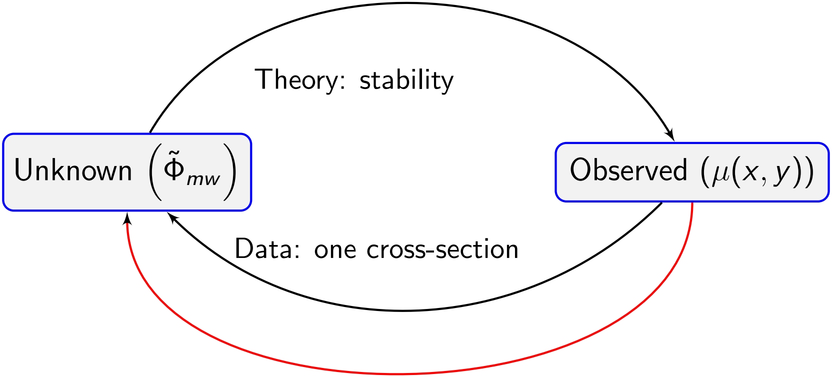
Identifying Joint Surplus



Identifying Joint Surplus



Identifying Joint Surplus



Restrictions: separability, distributional assumptions

Consequence

Consequence

Then (Chiappori-Salanié-Weiss 2012)

Theorem

Under (S), there exists $U(x,y)$ *and* $V(x,y)$ *such that* $U(x,y)+V(x,y)=\Phi(x,y)$ *and for any match* $(m \in x, w \in y)$

$$u_m = U(x,y) + \varepsilon_m(y) \text{ and } v_w = V(x,y) + \eta_w(x).$$

Two One-Sided Choice Models

Two One-Sided Choice Models

Moreover:

Two One-Sided Choice Models

Moreover:

Theorem

$$v(w) = \max_x (V(x, y) + \eta_w(x)).$$

Two One-Sided Choice Models

Moreover:

Theorem

$$v(w) = \max_x (V(x,y) + \eta_w(x)).$$

Proof:

$$\begin{aligned} v(w) &= \max_x \max_{m \in x} (\Phi(x,y) + \varepsilon_m(y) + \eta_w(x) - u(m)) \\ &= \max_x \left(\Phi(x,y) + \eta_w(x) - \min_{m \in x} (u(m) - \varepsilon_m(y)) \right) \\ &\equiv \max_x (\Phi(x,y) + \eta_w(x) - U(x,y)) \\ &\equiv \max_x (V(x,y) + \eta_w(x)). \end{aligned}$$

The general solution — Galichon-Salanié

The general solution — Galichon-Salanié

Assume that for each $w \in y$,
the vector $(\eta_w(x))_x$ has a multidimensional cdf $\mathbf{Q}_y$

The general solution — Galichon-Salanié

Assume that for each $w \in y$,
the vector $(\eta_w(x))_x$ has a multidimensional cdf $\mathbf{Q}_y$
and for each $m \in x$,
the vector $(\varepsilon_m(y))_y$ has a multidimensional cdf $\mathbf{P}_x$

The general solution — Galichon-Salanié

Assume that for each $w \in y$,
the vector $(\eta_w(x))_x$ has a multidimensional cdf $\mathbf{Q}_y$
and for each $m \in x$,
the vector $(\varepsilon_m(y))_y$ has a multidimensional cdf $\mathbf{P}_x$
Also assume "large markets": many $m \in x$, many $w \in y$.

Utilities

Utilities

Then **expected utilities** for a woman of type y, given all the $V(x, y)$, are

$$E\left(v(w) | w \in y\right) = E_{\mathbf{Q}_y} \max_x \left(V(x, y) + \eta_w(x)\right) \equiv H_y(V_{\cdot y})$$

Utilities

Then **expected utilities** for a woman of type y, given all the $V(x,y)$, are

$$E\left(v(w)|w \in y\right) = E_{\mathbf{Q}_y} \max_x \left(V(x,y) + \eta_w(x)\right) \equiv H_y(V_{\cdot y})$$

And the solution has total surplus:

$$\mathcal{W} = \sum_x p_x G_x(U_{x\cdot}) + \sum_y q_y H_y(V_{\cdot y})$$

for the equilibrium U and V.

Utilities

Then **expected utilities** for a woman of type y, given all the $V(x,y)$, are

$$E\left(v(w)|w \in y\right) = E_{\mathbf{Q}_y} \max_x \left(V(x,y) + \eta_w(x)\right) \equiv H_y(V_{\cdot y})$$

And the solution has total surplus:

$$\mathcal{W} = \sum_x p_x G_x(U_{x\cdot}) + \sum_y q_y H_y(V_{\cdot y})$$

for the equilibrium U and V.
Still many unknown quantities... the $U(x,y)$'s and $V(x,y)$'s.

Inverting E max for Given y

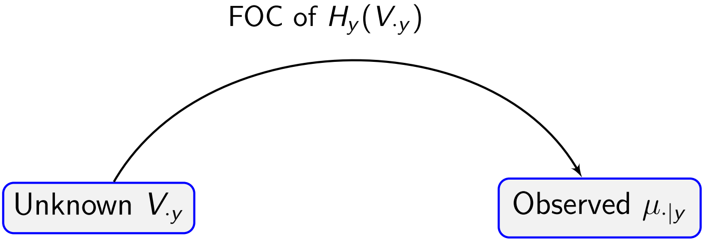
Unknown $V_{\cdot y}$

Inverting E max for Given y

Unknown $V_{\cdot y}$

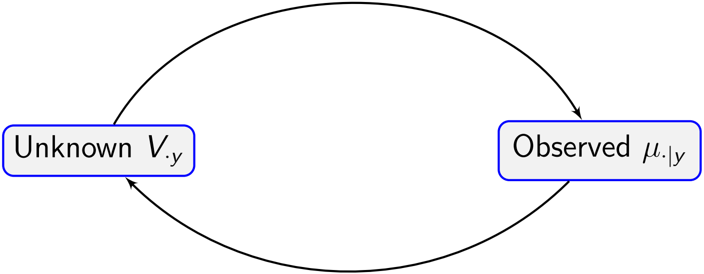
Observed $\mu_{\cdot|y}$

Inverting E max for Given y



Inverting $E\max$ for Given y

FOC of $H_y(V_{\cdot y})$



FOC of $H_y^*(\mu_{\cdot|y}) = \max_V (\mu \cdot V - H_y(V_{\cdot y}))$

Some Convex Analysis

Some Convex Analysis

The expected utility $H_y(V_{\cdot y})$ is convex; and the implied marriage patterns are

$$\mu(x, y) = \frac{\partial H_y}{\partial V(x, y)}.$$

Some Convex Analysis

The expected utility $H_y(V_{\cdot y})$ is convex; and the implied marriage patterns are

$$\mu(x,y) = \frac{\partial H_y}{\partial V(x,y)}.$$

We only need to invert!

Some Convex Analysis

The expected utility $H_y(V_{\cdot y})$ is convex; and the implied marriage patterns are

$$\mu(x, y) = \frac{\partial H_y}{\partial V(x, y)}.$$

We only need to invert!
f a convex function on a convex set $C \to \mathbb{R}$: it is continuous, almost everywhere differentiable

Some Convex Analysis

The expected utility $H_y(V_{\cdot y})$ is convex; and the implied marriage patterns are

$$\mu(x,y) = \frac{\partial H_y}{\partial V(x,y)}.$$

We only need to invert!
f a convex function on a convex set $C \to \mathbb{R}$: it is continuous, almost everywhere differentiable
define the Legendre-Fenchel (convex dual) transform:

$$f^*(y) = \max_{x \in C} (xy - f(x))$$

Convex Duality

Convex Duality

The LF transform f^* is finite on some convex set $C^* \subset \mathbb{R}$; and it is convex too
(as a convex (max) function of linear functions of y)

Convex Duality

The LF transform f^* is finite on some convex set $C^* \subset \mathbb{R}$; and it is convex too
(as a convex (max) function of linear functions of y)
Duality: where f and f^* are differentiable,

$$f'(x) = y \text{ iff } (f^*)'(y) = x.$$

Convex Duality

The LF transform f^* is finite on some convex set $C^* \subset \mathbb{R}$; and it is convex too
(as a convex (max) function of linear functions of y)
Duality: where f and f^* are differentiable,

$$f'(x) = y \text{ iff } (f^*)'(y) = x.$$

=a "convex inversion formula".

Applying Convexity

Applying Convexity

The Legendre-Fenchel transform of H_y is

$$H_y^*(\mu_{\cdot|y}) = \max_{V_{\cdot y}} \left(\sum_x \mu_{x|y} V(x,y) - H_y(V_{\cdot y}) \right).$$

Applying Convexity

The Legendre-Fenchel transform of H_y is

$$H_y^*(\mu_{\cdot|y}) = \max_{V_{\cdot y}} \left(\sum_x \mu_{x|y} V(x,y) - H_y(V_{\cdot y}) \right).$$

It is another convex function, and

$$V(x,y) = \frac{\partial H_y^*}{\partial \mu_{x|y}}$$

Applying Convexity

The Legendre-Fenchel transform of H_y is

$$H_y^*(\mu_{\cdot|y}) = \max_{V_{\cdot y}} \left(\sum_x \mu_{x|y} V(x,y) - H_y(V_{\cdot y}) \right).$$

It is another convex function, and

$$V(x,y) = \frac{\partial H_y^*}{\partial \mu_{x|y}}$$

can be estimated from the data.

Consequences

Consequences

Consequence 1:

$$\Phi(x,y) = U(x,y) + V(x,y) = \frac{\partial G_x^*}{\partial \mu_{y|x}} + \frac{\partial H_y^*}{\partial \mu_{x|y}}$$

just identifies the marital surplus if we know $\mathbf{P}_x$, $\mathbf{Q}_y$, and $\mu(x,y)$.

Consequences

Consequence 1:

$$\Phi(x,y) = U(x,y) + V(x,y) = \frac{\partial G_x^*}{\partial \mu_{y|x}} + \frac{\partial H_y^*}{\partial \mu_{x|y}}$$

just identifies the marital surplus if we know $\mathbf{P}_x$, $\mathbf{Q}_y$, and $\mu(x,y)$.

Consequence 2:
Expected utilities etc are easily computed

Consequences

Consequence 1:

$$\Phi(x,y) = U(x,y) + V(x,y) = \frac{\partial G_x^*}{\partial \mu_{y|x}} + \frac{\partial H_y^*}{\partial \mu_{x|y}}$$

just identifies the marital surplus if we know $\mathbf{P}_x$, $\mathbf{Q}_y$, and $\mu(x,y)$.

Consequence 2:
Expected utilities etc are easily computed
we can even recover the full distribution of $v_w | w \in y$.

Identification. . . sort of

Identification. . . sort of

We only have *conditional just identification*:

Identification. . . sort of

We only have *conditional just identification*:
for each possible choice of error distributions $\mathbf{P}_x$ and $\mathbf{Q}_y$,

Identification. . . sort of

We only have *conditional just identification*:
for each possible choice of error distributions $\mathbf{P}_x$ and $\mathbf{Q}_y$,
for every sequence of numbers $(\mu(x, y))$,

Identification. . . sort of

We only have *conditional just identification*:
for each possible choice of error distributions $\mathbf{P}_x$ and $\mathbf{Q}_y$,
for every sequence of numbers $(\mu(x, y))$,
there is **one** joint surplus Φ that rationalizes the matching patterns μ

Identification. . . sort of

We only have *conditional just identification*:
for each possible choice of error distributions $\mathbf{P}_x$ and $\mathbf{Q}_y$,
for every sequence of numbers $(\mu(x, y))$,
there is **one** joint surplus Φ that rationalizes the matching patterns μ
the model is **not** testable; we have too many degrees of freedom with the error terms.

Using Comparative statics

Using Comparative statics

$=$the only way we can test various specifications of errors, **given more cross-sections.**

Using Comparative statics

=the only way we can test various specifications of errors, **given more cross-sections.**

All separable models share some comparative statics,
so that we can test separability

Using Comparative statics

=the only way we can test various specifications of errors, **given more cross-sections.**
All separable models share some comparative statics,
so that we can test separability
But each of them has additional specific implications
so that we can test between them.

Observed Transfers

Observed Transfers

Salanié 2015: we could observe

Observed Transfers

Salanié 2015: we could observe

- **Case 1:** only the mean value of the transfer t_{mw} in each "cell" (x, y)—that is,

$$t_{xy} = E\left(t_{mw} | x, y\right)$$

Observed Transfers

Salanié 2015: we could observe

- **Case 1:** only the mean value of the transfer t_{mw} in each "cell" (x, y)—that is,

$$t_{xy} = E\left(t_{mw} | x, y\right)$$

- **Case 2:** for each realized match, the value of the transfer t_{mw}.

Observed Transfers

Salanié 2015: we could observe

- **Case 1:** only the mean value of the transfer t_{mw} in each "cell" (x, y)—that is,

$$t_{xy} = E\left(t_{mw} | x, y\right)$$

- **Case 2:** for each realized match, the value of the transfer t_{mw}.

Observed Transfers

Salanié 2015: we could observe

- **Case 1:** only the mean value of the transfer t_{mw} in each "cell" (x, y)—that is,

$$t_{xy} = E\left(t_{mw} | x, y\right)$$

- **Case 2:** for each realized match, the value of the transfer t_{mw}.

We need to restrict heterogeneity on pre-transfer utilities; we assume separability again,

$$\begin{aligned} U_m(w) &= a(x, y) + \varepsilon^a_m(y) + \eta^a_w(x) \\ V_w(m) &= b(x, y) + \varepsilon^b_m(y) + \eta^b_w(x). \end{aligned}$$

Transfers are Separable

Transfers are Separable

$$t_{mw} = T(x, y) + \eta^a_w(x) - \varepsilon^b_m(y);$$

Transfers are Separable

$$t_{mw} = T(x, y) + \eta^a_w(x) - \varepsilon^b_m(y);$$

and

$$T(x, y) = a(x, y) - \frac{\partial G^*_x}{\partial \mu_{y|x}} = \frac{\partial H^*_y}{\partial \mu_{x|y}} - b(x, y).$$

Identification

Identification

Observing only the mean value (**case 1**)

$$t(x,y) = T(x,y) + E\left(\eta_w^a(x) - \varepsilon_m^b(y) | (x,y)\right)$$

hardly helps at all.

Identification

Observing only the mean value (**case 1**)

$$t(x,y) = T(x,y) + E\left(\eta^a_w(x) - \varepsilon^b_m(y) | (x,y)\right)$$

hardly helps at all.

Case 2: if we observe all t_{mw} then we can test separability by

$$t_{mw} + t_{m'w'} = t_{mw'} + t_{m'w}.$$

Identification

Observing only the mean value (**case 1**)

$$t(x,y) = T(x,y) + E\left(\eta_w^a(x) - \varepsilon_m^b(y) | (x,y)\right)$$

hardly helps at all.
Case 2: if we observe all t_{mw} then we can test separability by

$$t_{mw} + t_{m'w'} = t_{mw'} + t_{m'w}.$$

and we can identify the distributions of ε^b and η^a if $\varepsilon^a \equiv 0$ and $\eta^b \equiv 0$,
that is if

$$U_m(w) = a(x,y) + \eta_w^a(x)$$
$$V_w(m) = b(x,y) + \varepsilon_m^b(y).$$

Conclusion: testing for complementarities

Conclusion: testing for complementarities

Becker 1973: given supermodular surplus

$$\Phi(x \vee z, y \vee t) + \Phi(x \wedge z, y \wedge t) \geq \Phi(x, t) + \Phi(z, y)$$

we have Positive Assortative Matching.

Conclusion: testing for complementarities

Becker 1973: given supermodular surplus

$$\Phi(x \vee z, y \vee t) + \Phi(x \wedge z, y \wedge t) \geq \Phi(x, t) + \Phi(z, y)$$

we have Positive Assortative Matching.
Reverse question: when can we infer complementarities in surplus?

Conclusion: testing for complementarities

Becker 1973: given supermodular surplus

$$\Phi(x \vee z, y \vee t) + \Phi(x \wedge z, y \wedge t) \geq \Phi(x, t) + \Phi(z, y)$$

we have Positive Assortative Matching.
Reverse question: when can we infer complementarities in surplus?
Hard if we do not know the distributions of unobserved heterogeneity;
may be possible with observed transfers in Case 2.